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C. Laroque, J. Himmelspach, R. Pasupathy, O. Rose, and A.M. Uhrmacher, eds

AN MVA APPROXIMATION FOR CONWIP PRIORITY MODELING

Guy L. Curry

Texas A&M University
Industrial and Systems Engineering
College Station, Texas 77843-3131, USA

Moonsu Lee

Korea University of Technology and Education
School of Industrial Management
Cheonan City, 337-708, KOREA

ABSTRACT

Constant work-in-process control (CONWIP) by product type is a strategy for improving the cycle time in multiple product factories. For realistic sized systems, a mean-value analysis (MVA) approximation methodology yields quick and accurate results. A processing step modeling paradigm is developed for the MVA methodology and applied to multiple-product reentrant-flow sequences. A variety of sequencing rules have been proposed in an attempt to improve the mean cycle times while maintaining the product throughput rates. A general priority scheme is developed for the MVA modeling approach which allows many of the sequencing rules to be implemented and evaluated under multiple product CONWIP control. Four priority schemes (FIFO, shortest expected processing time, shortest remaining processing time, and Wein's work-balance) are illustrated for a data set from the literature. The best priority scheme, work-balance, obtained a 41% mean processing time improvement over FIFO under push control and 37% under CONWIP control.

1 INTRODUCTION

Pull production strategies are an integral part of the concept in lean manufacturing (Nicholas 2011 and Black and Phillips 2012) of reducing work-in-process (WIP) and cycle time in manufacturing settings. The now famous Toyota production control approach is based on the concept of controlling WIP at each workstation. Modeling these systems have been studied extensively (Curry and Feldman 2011, Chapter 9) and is a difficult task compounded by the issue of proper limit settings at each workstation. Thus, in order to most effectively use the Toyota production control system, a complex multi-dimensional optimization problem must first be solved. A similar but much simpler approach is the use of a constant WIP for the factory or sub-factory grouping called CONWIP; which was studied by Spearman, Woodruff, and Hopp (1990) and described more fully in the Factory Physics book of Hopp and Spearman (1996). For the multiple product systems which we consider in this paper, a constant WIP limit per product is imposed on the production system instead of a single WIP limit for all products combined. For these systems, models of exact and approximation methods for large CONWIP settings have been studied by Reiser and Lavenberg (1980), (see Curry and Feldman 2011, Chapter 8 for an introduction to this approach) and more recently Park et al. (2000, 2002) extended MVA to allow for batch processing. Although batch processing and alternative routings are common phenomena in semiconductor wafer fabs, these issues are not addressed in this paper. The purpose of this paper is develop an analytical modeling approach for a class of priority sequencing algorithms within CONWIP control and to illustrate that these can have a significant impact on the production cycle times for multiple-product re-entrant-flow systems which are also standard characteristics of semiconductor fabs.

It has been demonstrated for re-entrant flow systems such as production systems that use a processing step paradigm that certain priority sequencing rules can significantly reduce the product and overall factory cycle time while maintaining the same throughput rates (Wein 1992). Wein's work was related to push production systems and used simulation as the verification tool for his proposed Work-Balance sequencing scheme. In a simulation case study, Curry and Feldman (2011, Chapter 8) illustrated that these priority schemes, particularly Work-Balance, also can make a significant impact under CONWIP control.

In this paper the CONWIP modeling paradigm is extended to incorporate general sequencing schemes applicable to each workstation within the factory. To accomplish this, a processing step approach is developed and then extended to incorporate general non-preemptive priority rules for sequencing jobs at each workstation based on their particular mean processing characteristics by processing step at that workstation. This family of sequencing algorithms includes first-in first-out (FIFO), shortest expected processing time at the current workstation (SEPT), shortest remaining expected processing time at all future workstations (SRPT) and Wein's Work-Balance (WBAL) algorithm. Significant differences between the mean cycle time performances of these various sequencing algorithms are illustrated via the general analytical sequencing model under CONWIP control and the analytical model is partially validated by comparing it to simulation model results. A variety of preemptive-resume priority scheme approximations and solution procedures have been proposed and compared in Bryant et al. (1984). Related literature on MVA preemptive-resume priority approximation schemes from computer performance modeling are Kaufman (1984), Bondi and Chuang (1988) and Leemans and Dedene (1996). In this paper, only non-preemptive priority schemes are considered. It should be noted that multi-class systems, general service times, batch processing, and differing priorities by customer class assumptions yield queueing networks for which product-form solutions do not apply (Leemans and Dedene 1996, Buitenhek et al. 2000, Park et al. 2002) and approximation methods must be utilized.

