*Proceedings of the 2005 Winter Simulation Conference*
*M. E. Kuhl, N. M. Steiger, F. B. Armstrong, and J. A. Joines, eds.*

# SIMULATION ANALYSIS OF CORRELATION AND CREDIT MIGRATION MODELS FOR CREDIT PORTFOLIOS

William J. Morokoff

New Product Research
Moody's KMV
99 Church St.
New York, NY 10007, U.S.A.

## ABSTRACT

The market for derivatives such as first-to-default baskets and CDO tranches on portfolios of corporate credit exposures (bonds, loans, default swaps, etc.) has grown rapidly in recent years. Various models for capturing portfolio correlation effects have been introduced, with Default Time models becoming the most widely used. While attractive for their relative simplicity and ability, in some cases, to allow fast computation of hedge ratios, there is increasing concern around the limitations and implications of these models. This paper uses simulation to study the effects of credit migration and correlation assumptions underlying the models for valuation of derivatives on credit portfolios.

## 1 INTRODUCTION

The credit derivatives markets have grown rapidly over the last decade. These financial instruments derive their value from basic credit contracts such as bond and loans, and their valuation often depends on the probability of the underlying credit issuer defaulting. Credit indices, basket default swaps and collateralized debt obligations (CDOs) are credit derivatives that depend on portfolios of credit instruments; as such, their valuation depends on the correlated nature of credit, and in particular the joint probabilities of default.

Measuring probabilities of corporate default has become a well developed art. Rating agencies such as Moody's Investors Service provide in depth qualitative analysis of a firm's management and financial viability in the process of assigning a rating. Quantitative credit research providers like Moody's KMV focus on structural and econometric models based on equity market data and historical default data to determine probabilities of default. Models based on credit market price can also be used to imply out default probabilities.

In this paper we will use the notation $CEDF(t)$ to refer to the cumulative Expected Default Frequency™ credit measure, which is the Moody's KMV terminology for the cumulative default probability of a firm over the period to time *t*. A discussion of the methodology used to determine the EDF credit measure can be found in Kealhofer (2003).

Determination of credit correlation has also been extensively studied (see Zeng and Zhang (2001) for one discussion). The term correlation itself is somewhat vague as it is frequently used to refer to dependence among a number of different quantities, including equity returns, bond price returns, and yield or spread changes. In this paper we will focus on asset return correlation, a concept derived from the original work of Merton (1974) on structural models. In this framework, a firm defaults when its total franchise value, or asset value, falls below some measure of its liabilities. A firm's asset value cannot be directly observed in the market, but it can be implied out from market cap, volatility and liability information. Asset return correlation measures the degree to which two firms' asset values move together.

For this paper, we will assume that a firm's cumulative default probability term structure is known, as are the pair-wise asset return correlations. In addition, we will assume a Gaussian copula model, defined below, to specify the default dependence across firms.

In addition to default probability and asset correlation, there is an additional factor that determines joint default behavior: credit migration. This effect has been discussed in Finger (2000). The question is how does a firm's credit quality (and therefore default probability) evolve over time. This may be modeled explicitly by specifying a transition process for a credit state variable, or implicitly by specifying a model that embeds assumptions about the credit migration process. In a structural model framework, geometric Brownian motion is often assumed for the evolution of the firm asset value, consistent with the Gaussian copula description of the asset return correlations. Assuming a constant liability structure for the firm (as specified by its initial default probability term structure) leads to the implicit credit migration models underlying the Default Time (single time step approximation) and Gaussian (multi-step) models described below. In contrast, to account for the evolving nature of firm liabilities, it is necessary to impose an explicit credit state migration process. In the following, we use the Distance to Default credit measure (DD), which is the number of standard deviations a firm's asset value is above the firm's default point, as the evolution variable, and use transition densities calibrated to historical data to determine the evolution process.

