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A MAGIC NUMBER VERSUS TRICKLE DOWN AGENT-BASED MODEL OF TAX POLICY

Shih-Hsien Tseng

Business and Entrepreneurial Management
Kainan University
No. 1, Kainan Road, Luzhu Shiang,
Taoyuan 33857 TAIWAN

Theodore T. Allen

Industrial and Systems Engineering
The Ohio State University
1971 Neil Avenue- 210 Baker Systems,
Columbus, OH 43221 USA

ABSTRACT

The purpose of this article is to explore the interaction of two opposing forces. The forces of wealth accumulation in sturring job creation and the force of satisfaction and the "magic number" in causing job destruction are explored." An agent based model is proposed to explore the potentially competing effects of two hypothesized economic forces. The first is "trickle down" economics in which job creation occurs when wealth accumulates. The second is the "magic number" effect in which retirement occurs when wealth accumulates. Also, considered is the so-called "substitution effect" in which less is produced when the tax burden is considered to be too high. The "magic number" agent-based model proposed here is then explored using design of experiments. Three types of experiments were performed to explore (but not validate) the effects of assumed conditions on system gross domestic product (GDP) and tax revenue predicted after 50 years of operations.

1 INTRODUCTION

Many current financial planners call attention to the "magic number" at which point a person has sufficient financial savings that retirement is possible (Updegrave 2010). For example, ING and T. Rowe Price both have on-line calculators available for potential retirees to use for planning (see references). Apparently, however, the phenomenon of magic numbers has received little, if any, attention in tax and economic growth policy discussions.

At the same time, credible economists have observed situations in which large groups of people transform what would likely have been income into some other form to avoid paying at tax rates that they consider excessive (Gruber and Saez 2002). The reductions in income on the order of 20% in response to high taxation is a popular subject in the press as well (Palmer 2011). Further, economists have also documented the potentially beneficial effects of technical progress in encouraging workers to retire when the workers perceive their skills are obsolete (Ahituv and Zeira 2000).

Yet, having productive workers retiring when they hit their magic numbers clearly has a negative on economic growth and gross domestic product (GPD). Workers with hard-to-train and/or hard-to-find skills and talents succeed so much earlier may then decide to retire and the skills are lost to the marketplace. For example, such famous people as Jodie Foster and Michael Jackson have said in interviews that they stopped contributing through their crafts to a great extent because they had enough money.

Meanwhile, the hypothesized "trickle down" job creation mechanism also operates based on wealth accumulation. Since the nature of the forces relate to individual decision-making, i.e., to create a job or to retire, agent-based model seems to fit the exploration well. The purpose of the proposed "magic number" agent-based models is to permit logical exploration of these hypothesized opposing forces.

In this article a simple agent-based model of a miniature economy is proposed. In recent years the agent-based model has been a new research method in social science. In addition, agent-based modeling and simulation is a new approach to understand the behavior of complex business and government systems (North and Macal 2007). It involves the development of rules for individuals for the environment. The focus on individuals and possible learning behaviors make agent-based modeling worthy of consideration. Several packages are available for generating agent-based model such as Netlogo and Repast. (Allen 2011). Thus, we utilize the characteristics of agent-based model and intend to replicate what might happen if a small number of workers of a certain type, e.g., manufacturers, lean six sigma black belts, or radiologists, entered a state and set up shop. Over time, these individuals, make money, pay taxes, in some cases move up in income and also hire others, and retire. The space in our model is the physical distance between shops in an imagined community, e.g., we could be picturing dentist offices in a suburban landscape. Our ability to make general statements about the economy is limited but our model can provide some degree of insight about the inherent conflict between the role in job creation of wealthy people and the magic number phenomenon. The applications of design of experiments techniques are intended to aid in interpreting the implications of the proposed magic number agent-based model.

The remainder of this paper is organized as follows. In Section 2, the structure of the proposed magic number agent-based model is described as well as the code in Netlogo. In Section 3, the applications of three types of design of experiments techniques are applied to aid in interpretation of the magic number model. One of the purposes of this paper is to illustrate design of experiments methods which explains why more than a single method is applied. Section 4 concludes with a discussion of the implications and opportunities for future research.

2 MAGIC NUMBER AGENT-BASED MODEL

2.1 The Model

The proposed model is described in this section. The model is based on the career evolution of individuals who earn and accumulate money. Note that students potentially interested in applying design of experiments would generally not need to develop their own model but could simply use a pre-existing simulation or game and/or construct a real-world experimental system.