2 PROCESSING STEP PARADIGM FOR CONWIP MODELING

The processing step paradigm for production systems definition is a realistic approach since most product production schemes are actually given in production steps with machines and processing requirements by step. For discussion, the following definitions and notation are utilized:

np is the number of products produced in the factory,
$ns(p)$ is the number of processing steps for product p,
$E[S^{p,s}]$ is the mean processing time for product p at step s,
$C^2[S^{p,s}]$ is the squared coefficient of variation of the processing time for product p at step s,
W^p is the number of jobs of product p allowed in the factory at one time (CONWIP limit),
$ws(p,s)$ is the workstation number at which processing occurs for product p on step s,
$\lambda^{p,s}$ is the mean flow rate of product p through processing step s,
$r^{p,s}$ is the relative visitation rates of product p to processing step s,
$u^{p,s}$ is the utilization factor of product p in processing step s on the machine at $ws(p,s)$,
$WIP^{p,s}$ is the WIP of product p at processing step s (thus, $\sum_s WIP^{p,s} = W^p$),
$CT^{p,s}$ is the cycle time (processing time plus queue time) of product p at processing step s, and
$\mathbf{1}(x = y)$ is an indicator function that has value one when the comparison in the parentheses is true and zero otherwise.

2.1 Modeling Approach

The basic idea is to obtain the cycle times $CT^{p,s}$ for product p through processing step s (performed at workstation $k = ws(p,s)$). To accomplish this, the approach is to compute the WIP by product at workstation k and to then estimate the cycle time by the number of jobs of each product and step ahead of this job at that workstation and multiple these numbers by their associated mean processing times. For closed queueing networks, the standard approach for estimating the WIP of a product at a workstation, or in this case in a processing step which then can be mapped to the associated workstation, is to distribute the fixed number of jobs of a product type across all of the steps in proportion to the amount of time spent in that step. Since a job type will not necessarily visit each step in the processing sequence in equal proportions due to rework and/or the possibility of probabilistic branching, then the relative number of visits to each step needs to be computed. Assuming that the cycle time at each step is known, $CT^{p,s}$, and the relative number of visits to each processing step, $r^{p,s}$, is also known, then the expected WIP, $WIP^{p,s}$, at that step by product class with total jobs in the system of W^p is given by

$$WIP^{p,s} = \frac{W^p r^{p,s} CT^{p,s}}{\sum_{j=1}^{ns(p)} r^{p,j} CT^{p,j}}. \tag{1}$$

For open queueing networks, the external inflows and the topology of the network determine the flow rates at each processing step. Then the relative number of visits to each processing step is computed by the step flows divided by the total external inflow for that product. However, for closed networks, there are no external flows in this sense since throughput is defined as the fraction of flow through a given workstation and then this flow is essentially transferred back to the initial processing step to initiate a new job (thus, keeping the system WIP constant). This process results in a mathematically dependent system of flow equations. To overcome this problem, one of the processing steps (herein always processing step one) is designed as the base processing step and a relative number of visits is determined for all other processing steps relative to this base step. Then the topology of the product's flow network is used to compute the remaining steps relative to this base step (normally set at a relative rate value of 1.0). Thus, the $r^{p,s}$ values of equation (1) are easily determined by product and processing step.

Given the relative work-in-process by job type and processing step at a workstation, then the queueing or waiting time for all full jobs ahead of a specific job type by priority class can be computed. This handles the queue time for this designated job except for the remaining processing time for the job on the processor at the arrival time. Under exponential processing time assumptions this would just be the mean processing time for that job type, but for general service times, the expected remaining time estimate (Gross and Harris, 1998) is

$$\frac{E\left[\left(S^{p,l}\right)^2\right]}{2E[S^{p,l}]} = E[S^{p,l}]^2 \left\{\frac{C^2[S^{p,l}]+1}{2E[S^{p,l}]}\right\} = E[S^{p,l}]\left\{\frac{C^2[S^{p,l}]+1}{2}\right\}.$$

These times are weighted by the relative utilization of the server associated with each product and processing step using this workstation. Note the number of servers at the workstation is assumed to be one. This single server assumption can be generalized but makes the analysis and notation much more complicated.