This paper uses simulation of the Default Time, Gaussian and DD Dynamics models in the context of valuation of a basket default swap to compare the effects of the credit migration process under the same assumptions on default probability and correlation. A discussion of similar issues in the context of a CDO collateral portfolio can be found in Morokoff (2003).

## 2 MODELING DEFAULT TIMES

This section describes the methodology most commonly employed today for simulating correlated defaults. It is known as the Default Time or Copula approach and is described by Li (2000) and Schmidt & Ward (2002).

For many credit derivatives the value depends not only on the probability of a default but also on the timing of the default over a given horizon. Default timing is determined from a default probability term structure which may be represented as a vector of cumulative default probabilities

$$\left(CEDF_1, CEDF_2, \cdots, CEDF_N\right)$$

specified at times

$$\left(T_1, T_2, \cdots, T_N\right).$$

The quantity $CEDF_i$ is interpreted to mean the probability of default in the interval $\left(0, T_i\right)$. Thus the $CEDF$ are increasing. This may be generalized to a time continuous default probability function $CEDF(t)$; however, default probabilities are usually report at discrete times , and a continuous function is obtained from interpolation.

One method of randomly sampling default times is known as the Default Time or Copula method. The idea is to randomly sample a uniform (0,1) variate $u$. Assuming that $T_N$ is the maturity, if $u > CEDF_N$ then the exposure does not default. If $CEDF_{i-1} < u \le CEDF_i$ then the exposure defaults in period $i$. This procedure is closely related to sampling a stopping time for a random process crossing a default boundary.

A key feature of this approach is the process for determining correlated default times. This requires sampling a set of correlated uniform variates $\left(u_1, \cdots, u_M\right)$, where $M$ is the number of exposures in the portfolio. This is done by specifying a copula function $C\left(u_1 \cdots, u_M\right)$, which is a probability distribution function defined on the $M$-dimensional unit cube. The copula function is often related to the asset return distribution function at time $T_N$, $F\left(R_1, \cdots, R_M\right)$, by the formula

$$C\left(u_1, \cdots, u_M\right) = F\left(F_1^{-1}\left(u_1\right), \cdots, F_M^{-1}\left(u_M\right)\right)$$

where $F_j^{-1}(\cdot)$ is the inverse of the marginal probability distribution for the $j^{th}$ exposure. However, any copula function may be used for this purpose. The most commonly used are Gaussian and T-copulas, although a variety of other methods, including Archimedean copulas, have been considered.

For the Gaussian copula, the sampling procedure is particularly simple. Based on the correlation matrix for the asset returns, a correlated sample of standard Normal variates $\left(\varepsilon_1, \cdots, \varepsilon_M\right)$ is sampled, either from a Cholesky decomposition of the correlation matrix or from a factor model decomposition. The uniform variates are then obtained from the formula

$$u_j = \Phi^{-1}\left(\varepsilon_j\right).$$

Here $\Phi$ is the one dimensional standard cumulative Normal distribution function.

If the factor modeling underlying the correlation structure has more than a few dimensions, it is necessary to use Monte Carlo simulation to sample correlated defaults and default times that are then used to evaluate expectation integrals such as the probability of having more than $k$ defaults or the expected value of the cash flows that are conditional on default losses. Under more restrictive assumptions on the correlation structure, semi-analytical solutions can be derived. For example, the latent variable approach, proposed by Vasicek (1987) for credit portfolio risk problems, has been extended to CDOs by Gregory and Laurent (2003). The idea is that there exists a low dimensional underlying latent variable $x$ such that conditional on $x$ the default probabilities and times for the exposures are independent. The law of conditional expectations then allows the portfolio properties of interest to be expressed as an expectation over $x$ of the portfolio properties of an independent portfolio. Often $x$ is taken to be one dimensional, so the problem reduces to a one dimensional quadrature.

## 3 MULTI-STEP SIMULATION

An alternative to the Default Time approach based on simulating the firm asset value as a stochastic random variable has been described by Hull and White (2001), Arvanitis and Gregory (2001) and Finger (2000). In this section we describe an implementation of this approach and describe a multi-step approach based on the empirically derived Distance to Default distributions.

