

## MINIMIZING OPPORTUNITY COST IN SELECTING THE BEST FEASIBLE DESIGN

Nugroho A. Pujowidianto

Hewlett-Packard Singapore  
138 Depot Road  
Singapore 109683, SINGAPORE

Loo Hay Lee

Industrial & Systems Engineering  
National University of Singapore  
10 Kent Ridge Crescent  
Singapore 119260, SINGAPORE

Chun-Hung Chen

Systems Engineering & Operations Research  
George Mason University  
4400 University Drive, MS 4A6  
Fairfax, VA 22030, USA

### ABSTRACT

Constrained ranking and selection (R&S) refers to the problem of selecting the best feasible design where both main objective and constraint measures need to be estimated via stochastic simulation. Despite the growing interests in constrained R&S, none has considered other selection qualities than a statistical measure called the probability of correct selection (*PCS*). In contrast, several new developments in other R&S literatures have considered financial significance as the selection quality. This paper aims to lay the foundation of using other selection qualities by attempting to minimize the opportunity cost in allocating the limited simulation budget. The opportunity cost is defined and two allocation rules which minimize its upper bound are presented together with a fully-sequential heuristic algorithm for implementation.

### 1 INTRODUCTION

We consider a constrained optimization where the number of alternatives is fixed. Due to uncertainties involved and the dynamic relationship between the parameters, both the main objective and constrained measures are estimated via stochastic simulation. As there is a finite number of alternatives, this problem falls under the Ranking and Selection (R&S) literatures. For introduction to R&S, excellent reviews are provided by Swisher et al. (2003), Kim and Nelson (2006), and Hong and Nelson (2009) while the examples of the problems can be found at [www.simopt.org](http://www.simopt.org) (Pasupathy and Henderson 2006; 2011). The goal of the initial works is to guarantee the desired level of probability of correct selection (*PCS*). This is done by determining the number of allocation needed or developing stopping rules which can reach more than the user-specified level of *PCS* when the procedure stops.

There is also another stream of works which attempt to optimize the simulation budget. This is because the use of simulation requires considerable simulation efforts. Therefore instead of looking at the worst-case scenario, it develops a R&S procedure for maximizing the desired selection quality such as *PCS* using an average-case analysis. The resulting procedures are thus generally more efficient (Branke et al. 2007).

There have been many recent developments on R&S with multiple performance measures to complement the R&S literatures which initially focus on the unconstrained case (Rinott 1978, Kim and Nel-

son 2001, Chen et al. 2000). The problems with multiple performance measures can be classified into two groups. If all multiple performance measures are equally important, the problem can be modeled as multi-objective optimization. Examples of simulation budget allocation procedures for multi-objective R&S can be found in Butler et al. (2001) which guarantee *PCS* or in Lee et al. (2010) and Teng et al. (2010) which maximize *PCS*. In some cases, it is more appropriate to use the constrained R&S model where there is one primary performance measure as the main objective while the rest of the secondary performance measures can be treated as constrained measures.

Despite the recent interests, all of the constrained R&S procedures focus only on *PCS*. Andradóttir et al. (2005) and Andradóttir and Kim (2010) laid the foundation of constrained R&S literatures by developing several procedures for guaranteeing *PCS* when there is a single constraint. The procedures consist of the feasibility determination phase and the selection phase which can be implemented either sequentially or simultaneously. For the case with multiple performance measures, the feasibility determination can be made more efficient as shown in Batur and Kim (2010). Integrating the two works yield in a valid procedure for selecting the best feasible design as proposed by Healey et al. (2013a). Healey et al. (2013b) and Healey et al. (2013c) provided some extensions by letting some designs to be dormant from the sampling process temporarily and by handling the issues in switching between alternatives. Another alternative is to convert the constrained R&S into its unconstrained version using multi-attribute utility theory as proposed by Morrice and Butler (2006). As all these procedures focus on guaranteeing *PCS*, there have also been some efforts for maximizing *PCS* in constrained R&S. Kabirian and Olafsson (2009) developed a heuristic algorithm based on the feasibility and optimality indicators without studying *PCS* analytically. Pujowidianto et al. (2009) and Lee et al. (2012) provided a closed-form allocation rule which asymptotically maximizes the lower bound of *PCS* that is developed using Bonferroni inequalities. The Bonferroni approximation is applicable for both the case of independent performance measures and the case where the performance measures are correlated. Hunter and Pasupathy (2010, 2012) attempted to maximize the *PCS* using large-deviations perspective by minimizing the rate of the probability of false selection (*PFS*,  $PFS = 1 - PCS$ ) for the case with independent performance measures. None of the previously mentioned works explicitly account for the correlation between the main objective and the constrained measures. This is handled by Hunter et al. (2011) and Pujowidianto et al. (2012).