Given the estimated queueing time for the designated product and step, the cycle time is merely the queue time plus the mean processing time for this product at this step. The interesting aspect of this modeling approach is that for CONWIP control these WIP values are functions of the product/step processing times and are obtained from equation (1). Note that the sum of the WIP values over all processing steps must equal the CONWIP parameter for that product, W^p, thus

$$W^p = \sum_{s=1}^{ns(p)} WIP^{p,s}.$$

Utilizing Little's Law (1961, 2011), $WIP = \lambda \times CT$, then using equation (1)

$$WIP^{p,s} = \lambda^{p,s} \times CT^{p,s} = \frac{W^p r^{p,s} CT^{p,s}}{\sum_{j=1}^{ns(p)} r^{p,j} CT^{p,j}} \tag{2}$$

and, thus, the flow rate of jobs of product *p* at processing step *s* is

$$\lambda^{p,s} = \frac{W^p r^{p,s}}{\sum_{j=1}^{ns(p)} r^{p,j} CT^{p,j}}. \tag{3}$$

A critical aspect of modeling CONWIP systems is that the number of jobs in the system of type *p* is fixed; hence the designated job of product type *p* at step *s* can only see $W^p - 1$ other jobs of that type; that is, it can not wait behind itself. Therefore, the designated job will be behind

$$WIP^{p,s} = \frac{\left(W^p - 1\right) r^{p,s} CT^{p,s}}{\sum_{j=1}^{ns(p)} r^{p,j} CT^{p,j}} \tag{4}$$

jobs of its own type *p* and

$$WIP^{i,s} = \frac{W^i r^{i,s} CT^{i,s}}{\sum_{j=1}^{ns(i)} r^{i,j} CT^{i,j}} \tag{5}$$

jobs of product type *i* for all $i \neq p$ (when step *s* of that product is processed at the common workstation *k*) . This leads to the cycle time estimate for product *p* at step *s* (note that processing occurs at workstation $k = ws(p,s)$) as

$$CT^{p,s} = E[S^{p,s}] + \sum_{i=1}^{np} \sum_{l=1}^{ns(l)} \frac{\left(W^i - \mathbf{1}\left(i = p\right)\right) \mathbf{1}(ws(i,l) = k) r^{i,l}}{\sum_{j=1}^{ns(i)} r^{i,j} CT^{i,j}} \left[E[S^{i,l}] CT^{i,l} + E[S^{i,l}]^2 \left\{ \frac{C^2[S^{i,l}] - 1}{2} \right\} \right]. \tag{6}$$

The first indicator term, $\mathbf{1}(i = p)$, in (6) reduces the number of jobs seen by one for product type *p*. The second indicator term in (6) omits processing steps which are not performed at workstation *k*. These equations are solved iteratively, using initial cycle time estimates of $CT^{p,s} = E[S^{p,s}]$. Note that in this iterative process the update should be only made after each complete iteration because of the summation term in the denominator. This iterative process is

$$CT^{p,s}(\text{next}) = f(CT^{p,s}(\text{current})). \tag{7}$$

Note that in equation (6) the priority scheme is first-in first-out (*FIFO*) since the total number of jobs in the workstation when the designated job arrives is estimated and no later arriving jobs are allowed to jump ahead of the designated job once it enters into the workstation. In the subsequence analysis, a more general approach is taken where later arriving jobs of a higher priority are allowed to get in front of the designated job.

Equation (6) is an extension of the Reiser and Lavenberg (1980) mean value analysis (MVA) model to the processing step paradigm. The exact MVA analysis recursively solves for a setting of the CONWIP levels based on values for systems with one less job for the designated job class and builds up the levels until the desired settings have been reached. Exact analysis is generally considered computationally inefficient for realistic situations with many products. Rather than try and solve these equations using the exact recursive approach, the approach of (7) is to iteratively solve these equations using a contraction mapping procedure.