While the default time approach captures the marginal default probabilities of each individual exposure correctly over the life of the simulation, substantial error may be introduced into the correlated default structure, depending on how the correlation structure and the underlying stochastic default process are viewed. Time series of asset, equity or debt price returns are usually based on daily or weekly time intervals. Given the relatively high default probability of most assets over time horizons of five years or longer, using a correlation structure based on weekly returns as a proxy for multi-year horizon correlations can lead to skewed results. In particular, the single step approach may not adequately capture the absorbing nature of the default state (i.e., the stochastic process has an absorbing boundary). Thus it is better to consider a simulation based on a sequence of shorter time steps that one single step to maturity.

It is possible to model the credit migration of a single asset as a continuous time stochastic process, such as geometric Brownian motion or an Ohrnstein-Uhlenbeck process, with an absorbing boundary implied by the cumulative default probability function $CEDF(t)$. In this formulation a free boundary problem PDE can be derived as described by Avellaneda and Zhu (2001). However, since data are not available to realistically determine $CEDF(t)$ as a time continuous function, the continuous approach does not add accuracy relative to a discrete approach as long as the correlated behavior of asset over the time step is consistent with the correlation modeling. In any case, unless a low-dimensional latent variable approach is applied, computation of the properties of a portfolio of many exposures will require a Monte Carlo simulation based on discrete time steps.

For analyzing a credit derivative, it is most convenient to use simulation time steps based on the payment dates associated with the contract. For one simulation step, the names defaulting during that period are identified, recoveries on defaulted names are determined, and interest and principal cash flows are assessed. If desired, the exact default time of an exposure can be sampled using the default time methodology described above within one simulation period. The key question for the simulation is thus whether the default occurs in a given period.

There are numerous approaches that can lead to multi-step simulations for correlated defaults depending on how the default process is modeled. We focus here on two methods related to structural models for which correlated default behavior is derived from the underlying firm asset value correlations. Both methods take as input the cumulative default function $CEDT_j(t)$ specified at discrete times $(T_1, \cdots, T_N)$ for each obligor in the collateral portfolio, indexed by $j$. In addition, the firm asset value correlation matrix for all obligors must be specified.

The first approach assumes that the asset value process for each obligor follows correlated geometric Brownian motion. The associated asset value (log) return process therefore follows a standard Brownian motion process. An obligor $j$ defaults during a period $(T_{i-1}, T_i]$ if the asset return $R_j^i$ at time $T_i$ is less than some threshold level $\alpha_j^i$, while $R_j^k > \alpha_j^k$ for all $k < i$ (i.e., there was no previous default). In a continuous time formulation, the function $\alpha_j(t)$ is the default boundary such that the default time is the stopping time of the Brownian motion process associated with crossing the boundary. Obviously the default thresholds must be related to the default probability. Specifically the relationship is

$$1 - P(R_j^1 > \alpha_j^1, \cdots, R_j^i > \alpha_j^i) = CEDF_j(T_i).$$

As this equation suggests, the determination of the default thresholds requires a non trivial calculation as it relates to inverting an $i-$variate cumulative Normal distribution (in the continuous case, the default boundary is the solution to a free boundary PDE). One approach that gets around the need to invert a multi-dimensional distribution is to determine the distribution of $R_j^{i-1}$, conditional on no defaults up to time $T_{i-1}$. Assuming we know this distribution and using the fact that

$$R_j^i = R_j^{i-1} + \varphi_j^i$$

where $\varphi_j^i$ is an increment independent of $R_j^{i-1}$ (since the return process is Brownian motion) with a Normal distribution, we can obtain by convolution the distribution of $R_j^i$, conditional on no defaults up to $T_{i-1}$, from the conditional distribution for $R_j^{i-1}$ and $\varphi_j^i$. We can then solve for the default threshold $\alpha_j^i$ from the equation

$$P\left(R_j^i \le \alpha_j^i \mid \text{ no defaults up to } T_{i-1}\right)\left(1 - CEDF\left(T_{i-1}\right)\right) = CEDF\left(T_i\right) - CEDF\left(T_{i-1}\right).$$