Figure 1 indicates the path of an individual explored in the proposed model. Every year or round the junior employee would earn a random amount of money S_1 which is assumed to be uniformly distributed with distribution U[\$0K, \$$S_1$K] where K stands for thousands of dollars. If they reached retirement age or their magic number, they would retire. If not, if they accumulated enough money (an indicator of success) they would be promoted. If promoted to senior level, they would earn the same S_1 plus an additional amount S_2 with distribution U[\$0K, \$$S_2$K].

In addition, there are articles discussed the situation that rich people may find ways to avoid paying taxes such as transform what would likely have been income into some other form. (Gruber and Saez 2002, Osborne 2012, Martel 2013). Thus, we assume that a senior worker they would have the opportunity to become “jaded” at which point they would issue a “whine” and also reduce their income by 20% to avoid taxes. The whine would last for 3 years affecting the local political climate. Another possibility for senior workers is the creation of a job, i.e., the spawning of another junior worker. This also occurs when a net worth or wealth threshold is reached in accordance with “trickle down” economic theory.

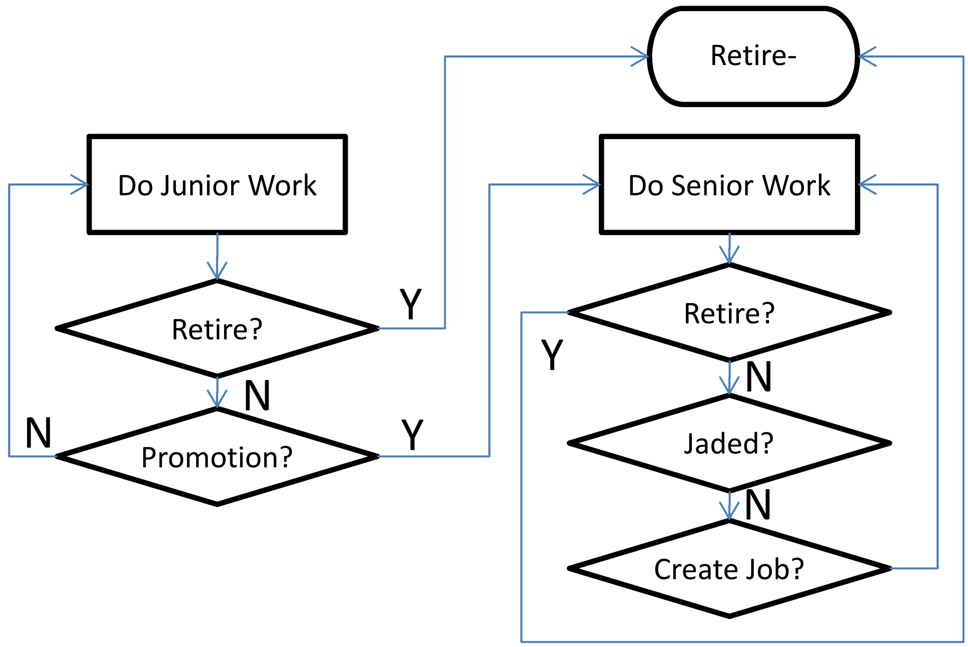

Figure 1: Career path investigated in the magic number model.

This model does not account for those "Type II" workers who have no wealth threshold, i.e., their magic number is infinity. The positive view of such people is that they are not motivated by money. The negative view of such people is that they are infinitely greedy and have lost perspective. Figure 2 illustrates the Type I assumption included in the model. Also, there is an assumption that a fraction r of what is made is saved. In our simulations, we assume $r = 0.5$ which might make sense if one includes home mortgage and equity accumulation. Retirement is assumed to occur at 45 years if the magic number has not been reached.

The model is run for 50 years (rounds) and the total amount of wealth produced and the total tax revenue are derived. It is assumed that 50 years is about as far a horizon as any policy adviser might consider. Note that the model starts all 10 individuals as having 0 work years (i.e., 20 years old) and junior status. Note also, the model assumes that there is no cap on how big markets might be or evolution of average wages for junior or senior workers.