2.2 General Priority Scheme

In a general priority processing scheme with non-preemptive processing, a job of type *p* at step *s* performed in workstation *k* sees on average the number of jobs as in the *FIFO* analysis above on arrival into the workstation. However, while this job is waiting in the queue jobs of higher priority will arrive at a rate $\lambda^{i,l}$ during the designated job's cycle time $CT^{p,s}$. Hence the number of these jobs that will also get in front of the designated job is $\lambda^{i,l} CT_q^{p,s}$. The subscript *q* notation indicates that these arrivals must occur before the job gets onto the server (while it is in the queue). Note that jobs of the same priority will not get in front of the designated job since the scheme is *FIFO* within priority classes. Therefore, there will be a third term added to the above cycle time estimate which accounts for later arriving jobs of "higher" priority. In this analysis, priorities are low to high in preference; that is, jobs of priority 1 are processed ahead of jobs of priority class 2, etc. All that is needed to accommodate for priorities is a third indicator term to determine the relative priority of a job of type *i* at step l with respect to the designated product *p* at step *s*. Of course these comparisons are restricted to jobs that are being processed at the same workstation $k = ws(p,s)$. The cycle time equation for product *p*, step *s* with work performed at workstation $k = ws(p,s)$ and priorities are low to high in order of preference is given in general by:

$$\begin{aligned} CT^{p,s} &= E[S^{p,s}] + \sum_{i=1}^{np}\sum_{l=1}^{ns(i)} \mathbf{1}\left(ws(i,l)=k\right)u^{i,l}\left[E[S^{i,l}]\left\{\frac{C^2(i,l)+1}{2}\right\}\right] \\ &+ \sum_{i=1}^{np}\sum_{l=1}^{ns(i)} \mathbf{1}\left(pri(i,l)\le pri(p,s)\right)\mathbf{1}\left(ws(i,l)=k\right)\left(WIP^{i,l}-u^{i,l}\right)E[S^{i,l}] \\ &+ \sum_{i=1}^{np}\sum_{l=1}^{ns(i)} \mathbf{1}\left(pri(i,l)< pri(p,s)\right)\mathbf{1}\left(ws(i,l)=k\right)\left[\lambda^{i,l}\left(CT^{p,s}-E[S^{p,s}]\right)E[S^{i,l}]\right]. \end{aligned} \tag{8}$$

The last term in the last line of equation (8) represents the cycle time in the queue, $CT_q^{p,s} = CT^{p,s} - E[S^{p,s}]$. The arrival rate of jobs of higher priority times the waiting time in the queue for job (*p*, *s*) results in the number of jobs that enter the queue after the designated job and are allowed to get in front of it. Note that the utilization factor *u* is the arrival rate of jobs of that type times the service time of that job type

$$u^{i,j} = \lambda^{i,j} E[S^{i,j}].$$

This equation, after substituting the work-in-process and arrival rates in terms of the cycle time parameters yields:

$$CT^{p,s} = E[S^{p,s}] + \sum_{i=1}^{np}\sum_{l=1}^{ns(i)} \frac{\left(W^i - \mathbf{1}(i=p)\right)\mathbf{1}\left(ws(i,l)=k\right)r^{i,l}}{\sum_{j=1}^{ns(i)} r^{i,j}CT^{i,j}} \left[E[S^{i,l}]^2 \left\{ \frac{C^2(i,l)+1}{2} \right\} \right]$$
$$+ \sum_{i=1}^{np}\sum_{l=1}^{ns(i)} \frac{\mathbf{1}\left(pri(i,l) \le pri(p,s)\right)\left(W^i - \mathbf{1}(i=p)\right)\mathbf{1}\left(ws(i,l)=k\right)r^{i,l}}{\sum_{j=1}^{ns(i)} r^{i,j}CT^{i,j}} \left[E[S^{i,l}]\left(CT^{i,l} - E[S^{i,l}]\right)\right]$$
$$+ \sum_{i=1}^{np}\sum_{l=1}^{ns(i)} \frac{\mathbf{1}\left(pri(i,l) < pri(p,s)\right)\left(W^i - \mathbf{1}(i=p)\right)\mathbf{1}\left(ws(i,l)=k\right)r^{i,l}}{\sum_{j=1}^{ns(i)} r^{i,j}CT^{i,j}} \left[E[S^{i,l}]\left(CT^{p,s} - E[S^{p,s}]\right)\right]. \quad (9)$$

The system of equations (9) are solved iteratively, again using initial cycle time estimates of $CT^{p,s} = E[S^{p,s}]$.

3 COMPARING PRIORITY SCHEMES

The data used for this analysis is from Wein (1992). The system consists of three products to be produced in a three workstation facility each with a single machine. The part type routings through the workstations are given Table 1.