Once $\alpha_j^i$ has been determined, the distribution of $R_j^i$ conditional on no defaults up to $T_i$ can be determined by truncating the distribution of $R_j^i$ conditional on no defaults up to time $T_{i-1}$. By repeated application of this procedure, the entire set of default thresholds can be determined. The main computational cost is associated with the convolution. This can be handled easily with the fast Fourier transform algorithm, which is effective since the conditional distribution is always convolved with a Normal distribution. Monte Carlo simulation can also be used to determine the default thresholds by determining the levels that will give the lead to the correct percentage of simulation paths to default in each period.

Once the default thresholds are determined, the simulation proceeds by sampling correlated Brownian motion paths for the asset returns at the specified times. Default occurs for a given obligor during the first period for which its return falls below the associated threshold. For names that don't default, conditional default probabilities at each time step can be used as input in valuation algorithms to provide consistent, correlated mark-to-model pricing for the collateral. We will refer to approach as the Gaussian multi-step method (Gaussian method for short).

As mentioned above, the assumption of geometric Brownian motion for the asset value process often does not adequately capture how a firm's credit quality changes over time because it does not take into account the associated changes in liability structure. It is known that as firms do well (e.g. as the asset value of the firm increases), they tend to take on more debt, thereby keeping their credit quality more stable over time. For example, a Baa rated firm will tend to maintain that rating by borrowing more when opportunities arise. It would be unusual for such a firm to grow without adding leverage to become a Aaa rated. However, this tends to be the consequence of the geometric Brownian motion model: over longer time horizons, firms that do not default undergo systematic improvement in their credit quality.

To capture the effects of changes to both asset value and liability structure on credit quality in long horizon multi-step simulations, at MKMV we have developed a multi-step simulation based on the Distance to Default transition densities. We now consider the implementation of this second, empirically-based method.

A key point to consider when working with historically observed data is the need to bucket the data in order to build a suitable sample size. For example, the first step in determining the probability of transitioning from a $DD$ value of 3 over a one year horizon to a $DD$ value of 4 is to identify all names in the historical sample that have at some time point a $DD$ value of 3. However, since $DD$ is determined as a continuous variable, it is unlikely that any of the sample will have a $DD$ value of exactly 3. Thus it is necessary to repose the question as to the probability of transition from a bucket, or interval, containing the $DD$ value 3 to a $DD$ value less than 4. The distribution of arrival $DD$'s after one year does not necessarily have to be bucketed – a parametric distribution for the cumulative transition probability distribution can be selected and the actual data used to estimate the distribution's parameters. However, for use in a multi-step simulation, it is convenient to work with the transition probabilities from one bucket to another bucket in the form of a transition matrix. The multi-step simulation is then carried out as a discrete Markov chain by repeated application of the transition matrix to an initial state vector. The size of the transition matrix, which is determined by the size of the $DD$ buckets, is chosen to balance the desire for high resolution in $DD$ space with the need to minimize the statistical errors arising from small sample sizes. Ultimately this is a question of the size of the original data set. The MKMV simulation is based on 10 years of monthly data on over 9000 firms.

There are a number of important observations to be made about the $DD$ transition matrix. First, the default state, conveniently labeled as $DD = 0$, is an absorbing state. The total probability of transitioning to this default state over a given time period is the forward EDF. This EDF is different for each firm; however, the transition matrix was determined by pooling data on many firms. Thus the transition matrix must be viewed as firm aggregate behavior. In order to capture the firm-specific behavior dictated by the input EDF term structure for each firm, it is necessary to make a firm-specific calibration of the transition matrix. The calibration consists of satisfying the constraint that over a given time period, the probability of transitioning from a non-default state to the default state must be the unconditional (or more precisely, conditional only on data specified at $T_0$) forward default probability:

$$FWD\ EDF(T_{i-1},T_i) = \frac{CEDF(T_i) - CEDF\left(T_{i-1}\right)}{1 - CEDF\left(T_{i-1}\right)}.$$