In some cases, the *PCS* is unable to represent the decision maker's interest. This is because the decision maker may be more concerned of economic criteria than statistical significance. For example, 10% *PCS* with \$1 loss for every false selection may be preferred to 90% *PCS* with \$1 million loss in the event that false selection occurs. This concept was first addressed by Chick and Inoue (2001) by minimizing the expected opportunity cost (*EOC*) in the context of selecting the best in an unconstrained optimization. Lee et al. (2007) provided a procedure for minimizing the opportunity cost of the observed Pareto and non-Pareto set for a multi-objective R&S. However, none has used the opportunity cost (*OC*) concept in a constrained R&S. Table 1 summarizes the development of R&S and the gap in the context of constrained R&S.

This paper aims to lay the foundation of constrained R&S based on economic criteria by minimizing the upper bound of *OC*. The insights on the difference between the allocation rule when the criteria is *OC* and when *PCS* is the concern are discussed. This is a useful contribution which will lead the development of other simulation budget allocation procedures which focus on the financial significance. For example, in the context of unconstrained optimization, Chick et al. (2010) attempted to maximize the expected value of information of individual alternative if there can only be one sample to be allocated in the next stage of sampling process. Chick and Gans (2009) and Chick and Frazier (2012) pioneered the modeling of the simulation budget allocation procedure which looks at the stream of expected rewards. Therefore, instead of myopically allocate the budget via one-step lookahead, they consider the overall rewards using dynamic programming.

Table 1: Overview of R&S and the Research Gap in Constrained R&S

	Guaranteeing Desired PCS	Maximizing Desired Selection Quality	
		PCS	OC
Unconstrained optimization	Rinott (1978), Kim and Nelson (2001), Nelson et al. (2001)	Chen et al. (2000, 2010)	Chick and Inoue (2001a, 2001b), Chick and Wu (2005), He et al. (2007)
Constrained optimization	Andradóttir et al. (2005), Andradóttir and Kim (2010), Morrice and Butler (2006)	Pujowidianto et al. (2009, 2012), Hunter and Pasupathy (2010, 2012), Lee et al. (2012)	?
Multi-objective optimization	Butler et al. (2001)	Lee et al. (2010), Teng et al. (2010)	Lee et al. (2007)

The organization of this paper is as follows. Section 2 formulates the simulation budget allocation problem and defines the opportunity cost in the context of constrained optimization. Section 3 presents the allocation rules which minimize the upper bound of the total opportunity cost (*TOC*). The sequential heuristic algorithm for implementation is provided in Section 4. Section 5 concludes this paper.

## 2 PROBLEM FORMULATION

### 2.1 Selection Problem

We consider the problem of selecting the best feasible design from  $k$  alternatives. Let  $X_i$  be the random variable of the output of the main objective of design  $i$  while  $Y_{il}$  is the random variable of the output of the  $l$ th constraint measure,  $i = 1, 2, \dots, k, l = 1, 2, \dots, s$ .  $X_{ij}$  and  $Y_{ilj}$  are the  $j$ th replications of  $X_i$  and  $Y_{il}$  respectively. The main objective value is  $h_i = E[X_{ij}]$  and the  $l$ th constraint measure value is  $g_{il} = E[Y_{ilj}]$ . The variances are  $\sigma_{h_i}^2 = Var[X_{ij}]$  and are  $\sigma_{g_{il}}^2 = Var[Y_{ilj}]$ . We assume that the performance measures are normally distributed. The output is independent from design to design as well as from replication to replication. In addition, the performance measures are independent from one to another.