Table 1: Product routing steps through the various workstations

PRODUCT	Step 1	Step 2	Step 3	Step 4	Step 5
1	3	1	2		
2	1	2	3	1	2
3	2	3	1	3	

The mean processing times (minutes) by product and processing step $E[S^{p,s}]$ are in Table 2 and the processing times squared coefficient of variations (SCV) are all 1 because the processing times were all assumed to be exponentially distributed by Wein (1992). The assumption of all SCV's being the same and exponential is immaterial to the analysis. The equations use SCV values by product and step $C^2[S^{p,s}]$.

Table 2: Mean processing times by product and processing step

PRODUCT	Step 1	Step 2	Step 3	Step 4	Step 5
1	6	4	1		
2	8	6	1	2	7
3	4	9	4	2	

3.1 FIFO Processing Control

For a FIFO processing control scheme, all priorities by product and processing step, $pri(p,s)$, are considered the same: here all are given the value of 1. The following table gives the FIFO priority values (this particular table is included merely for comparison with the other priority schemes and to illustrate that the general approach yields a variety priority schemes as special instances):

Table 3: FIFO Priority Scheme

PRODUCT	Step 1	Step 2	Step 3	Step 4	Step 5
1	1	1	1		
2	1	1	1	1	1
3	1	1	1	1	

The mean cycle time results by product and total factory performance is given in Table 4. The product performance information is displayed in Table 5. The overall data is a Total WIP of 20, a system throughput rate of 8.91 jobs/hour and a factory overall cycle time of 134.63 minutes. The simulation overall time for the FIFO priority scheme was 133.3 minutes (standard deviation 41.34) and the relative error is 0.98% for the MVA approximation approach.

Table 4: Cycle Times (minutes) by Product and Processing Step. Analytical results are on top with simulation sample means and standard deviations given below in parenthesis (Mean, Standard Deviation)

PROD.	TYPE	Step 1	Step 2	Step 3	Step 4	Step 5
1	Anal. Simul.	39.550 (39.64, 25.55)	32.634 (30.85, 24.14)	27.921 (31.61, 25.26)		
2	Anal. Simul.	35.837 (31.87, 22.88)	30.734 (32.29, 24.35)	36.698 (36.46, 26.19)	29.837 (28.24, 23.00)	31.734 (34.91, 24.42)
3	Anal. Simul.	30.332 (34.80, 25.42)	41.737 (36.70, 25.00)	32.994 (30.82, 24.85)	34.737 (31.89, 23.95)	

3.2 SEPT Processing Control

The SEPT sequence has jobs sequenced by the shortest expected processing time at the given work-station. For the processing time data of this example, the SEPT processing step priorities are given Table 6. The mean cycle time results by product and total factory performance is given in Table 7. The product performance information is displayed in Table 8. The overall data is a Total WIP of 15, a system throughput rate of 9.313 jobs/hour and a factory overall cycle time of 96.640 minutes. The simulation overall time for the SEPT priority scheme was 97.45 minutes (standard deviation 65.02) and the relative error is 0.88% for the MVA approximation approach.

Table 5: Overall System Performance Data by Product for the FIFO Scheme. Analytical results are on top with simulation sample means and standard deviations given below in parenthesis (Mean, Standard Deviation)

PROD.	TYPE	Throughput (jobs/hr)	Cycle Time (min)	WIP (jobs)
1	Anal. Simul.	2.997	100.105 (102.07, 25.89)	5
2	Anal. Simul.	2.912	164.839 (163.76, 31.20)	8
3	Anal. Simul.	3.004	139.800 (134.24, 39.84)	7

Table 6: SEPT Priority Scheme

PRODUCT	Step 1	Step 2	Step 3	Step 4	Step 5
1	3	3	1		
2	4	3	1	1	4
3	2	4	2	2	

Table 7: Cycle Times (minutes) by Product and Processing Step. Analytical results are on top with simulation sample means and standard deviations given below in parenthesis (Mean, Standard Deviation)

PROD.	TYPE	Step 1	Step 2	Step 3	Step 4	Step 5
1	Anal. Simul.	15.462 (18.04, 12.79)	15.292 (17.21, 16.32)	6.167 (6.42, 6.24)		
2	Anal. Simul.	69.614 (66.06, 49.05)	18.893 (20.73, 18.85)	7.701 (8.29, 7.87)	7.050 (8.26, 7.17)	59.877 (59.24, 43.99)
3	Anal. Simul.	10.631 (12.08, 9.16)	65.461 (54.13, 35.55)	11.291 (12.28, 9.83)	8.758 (9.69, 8.97)	