There are numerous ways this constraint could be enforced. One simple approach is to rescale all the original, firm aggregate transition probabilities to default by a single factor such that their sum, weighted by the unconditional probabilities of being in each non-default state at time $T_{i-1}$, matches the forward EDF. Once the transition probabilities are adjusted by this scaling, the unconditional probabilities for each state at time $T_i$ can be determined, thereby allowing the calibration for the next time step. This is equivalent to the convolution and truncation steps employed for the geometric Brownian motion model.

A second consideration for the transition matrix is whether the underlying data supports the model of a Markov process. Not surprisingly, the firm-aggregate transition matrices for time horizons of 6 months, 1 year, 2 years, 5 years, etc., derived from the data do not fit perfectly in a Markov framework. In other words, the one year matrix is not exactly the convolution of the 6 month matrix with itself; nor is the five year transition matrix exactly the five-fold convolution of the one year transition matrix. The agreement of these transition matrices is however sufficient, particularly given the complexity of the underlying factors which drive credit migration of firms as well as the firm-aggregate nature of the transitions themselves, to warrant the approximation by a single, Markov transition matrix, which is determined by optimally fitting, in a least-squares sense, one matrix (and its convolutions) to the empirical transition matrices. This avoids the exceptionally difficult task of specifying and calibrating a non-Markov process for the credit migration.

Once the transition matrix is specified for each obligor at each time step, the simulation proceeds by sampling from $F_i\left(DD \mid DD_{i-1}\right)$, the probability distribution of $DD$ states at time $T_i$ determined from the appropriate probability distribution (as given by the transition matrix) conditional on the $DD$ state at time $T_{i-1}$. By interpolation from the cumulative probabilities for the discrete transition matrix $DD$ states, $F_i\left(DD \mid DD_{i-1}\right)$ can be assumed to be a continuous, non-decreasing function with inverse $F_i^{-1}(u)$ defined on the unit interval $[0,1]$. For values of $u$ in the interval $[0, P(DD_{i-1} \to 0)]$ (i.e., between 0 and the conditional probability of defaulting), it follows that $F_i^{-1}(u)=0$. We introduce correlations among obligors by assuming multi-variate Brownian motion for the asset return process and sampling the correlated asset return increments according to the specified asset return correlation matrix. The cumulative Normal distribution function is then used to map the sampled asset return increments to the unit interval; this value is then used as the argument for $F_i^{-1}(u)$. More precisely, the $DD$ sample for obligor $j$ at time $i$ is given by

$$DD = F_i^{-1}\left(\Phi\left(\varepsilon_j\right)\right)$$

where the $\varepsilon_j$ are the normalized, correlated Normal samples of asset returns.

For a low enough asset return sample , the default state of $DD=0$ is sampled. In this case, a random recovery may be drawn from an appropriate distribution of recovery rates. If the obligor does not default, the sampled $DD$ state at $T_i$ can be used to determine a conditional EDF term structure looking forward that can be used to discount future cash flows according to their credit risk in order to obtain a price for the exposure at time $T_i$.

## 4 DEFAULT TIME VS. GAUSSIAN MIGRATION

In this section we compare the continuous time Gaussian migration model to the closely related single step Default Time model in terms of the implied Joint Default Frequency (*JDF*) for a pair of identical credits over a given 5 year horizon $T$. The purpose is to compute using Monte Carlo simulation the relative difference of the JDF under the two models over a range of default probabilities and asset correlations.