Without loss of generality, the goal of the selection problem is to select the design with the smallest main objective value where all of the constraint measures should be below the respective constraint limits  $\gamma_l$ . The constrained R&S problem is therefore to select the best feasible design  $b$  as follows

$$b = \arg \min_{i=1,2,\dots,k} h_i \text{ subject to } g_{il} \leq \gamma_l, l = 1, 2, \dots, s. \quad (1)$$

In practice, the performance measures need to be estimated via simulation. Let  $n_i$  be the number of replications for design  $i$ . Thus, the performance measures can be estimated by the sample means,  $\hat{H}_i = \bar{x}_i = \sum_{j=1}^{n_i} X_{ij}/n_i$  and  $\hat{G}_{il} = \bar{y}_{il} = \sum_{j=1}^{n_i} Y_{ilj}/n_i$ . The selection is based on the sample mean and so the selected alternative  $\mathfrak{D}$  is

$$\mathfrak{D} = \arg \min_{i=1,2,\dots,k} \hat{H}_i \text{ subject to } \hat{G}_{il} \leq \gamma_l, l = 1, 2, \dots, s. \quad (2)$$

## 2.2 Simulation Budget Allocation Problem

Let  $\beta$  be the simulation budget. The objective is to minimize the total opportunity cost ( $TOC$ ) and so the simulation budget allocation problem is

$$\min_{n_1, n_2, \dots, n_k} TOC \text{ subject to } \sum_{i=1}^k n_i = \beta, n_i \geq 0. \quad (3)$$

## 2.3 Definition of Opportunity Cost ( $OC$ )

As there are multiple performance measures, there are several opportunity costs. First, there is an opportunity cost in terms of the main objective if the selected alternative is inferior to the best feasible design. Secondly, there is an opportunity cost for accounting the loss by selecting an infeasible design as the best.

Let  $OC_h$  be the opportunity cost in terms of main objective. The opportunity cost is the difference in means of the main objective values between the selected alternative and the best as follows

$$OC_{h_{\mathcal{D}}} = \max(h_{\mathcal{D}} - h_b, 0). \quad (4)$$

In addition, let  $OC_{g_l}$  be the opportunity cost in terms of the  $l$ th constraint measure. The opportunity cost is the difference between the mean of the constraint measure with the respective constraint limit multiplied by weight factors  $\omega_l$ ,  $l = 1, 2, \dots, s$ . The weight factor is used to monetize the penalty of violating the constraint measures. For example, if the goal is to minimize the total operations cost while ensuring the average waiting time is less than certain limit, the weight factor would be the cost of the excess in waiting time.

$$OC_{g_{\mathcal{D}l}} = \omega_l \max(g_{\mathcal{D}l} - \gamma_l, 0). \quad (5)$$

If appropriate,  $OC_{g_{\mathcal{D}l}}$  can be modified into a step function. This is useful when there is different penalties of violating the constraint limit for each specific ranges.

The total opportunity cost ( $TOC$ ) is therefore

$$TOC = \sum_{i \neq b} \{ [(OC_{h_i} + \sum_{l=1}^s OC_{g_{il}}) | \mathcal{D} = i] P\{\mathcal{D} = i\} \}, \quad (6)$$

where

$$P\{\mathcal{D} = i\} = P\{ \cap_{l=1}^s (\hat{G}_{il} \leq \gamma_l) \cap [ \cap_{j \neq i} [ \cup_{l=1}^s (\hat{G}_{jl} > \gamma_l) \cup (\hat{H}_i < \hat{H}_j) ] ] \}. \quad (7)$$

Note that given a design is selected as the best, the total opportunity cost in (6) is the same as the case where the constraint measures are incorporated into the main objective to have an unconstrained optimization. However, the probability of design  $i$  being the best is different. In this paper, a design can only be selected as the best if it is feasible with respect to all constraint limits. In the case where the decision maker converts the constraints into the main objective, there is a possibility that an infeasible design is the best design if it is close to the constraint limits and has a significantly better main objective than all feasible designs.