Table 8: Overall System Performance Data by Product for the SEPT Scheme. Analytical results are on top with simulation sample means and standard deviations given below in parenthesis (Mean, Standard Deviation)

PROD.	TYPE	Throughput (jobs/hr)	Cycle Time (min)	WIP (jobs)
1	Anal. Simul.	3.250	36.922 (41.67, 21.30)	2
2	Anal. Simul.	2.942	163.134 (162.55, 61.38)	8
3	Anal. Simul.	3.120	96.141 (88.19, 37.75)	5

SRPT Processing Control

The SRPT scheme sequences jobs in the order of the shortest remaining expected processing time summed over all remaining steps. The SRPT processing step priorities are given Table 9.

Table 9: SRPT Priority Scheme

PRODUCT	Step 1	Step 2	Step 3	Step 4	Step 5
1	3	1	1		
2	4	3	2	3	2
3	4	4	2	1	

The mean cycle time results by product and total factory performance is given in Table 10. The product performance information is displayed in Table 11. The overall data is a Total WIP of 23, a system throughput rate of 9.669 jobs/hour and a factory overall cycle time of 142.723 minutes. The simulation overall time for the SRPT priority scheme was 145.18 minutes (standard deviation 139.29) and the relative error is 1.71% for the MVA approximation approach.

Table 10: Cycle Times (minutes) by Product and Processing Step. Analytical results are on top with simulation sample means and standard deviations given below in parenthesis (Mean, Standard Deviation)

PROD.	TYPE	Step 1	Step 2	Step 3	Step 4	Step 5
1	Anal. Simul.	15.645 (16.77, 12.92)	9.245 (10.22, 8.26)	6.066 (6.34, 6.25)		
2	Anal. Simul.	79.206 (55.83, 46.12)	22.989 (30.61, 23.89)	9.277 (8.47, 8.89)	19.548 (21.64, 23.35)	14.236 (18.84. 14.39)
3	Anal. Simul.	101.382 (132.55, 178.93)	164.000 (141.69, 82.33)	15.361 (16.60, 13.46)	8.801 (9.10, 8.00)	

Table 11: Overall System Performance Data by Product for the SRPT Scheme. Analytical results are on top with simulation sample means and standard deviations given below in parenthesis (Mean, Standard Deviation)

PROD.	TYPE	Throughput (jobs/hr)	Cycle Time (min)	WIP (jobs)
1	Anal. Simul.	3.876	30.956 (33.32, 16.51)	2
2	Anal. Simul.	2.891	145.258 (135.30, 47.24)	7
3	Anal. Simul.	2.901	289.532 (300.04, 151.22)	14

3.3 Work-Balance (WBAL) Processing Control

The WBAL scheme sequences jobs in the order of the shortest remaining expected processing time at this workstation. The WBAL processing step priorities are given Table 12.

Table 12: WBAL Priority Scheme.

PRODUCT	Step 1	Step 2	Step 3	Step 4	Step 5
1	3	3	1		
2	4	4	1	1	3
3	2	4	2	2	

The mean cycle time results by product and total factory performance is given in Table 13. The product performance information is displayed in Table 14. The overall data is a Total WIP of 13, a system throughput rate of 9.20 jobs/hour and a factory overall cycle time of 84.784 minutes. The simulation overall time for the WBAL priority scheme was 85.26 minutes (standard deviation 53.70) and the relative error is 0.60% for the MVA approximation approach.

Table 13: Cycle Times (minutes) by Product and Processing Step. Analytical results are on top with simulation sample means and standard deviations given below in parenthesis (Mean, Standard Deviation)

PROD.	TYPE	Step 1	Step 2	Step 3	Step 4	Step 5
1	Anal. Simul.	14.997 (17.45, 12.41)	14.655 (17.33, 16.01)	6.048 (6.48, 6.30)		
2	Anal. Simul.	59.840 (55.59, 44.30)	51.382 (46.43, 38.00)	7.506 (7.89, 7.68)	6.892 (8.59, 7.38)	19.808 (24.66, 20.12)
3	Anal. Simul.	10.303 (11.89, 9.08)	51.848 (39.72, 29.26)	10.917 (12.37, 10.13)	8.278 (8.66, 8.34)	