The problem may be posed as a question of stopping times for two identical, correlated Brownian motion processes $x_1(t),\ x_2(t)$, which represent the cumulative asset return on the two firms. The default boundary for the assets $f(t)$ is assumed known. We define the following quantities for these assets:

$$E\left(x_1(t)\cdot x_2(t)\right) = \rho\ t$$
$$\tau_i = \min\left(t \mid x_i(t) \le f(t)\right)$$
$$CEDF_i(T) = P\left(\tau_i \le T\right)$$
$$\alpha_i(T):\quad P\left(x_i(T) \le \alpha_i(T)\right) = P\left(\tau_i \le T\right)$$

The specified default boundary determines the cumulative default probability to $T$. This in turn determines the default threshold $\alpha(T)$ used in the Default Time model to decide if a default has occurred. The parameter $\rho$ is the asset return correlation for the Gaussian process as well as the Default Time correlation.

The Joint Default Frequency for each model can be defined as

$$JDF_{\text{Gaussian}} = P(\tau_1 \le T \text{ and } \tau_2 \le T)$$
$$JDF_{\text{Default Time}} = P\begin{pmatrix} x_1(T) \le \alpha_1(T) \\ \text{and } x_2(T) \le \alpha_2(T) \end{pmatrix}.$$

For the Default Time model, the *JDF* is given by the bivariate cumulative Normal function. There is also an analytic representation of the Gaussian *JDF* in terms of an infinite series of functions. The proposition we would like to establish is that

$$JDF_{\text{Gaussian}}(T) \le JDF_{\text{Default Time}}(T).$$

It is easy to show that for two identical process, this relationship holds as an identity for correlation 0 and correlation 100%. As far as we are aware, there is not a direct proof of this relationship in the general case. For the purposes of this paper, therefore, the *JDF* for the Gaussian model will be computed by simulation using weekly time steps out to a 5 year horizon. More precisely, we would like to study the function

$$\frac{JDF_{\text{Default Time}}}{JDF_{\text{Gaussian}}}\left(CEDF,\rho,T\right).$$

To simplify the parameterization, we will assume that there is a constant annualized default rate $r$. In this case, the cumulative default probability is given by

$$CEDF(t)=1-(1-r)^{t}.$$

Figure 1 shows a plot of the ratio of the *JDF*s for the two models at different *CEDF(T)* levels over a correlation range from 0 to 100%. Allowing for simulation error, we observe a general symmetric, parabolic shape of the function, matching as expected at a ratio of 1 at the extreme correlations of 0 and 100%, with the peak difference between the models occurring around a correlation of 50%. The plot shows that the relative difference of the models increases substantially as the default probability decreases. It should be noted, however, that at low default probabilities, the joint default frequencies are very low, so that the absolute difference in level between the models gets smaller as the default probability decreases.

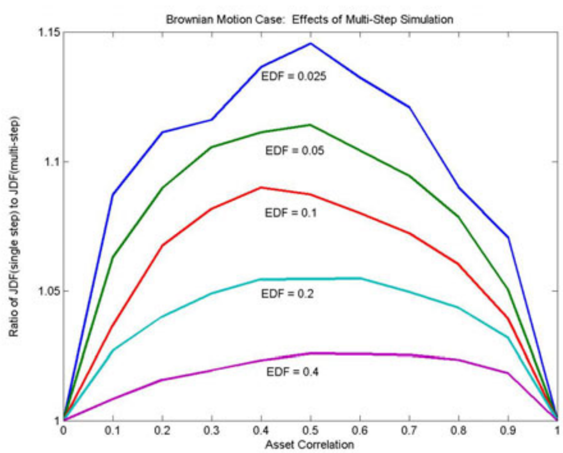


Figure 1: Ratio of Joint Default Frequencies for Default Time and Gaussian Credit Migration Models

For pricing purposes, risk-neutral default probabilities are used. Mid 2005 levels for a typical investment grade credit indicate a 5 year cumulative risk-neutral default probability of around 0.05. The plot indicates that for a typical range of correlations observed, there is around 10% relative difference in the pair-wise joint default frequencies implied by the Default Time and Gaussian models.