## 3 ALLOCATION RULE

This section provides some insights on the allocation rule that aims to minimize the estimated total opportunity cost as discussed in Section 3.1. For ease of illustration, Section 3.2 considers a special case with three designs when there is a single constraint. Two allocation rules are derived. Section 3.3 uses the asymptotic framework as the simulation budget goes to infinity,  $\beta \rightarrow \infty$ . This is because there are usually

certain good properties when the problem is analyzed in the asymptotic context. Section 3.4 proposes an alternative allocation rule which considers the total opportunity cost in the next stage of sampling when all additional samples are allocated to only one design. Section 3.5 provides insights to generalize the results into the common scenarios with more than three designs in the presence of multiple constraints.

### 3.1 Estimated Total Opportunity Cost (ETOC)

There is no closed-form expressions for  $TOC$  in (6). It can be evaluated using Monte Carlo Simulation. However, we do not want to add the computational burden as the computational budget is limited. This is addressed by minimizing an upper bound to  $TOC$  called the estimated total opportunity cost (ETOC). The probability of design  $i$  selected as the best can be bounded from above as follows

$$P\{\mathcal{D} = i\} \leq \min\{\min_l P(\hat{G}_{il} \leq \gamma_l), \min_{j \neq i} P[\cup_{l=1}^s (\hat{G}_{jl} > \gamma_l) \cup (\hat{H}_i < \hat{H}_j)]\}. \quad (8)$$

Assuming that the main objective and the constraint measures are independent,

$$P\{\mathcal{D} = i\} \leq \min\{\min_l P(\hat{G}_{il} \leq \gamma_l), \min_{j \neq i} [P(\sum_{l=1}^s \hat{G}_{jl} > \gamma_l) + P(\hat{H}_i < \hat{H}_j)]\}. \quad (9)$$

Therefore, the estimated total opportunity cost  $ETOC$  is given by

$$TOC \leq ETOC = \sum_{i \neq b} \{(OC_{h_i} + \sum_{l=1}^s OC_{g_l}) \times \min\{\min_l P(\hat{G}_{il} \leq \gamma_l), \min_{j \neq i} [P(\sum_{l=1}^s \hat{G}_{jl} > \gamma_l) + P(\hat{H}_i < \hat{H}_j)]\}\}. \quad (10)$$

The approximate simulation budget allocation rule becomes

$$\min_{n_1, n_2, \dots, n_k} ETOC \text{ subject to } \sum_{i=1}^k n_i = \beta, n_i \geq 0. \quad (11)$$

### 3.2 The Case with Three Designs

Let design 1 be the best feasible design, i.e.  $b = 1$  where  $h_1 < \min_{i \neq 1} h_i$ . As there is only one constraint,  $g_{1l} = g_1 \leq \gamma$ . Design 2 represents a design that is feasible but worse than the best design in terms of the main objective, i.e.  $h_1 < h_2, g_2 \leq \gamma$ . Design 3 has a better main objective value than the best design but it is infeasible, i.e.  $h_1 > h_3, g_3 > \gamma$ . The total opportunity cost ( $TOC$ ) in this case becomes

$$TOC = [(h_2 - h_1)|\mathcal{D} = 2]P\{\mathcal{D} = 2\} + [\omega(g_3 - \gamma)|\mathcal{D} = 3]P\{\mathcal{D} = 3\}. \quad (12)$$

