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TESTING STOCHASTIC ORDER FOR RELIABILITY ANALYSIS OF COMPLEX SYSTEMS

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ABSTRACT

System reliability plays a critical role in the comparison of complex stochastic systems. The reliability of a system can be articulated by its survivability or conditional survivability function. Systems' survivability may be compared based on a point measure such as the expected survivability. However, a point based comparison does not take advantage of all the available information. Here the interest is in the comparison of survival functions based on the stochastic order. The survival functions are assumed to be estimated via simulation. A statistical sequential procedure is presented for selecting the most reliable system with a guarantee of the best system selection.

1 INTRODUCTION

Survival function is a cornerstone of most reliability analysis. In mission critical systems, such as telecommunication and electrical power systems, the survival function provides an indication of the probability of providing the intended mission over the life of the mission. Although the predominant reliability measures are first and second moment measures, moments do not capture the full characteristics of a reliability distribution (Meeker and Escobar 1998). Stochastic order entails comparing systems based on their reliability distributions rather than a single point of the distributions (Shaked and Shanthikumar 2007).

Obtaining the survival function of a complex system is often mathematically intractable. Simulation is the most commonly used method in studying the survival function of complex systems. Estimating survival functions via simulation requires proper statistical analysis because of the sampling error involved. We present a statistical sequential procedure to determine the most reliable system from K systems based on the comparison of their survival functions using a stochastic order.

2 METHODOLOGY

The survival function $S(t)$ is the probability that a system is functioning at time t: $S(t) = \Pr(T \geq t)$ for $t \geq 0$. S_1 dominates S_2 in terms of stochastic order if $S_1(t) \geq S_2(t)$ for all t and there is at least one t for which a strong inequality holds. This means that the probability of having a lifetime longer than t for System 1 is larger than that for System 2. The comparison of survival functions can lead to three distinct comparison decisions: i) equality, ii) dominance, and iii) nondominance.

Let p_i^b be the cumulative probability of system i corresponding to bin b, $b = 1, 2, \ldots, B$. The following proportion hypothesis test is performed between S_1 and S_2: $H_0^b : p_1^b - p_2^b = 0$; $H_{11}^b : p_1^b - p_2^b > 0$; $H_{12}^b : p_1^b - p_2^b < 0$. The proportion IZ parameter δ_p is the smallest practical difference worth detecting between the survival functions of two systems. To control the probability of Type II error, the alternative hypotheses are transformed into $H_{11}^b : p_1^b - p_2^b \geq \delta_p$ and $H_{12}^b : p_1^b - p_2^b \leq -\delta_p$.

Paulson (1964) presents a sequential procedure for the mean hypothesis test: $H_0^\mu : \mu_1 - \mu_2 = 0$; $H_{11}^\mu : \mu_1 - \mu_2 \geq \delta_\mu$; $H_{12}^\mu : \mu_1 - \mu_2 \leq -\delta_\mu$. The mean-IZ parameter δ_μ is the smallest practical difference

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worth detecting between the means. Under the assumption that the observations are independent and identically distributed (IID) normal, the mean hypothesis test procedure is shown to satisfy the prespecified probability of Type I and II error conditions. Here, we will adapt this sequential procedure to solve the proportion hypothesis test. In sequential procedures, more simulation replications are needed only if a selection decision cannot be made at the current stage. This approach reduces the overall computational effort in terms of the number of simulation observations needed to make a selection decision.

In order to decide which system is dominant in stochastic order, the proportion hypothesis test is performed for every bin b, $b = 1,\ldots,B$. To achieve a $1-\alpha$ probability of correct selection guarantee in the comparison of K systems, the probability of Type I error α_b and Type II error β_b of each proportion hypothesis test is set to $\alpha/(KB)$ using the Bonferroni inequality.

In the proposed procedure, the cumulative bin counts from samples of N observations are considered basic observations. These basic observations across systems are independent because they originate from different samples. Since they have Binomial distribution with parameters N and p_i^b, they are approximately normal with mean Np_i^b and variance $Np_i^b(1-p_i^b)$ when N is large enough. Hence, the assumption of the IID normality of the basic observations is satisfied. The recommended value for the sample size N is 30.

In the Initialization step, r_0 independent samples of N IID observations are obtained from all simulated systems. The recommended value for r_0 is 10. The number of bins B is set to r_0. The recommended value for the constant parameter d is $3N\delta_p/8$. The smallest observation s and the largest observation ℓ of all observations are determined. The bin size h is determined by dividing the difference $\ell - s$ by the number of bins B. The lower bound of the first bin $L(1)$ is set to s. The subsequent bin lower bounds $L(b)$ are set to $L(b-1)+h$. The number of stages r is set to r_0. The cumulative bin counts $C_i^b(r)$ are determined by counting the number of observations larger than the lower bound $L(b)$ at stage r.

In the Calculation step, the estimates of the proportions $p_i^b(r)$ are computed by $C_i^b(r)/N$. The estimates of the variances of the cumulative bin counts are computed by $V_i^b(r) = Np_i^b(r)\left(1-p_i^b(r)\right)$. In the Comparison step, comparisons are performed for every bin of system pair (i,j) using the following bounds:

$$IB = \frac{\left[V_i^b(r)+V_j^b(r)\right]a(\beta_B)}{2d} - r(N\delta_p - d) \text{ and } OB = \frac{\left[V_i^b(r)+V_j^b(r)\right]a(\alpha_B/2)}{2d} + rd,$$

where $a(\alpha) = [(1/\alpha)^{2/f}-1](f/2)$ and $f = r-1$. An equivalence, dominance, or nondominance decision can be made based on the results of these comparisons. If a system is found to be inferior, it is eliminated. In the Termination step, if a single system is left or all the remaining system pairs have been declared equal or nondominant, the procedure terminates. Otherwise, the procedure is inconclusive at stage r. N IID observations are obtained from the systems still under consideration, and the stage counter r is increased by one. The cumulative bin counts are updated, and the procedure goes back to the Calculation step.

3 CONCLUSION

Since estimating the survival function of complex stochastic systems is intractable by analytical techniques, systems are usually simulated to estimate their survival functions. The estimated survival functions involve sampling variation; hence, statistical procedures are needed to compare and select the most reliable system with a guarantee of correct selection. A sequential procedure for the comparison of survival functions based on the stochastic order is proposed. The experimental results show that the proposed procedure selects the best system from K systems with a prespecified probability of correct selection.

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