## 5 BASKET DEFAULT SWAP EXAMPLE

We now consider the effects of credit migration models on pricing a five name *kth*-to-default basket swap. In this contract the buyer agrees to pay a percentage $x$ of a notional amount (taken here to be 1) in the event that at least $k$ names in the basket default; in exchange the seller agrees to pay the buyer periodic coupons (called spread) up until the point of the *kth* default or until the maturity of the contract. In most contracts the amount $x$ is uncertain and depends on the recovery associated with the default events. For pricing purposes, it is generally considered a constant (the expected value of a random loss taken to be independent of the random default events); for this study, we will set $x$ to be 1 (full notional must be paid) as all prices scale linearly with $x$. The price of the basket default swap is quoted as the fair spread that must be paid to the buyer to balance the present value of the coupon side with the present value of the loss side.

We consider here the spread required on a first-to-default basket and a second-to-default basket as a function of correlation, under the Default Time, Gaussian and DD Dynamics credit migration models. The required spread is a function of the probability of the first (second) default occurring during a specific period. This probability is computed by simulating the joint default behavior of the basket of names under the different migration models.

The example here considers quarterly periods over a five year horizon for which the buyer of the basket receives $s/4$ at the end of each period ($s$ being the annualized spread) in which there have been fewer than $k$ defaults ($k$ = 1 or 2). If the *kth* default occurs during a period, it is assumed to occur at the mid-point of the period; thus the buyer receives $s/8$ but must pay 1. The discounted cash flows are computed by assuming a flat interest rate term structure of 3% annualized with continuous compounding.

For simplicity the basket consists of five identical credits. For pricing purposes we consider a risk-neutral cumulative default probability term structure of [.003 .009 .019 .034 .049] corresponding to years 1 through 5. This corresponds to an annualized risk-neutral default probability term structure of [.003 .0045 .0065 .0085 .0010], which is typical of an average investment grade credit in mid 2005. The quarterly cumulative default probabilities are obtained by linearly interpolating on $log(1-CEDF(t))$.

Figure 2 shows the required spread for the first to default basket for the three models as a function of asset correlation ranging from 0 to 50%. As expected, under the assumption of zero correlation, all models produce the same spread. It is also intuitive that as correlation increases the required spread decreases for all models. This is because the probability of having zero defaults (i.e., the probability that the buyer does not have to pay anything) increases with correlation – thus the buyer requires less spread to balance the potential loss. The probability of zero defaults is minimized when the names are independent. The Default Time model shows the greatest sensitivity to increasing correlation consistent with the over-emphasis of correlation shown in Figure 1. The Gaussian model exhibits similar behavior, but indicates less sensitivity at the higher correlations. The corresponds to the presence of the absorbing default boundary in the Gaussian model (not present in the Default Time model) that leads to lower effective default correlation. The most dramatic difference, however, is shown by the DD Dynamics model. Here the credit migration model implies significantly less sensitivity to increasing correlation and correspondingly less effective default correlation. The empirically determined transition densities effectively put the breaks on names as the approach default in a way such that while the default probabilities remain the same, the default correlations derived from the multi-step simulation are reduced compared with the other models. Consider another way, if the market price for this basket is 100 basis points, the Default Time model would indicate a correlation of 32%, the Gaussian model would indicate 34% correlation, while the DD Dynamics model implies around 42% correlation.

The second to default basket swap is considered in Figure 3. Again at zero correlation, all the models give the same spread. However, the required spread for the second to default basket now increases as correlation increases, indicating that the probability of having at least two defaults increases as correlation increases. As with the first to default basket, the Default Time model shows