The idea of the upper bound of  $P\{\mathcal{D} = i\}$  is to look at the individual probability that is critical in determining its probability of being selected as the best. For example, design 2 is feasible while design 3 is infeasible. Therefore, there is a higher chance for design 2 to appear as feasible and to beat design 3 as design 3 is likely to be infeasible. In this case, the probability to be focused is the probability of design 2 beating the best design 1, that is either when design 2 appears to be better than design 1 in terms of the main objective or design 1 appears to be infeasible. Similarly, design 3 has a high chance to appear to be better than designs 1 and 2 in terms of the main objective. Therefore, its probability of being the best is mostly influenced by its probability of being feasible. The estimated  $TOC$  ( $ETOC$ ) can then be given by

$$ETOC = [(h_2 - h_1)|\mathcal{D} = 2][P(\hat{G}_1 > \gamma) + P(\hat{H}_2 < \hat{H}_1)] + [\omega(g_3 - \gamma)|\mathcal{D} = 3]P(\hat{G}_3 \leq \gamma). \quad (13)$$

Let  $\phi$  and  $\Phi$  be the probability density function (pdf) and the cumulative distribution function (cdf) for the standard normal distribution,  $\phi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$ ,  $\Phi(t) = \int_{-\infty}^t \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$ . Given that  $N(\mu, \sigma^2)$  denotes a normal distribution with mean  $\mu$  and variance  $\sigma^2$ , let  $f(x_{1,2})$  be the pdf of  $N(h_1 - h_2, \hat{\sigma}_{h_{1,2}}^2)$ ,  $\hat{\sigma}_{h_{1,2}}^2 = \left(\frac{\sigma_{h_1}^2}{n_1} + \frac{\sigma_{h_2}^2}{n_2}\right)$ ,  $f(y_{1,\gamma})$  be the pdf of  $N(g_1 - \gamma, \hat{\sigma}_{g_1}^2)$ ,  $\hat{\sigma}_{g_1}^2 = \frac{\sigma_{g_1}^2}{n_1}$ ,  $f(y_{\gamma,3})$  be the pdf of  $N(\gamma - g_3, \hat{\sigma}_{g_3}^2)$ , and  $\hat{\sigma}_{g_3}^2 = \frac{\sigma_{g_3}^2}{n_3}$ . Therefore,

$$[(h_2 - h_1)|\mathfrak{D} = 2]P(\hat{G}_1 > \gamma) = (h_2 - h_1) \int_0^{+\infty} f(y_{1,\gamma}) dy_{1,\gamma} = (h_2 - h_1) \Phi\left(\frac{g_1 - \gamma}{\hat{\sigma}_{g_1}}\right), \quad (14)$$

$$[(h_2 - h_1)|\mathfrak{D} = 2]P(\hat{H}_2 < \hat{H}_1) = (h_2 - h_1) \int_0^{+\infty} f(x_{1,2}) dx_{1,2} = (h_2 - h_1) \Phi\left(\frac{h_1 - h_2}{\hat{\sigma}_{h_{1,2}}}\right), \quad (15)$$

$$[\omega(g_3 - \gamma)|\mathfrak{D} = 3]P(\hat{G}_3 \leq \gamma) = \omega(g_3 - \gamma) \int_0^{+\infty} f(y_{\gamma,3}) dy_{\gamma,3} = \omega(g_3 - \gamma) \Phi\left(\frac{\gamma - g_3}{\hat{\sigma}_{g_3}}\right), \quad (16)$$

and

$$ETOC = (h_2 - h_1) \left[ \Phi\left(\frac{g_1 - \gamma}{\hat{\sigma}_{g_1}}\right) + \Phi\left(\frac{h_1 - h_2}{\hat{\sigma}_{h_{1,2}}}\right) \right] + \omega(g_3 - \gamma) \Phi\left(\frac{\gamma - g_3}{\hat{\sigma}_{g_3}}\right). \quad (17)$$

Two of the possible ways to minimize *ETOC* are described in Sections 3.3 and 3.4. Section 3.3 shows how *ETOC* can be asymptotically minimized while Section 3.4 aims to maximize the difference between *ETOC* before and after allocating some additional simulation budget to a design.