Table 14: Overall System Performance Data by Product for the WBAL Scheme. Analytical results are on top with simulation sample means and standard deviations given below in parenthesis (Mean, Standard Deviation)

PROD.	TYPE	Throughput (jobs/hr)	Cycle Time (min)	WIP (jobs)
1	Anal. Simul.	3.361	35.699 (41.26, 21.02)	2
2	Anal. Simul.	2.888	145.428 (143.08, 43.91)	7
3	Anal. Simul.	2.950	81.346 (72.65, 32.43)	4

4 RESULTS AND CONCLUSIONS

An analytical model was developed for a multiple-product factory operating under individual product CONWIP control and processing data given in a processing step paradigm. The four priority schemes presented by Wein (1992) (WBAL, FIFO, SEPT and SRPT) are all illustrated using his example data set. Analytical and simulation results for these examples demonstrate that the priority modeling approach is reasonably accurate. The general priority scheme is robust and encompasses a large collect of standard priority schemes as well as the potential as an evaluation tool for new priority schemes. Any non-preemptive processing scheme which gives a priority to a product at each processing step can be modeled. These step priorities are model-data input along with the problem data. The service times are assumed to be generally distributed with mean and squared coefficient of variation allowed for each processing. The main limitation of this particular model is the assumption of a single machine at each workstation. Multiple machine FIFO models exist (see Curry and Feldman 2011, Chapter 8) but have not been tested for the processing step priority scheme developed herein. The literature (Bondi and Whitt 1986, Eager et al. 2000) indicates that under high service time variability the MVA approximations on which this model is based can be significantly improved by interpolation procedures.

For the example priority schemes and the Wein problem, the CONWIP levels were determined by a manual search procedure. Methods of finding the best CONWIP/kanban settings have been studied by Altiok and Stidham (1983) for kanban systems and in Curry and Feldman (2011, Chapter 8) a tabu-search heuristic was developed for CONWIP parameter settings for a problem similar to that in this paper. Ryan and Choobineh (2003) studied the CONWIP units problem in a job shop setting.

For the push release procedure the WBAL priority scheme yielded a mean cycle time for the factory of 104.3 min. (based on the Curry and Feldman (2011) simulation study). The FIFO scheme resulted in a mean cycle time for the factory of 175.3 min. Thus the push priority improvement of WBAL versus FIFO is in the order of 40.5%. For this CONWIP study, the results are 84.8 min. versus FIFO 134.6 min., an improvement of 37.0%, SRPT 142.7 min., an improvement of 40.6%, and for SEPT (96.6 min.) an improvement was 12.2%.

These comparisons are given to illustrate the general priority model scheme for product level CONWIP control and are not a definitive study of the comparisons between the four priority schemes. The same data that was used in the previous two comparisons (Wein 1992, and Curry and Feldman 2011) was used for this illustration but the individual product CONWIP control process did not allow a really good comparison because of the granularity of the CONWIP control and the same throughput goals could not be obtained for the different priority schemes using this data set.

Essentially any priority scheme using processing step data and implemented at the workstation queue sequencing level can be evaluated using this analytical modeling approach. The four priority schemes studied do allow for a good illustration of the general nature and the accuracy of the CONWIP priority approximation modeling scheme. All of the priority schemes overall results were within 1.7% (3 of the 4 were 1% or less) in comparisons with simulation model results using the MOR/DS language of Curry et al. (1989). However at the individual product level, the accuracy is not that good possibly because of the priority scheme's impact on low priority segments resulting in high variance and MVA approximation methods not generally accounting for variance.

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AUTHOR BIOGRAPHIES

GUY L. CURRY is a Professor of Industrial and Systems Engineering at Texas A&M University. He received his B.S. in Mathematics from the University of Oklahoma, an M.S. degrees in Mathematics from Wichita State University, and the Ph.D. degree in Industrial Engineering from the University of Arkansas. His academic interests are optimization, simulation and queueing network approximations. He has published articles in over 30 referred journals and is the coauthor of three books. His email address is g-curry@tamu.edu.

MOONSU LEE is an Associate Professor of School of Industrial Management at Korea University of Technology and Education. Dr. Lee received his B.S. in Industrial Engineering from Hanyang University, Korea, an M.S. and Ph.D. degrees in Industrial Engineering from Texas A&M University. His academic interests are in the area of manufacturing system analysis, queueing network approximations and simulation. His email address is mslee@koreatech.ac.kr.