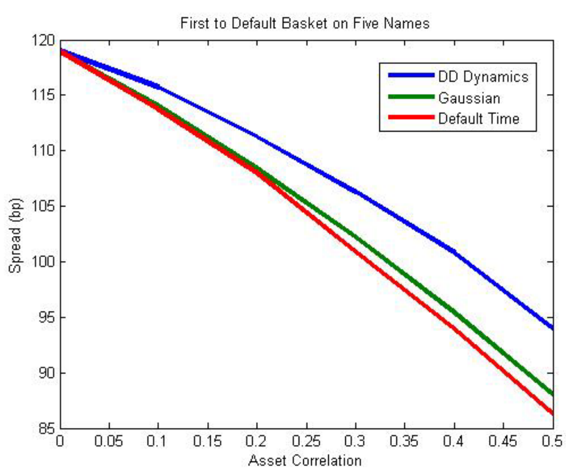


Figure 2: First to Default Basket Required Spread

the greatest sensitivity to increasing correlation, while the DD Dynamics model shows significantly less sensitivity. If the market spread is 20 basis points for this basket, the implied correlation is 26% for the Default Time model, 29% for the Gaussian model and 39% for the DD Dynamics model.

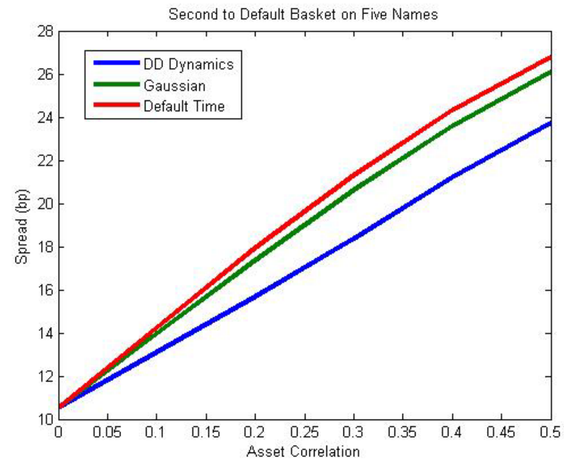


Figure 3: Second to Default Basket Required Spread

## ACKNOWLEDGMENTS

The work in this paper was built upon a great deal of research carried out by a number of people in the research group at Moody's KMV. In particular, special thanks are due to Yim Lee, Saiyid Islam, Michele Freed, David Cao and Frank Li.

## REFERENCES

Arvanitis, A and J Gregory. 2001. *Credit: The complete guide to pricing, hedging and risk management*. Risk Publications.

Avellaneda, M and J Zhu. 2001. Distance to default. *Risk* (December).

Finger, C. 2000. A comparison of stochastic default rate models. *RiskMetrics Journal* 1 (November), 49-73.

Gregory, J and Laurent, J. 2003. I will survive. *Risk*. (June), 103-107.

Hull, J. and A. White. 2001. Valuing credit default swaps II: modeling default correlations. *Journal of Derivatives* 8 (3), 12-22.

Kealhofer, S. 2003. Quantifying credit risk I: Default prediction. *Financial Analyst Journal* (January/February), 30-44.

Li, D. 2002. On default correlation: A copula function approach. *Journal of Fixed Income* 9 (4), 43-54.

Merton, R. 1974. On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance* 29 (2).

Morokoff, W.J. 2003. Simulation methods for risk analysis of collateralized debt obligations [online]. Available via <http://www.wintersim.org/prog03.htm#RA> [accessed August 9, 2005].

Schmidt, W. and I. Ward. 2002. Pricing default baskets. *Risk* (January), 111-114.

Vasicek, 0. 1987. Probability of loss on loan portfolio [online]. Available via <http://www.moodyskmv.com/research/portfoliotheory.html> [accessed August 9, 2005].

Zeng, B., and J. Zhang. 2001. An empirical assessment of asset correlation models. Available online via <http://www.moodyskmv.com/research/portfoliotheory.html> [accessed August 9, 2005].

## AUTHOR BIOGRAPHY

**WILLIAM MOROKOFF** is a senior director at Moody's KMV in the research department where he heads the New Product Research Group. His research focuses on Monte Carlo simulation applied to risk management and derivatives pricing in finance. He holds a Ph.D. from the Courant Institute at New York University. He can be contacted at <morokoff@mkmv.com>.