### 3.3 Asymptotic Allocation

The approximate simulation budget allocation problem can be solved by deriving the Karush-Kuhn-Tucker (KKT) conditions to the problem in (11). By taking the natural log in solving the KKT conditions and letting the simulation budget tend to infinity, i.e.  $\beta \rightarrow \infty$ , only the terms inside  $\Phi(\cdot)$  remain. In other words, the effect of the individual loss is dissolved. As a result, the allocation rule will be the same as that for maximizing the probability of correct selection *PCS* in Lee et al. (2012). The next section attempts to quantify the effect of the opportunity cost explicitly by looking at one step ahead.

### 3.4 One-step Lookahead Allocation

In this approach, we are interested in how to allocate additional samples  $\tau_i$  to a single design  $i$  given that  $n_i$  samples have been observed for each design  $i = 1, 2, \dots, k$ . This is similar to the approach by He et al. (2007) for unconstrained optimization. Given the additional sample to design  $i$ ,  $ETOC_i$  for the case with three designs can be updated as follows,

$$ETOC_i = (h_2 - h_1) \left[ \int_0^{+\infty} f^*(y_{1,\gamma}) dy_{1,\gamma} + \int_0^{+\infty} f^*(x_{1,2}) dx_{1,2} \right] + \omega(g_3 - \gamma) \int_0^{+\infty} f^*(y_{\gamma,3}) dy_{\gamma,3}. \quad (18)$$

The idea is to update the variance of the pdf to consider the additional sample  $\tau_i$ . For example if  $i = 1$ ,  $f^*(x_{1,2})$  is the pdf of  $N(h_1 - h_2, \hat{\sigma}_{h_{1,2}}^2 = \frac{\sigma_{h_1}^2}{n_1 + \tau_1} + \frac{\sigma_{h_2}^2}{n_2})$ ,  $f^*(y_{1,\gamma})$  be the pdf of  $N(g_1 - \gamma, \hat{\sigma}_{g_1}^2 = \frac{\sigma_{g_1}^2}{n_1 + \tau_1})$  while there is no changes to  $f(y_{\gamma,3})$  as  $f^*(y_{\gamma,3})$  remains to be the pdf of  $N(\gamma - g_3, \hat{\sigma}_{g_3}^2 = \frac{\sigma_{g_3}^2}{n_3})$ . If  $i = 2$ , only  $f^*(x_{1,2})$  is updated where  $\hat{\sigma}_{h_{1,2}}^2 = \frac{\sigma_{h_1}^2}{n_1} + \frac{\sigma_{h_2}^2}{n_2 + \tau_2}$  while only  $f^*(y_{\gamma,3})$  is updated if  $i = 3$  by changing the estimated variance  $\hat{\sigma}_{g_3}^2 = \frac{\sigma_{g_3}^2}{n_3 + \tau_3}$ .

The allocation rule is then to allocate the additional sample only to a design of which the difference between  $ETOC_i$  and the current  $ETOC$  is the highest.

### 3.5 Generalization

Sections 3.2 to 3.4 focus the discussion on the case with three designs when there is a single constraint. In general, there are often more than three designs and multiple constraints are present. The one-step lookahead allocation rule is expected to be similar. The presence of many constraints and designs will require one to compare the probabilities to derive the upper bound of the probability of being the best. This can be addressed by computing the respective opportunity costs. The probability to be focused is then the one that has the least opportunity cost so as to ensure that the upper bound of the  $TOC$  is tight.

## 4 SEQUENTIAL ALGORITHM

This section presents a fully-sequential heuristic algorithm for implementing the allocation rule. In practice, the means and variances can be estimated using sample mean and sample variance based on  $n_0$  initial replications for all of the designs. Afterwards, the sampling budget is increased by  $\tau = \sum_{i=1}^k \tau_i = 1$ . The algorithm is described as the following

```

INPUT      number of designs  $k$ , the total simulation budget  $\beta$ , the initial number of replications
            $n_0$ , the additional budget in each stage  $\tau$ , and;
INITIALIZE  $r \leftarrow 0$ ;
           Perform  $n_0$  samples for all designs;  $n_{1,r} = n_{2,r} = \dots = n_{k,r} = n_0$ .
LOOP WHILE  $\sum_{i=1}^k n_{i,r} < \beta$  DO
  UPDATE   Calculate sample means and sample standard deviation using the new simulation output;
           select the sample best feasible design  $\mathfrak{D}$ 

  ALLOCATE Increase the sampling budget by  $\tau = 1$  and select the design  $i^*$  to receive the additional
           one sample

  SIMULATE Perform additional one sample for design  $i^*$ ; update  $n_{i^*,r+1} = n_{i^*,r} + 1$ ;  $r \leftarrow r + 1$ .
END OF LOOP
    
```

For the asymptotic allocation, the allocation rule is the same as that in Lee et al. (2012). In this case,  $i^* = \arg \max_i (n_{i,r+1} - n_{i,r})$  while  $i^* = \arg \max_i |ETOC_i - ETOC|$  for the one-step lookahead allocation rule.

## 5 CONCLUSION

This paper revisits the problem of constrained ranking and selection where the performance metric of interest is the total opportunity cost instead of the probability of correct selection. This is to cater to the decision makers who are more interested in the economic value of the decision instead of the statistical significance. The problems are formulated and two possible allocation rules are presented. The asymptotic

allocation turns out to be equivalent to that for probability of correct selection. Similar to the case of unconstrained optimization, the one-step lookahead allocation is able to consider the total opportunity cost by looking at each hypothetical value for allocating samples for a single design before the actual sampling. This research can be extended by considering the framework by Chick and Frazier (2012) which looks at the entire future stream of rewards instead of only one-step ahead. In addition, the empirical value of new allocation rules considering the total opportunity cost needs to be investigated.

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## AUTHOR BIOGRAPHIES

**NUGROHO A. PUJOWIDIANTO** is an R&D Writing System Engineer in the Business Printing Division, Printing and Personal Systems Group at Hewlett-Packard Singapore. He received his B.Eng. (Mechanical Engineering) degree from Nanyang Technological University in 2006 and his Ph.D. degree from the Department of Industrial and Systems Engineering, National University of Singapore in 2013. He is a member of IEEE and was previously a teacher at Pioneer Secondary School under the Ministry of Education, Singapore. His research interests include simulation optimization and its application in health care. His email address is [nugroho@hp.com](mailto:nugroho@hp.com).

**LOO HAY LEE** is an Associate Professor and Deputy Head (Research) in the Department of Industrial and Systems Engineering, National University of Singapore. He received his B.S. (Electrical Engineering) degree from the National Taiwan University in 1992 and his Ph.D. degree in 1997 from Harvard University. He is currently a senior member of IEEE, a member of ORSS and INFORMS. He is the associate editor for IIE Transactions, Flexible Services and Manufacturing Journal, the Asia Pacific Journal of Operational Research, the co-editor for Journal of Simulation and is a member in the advisory board for OR Spectrum. His research interests include simulation-based optimization, and maritime logistics. His email address is [iseleelh@nus.edu.sg](mailto:iseleelh@nus.edu.sg).

**CHUN-HUNG CHEN** is a Professor in the Department of Systems Engineering and Operations Research at George Mason University. He received his Ph.D. from Harvard University in 1994. His research interests are mainly in development of very efficient methodology for simulation and optimization and its applications. Dr. Chen has served as Co-Editor of the Proceedings of the 2002 Winter Simulation Conference and Program Co-Chair for 2007 Informs Simulation Society Workshop. He is currently an associate editor of IEEE Transactions on Automatic Control, area editor of Journal of Simulation Modeling Practice and Theory, associate editor of International Journal of Simulation and Process Modeling, and simulation department editor for IIE Transactions. His email address is [cchen9@gmu.edu](mailto:cchen9@gmu.edu) and his web page is <http://mason.gmu.edu/~cchen9>.