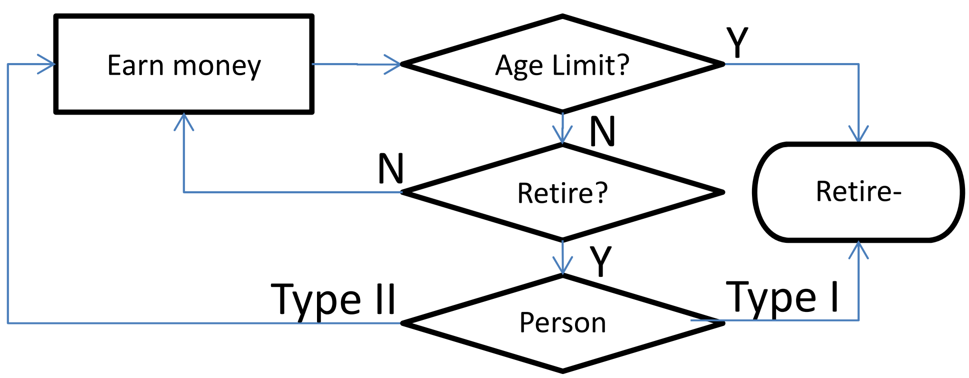

Figure 2: Illustration of the Type I person assumption.

2.2 The Code

The model was coded using the Netlogo environment, which is free. The Netlogo proposed by Uri Wilensky is an agent-based model programming environment built on the programming language JAVA. The model is implemented in Netlogo with individuals following the simple rules relating to wealth accumulation, job creating, and retirement. Figure 3 shows a screenshot of the magic number model coded using Netlogo. In Figure 3, junior workers are green and senior workers are blue. Whines are pink. The left side of Figure 3 are control bars that can be used to adjust the setting of factors based on our design of experiment and the bottom side are responses from the simulation. The code is available from the author.

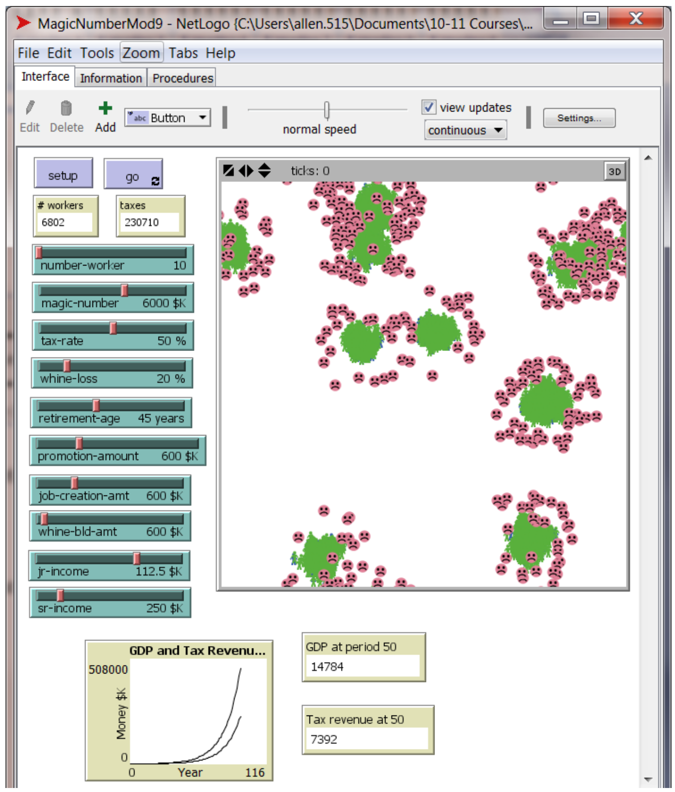

Figure 3: Screenshot of the developed Netlogo implementation of the magic number model.

3 EXPERIMENTAL DESIGN APPLICATIONS

In this section, three applications of design of experiments techniques to help interpret the implications of the magic number model are presented. These applications can provide limited validation and verification of the magic number model. This follows because if the results are too counter-intuitive, they might indicate a coding error (verification error) or a wrong assumption (validation error).

3.1 Standard Screening Using a Regular Fractional Factorial

Seven factors were varied using a regular fractional factorial (e.g., Allen 2010, pp. 289-313) as shown in Table 1. The right-hand-side of Table 1 shows the two responses which are single replicates from the magic number agent-based model.

Table 1: Fractional factorial experimental design and response values.

Magic Number	Tax Rate	Promotion Amount	Job Creation Amount	Jaded Threshold	Jr. Max. Income (S_1)	Sr. Max. Added Income (S_2)	GDP at 50 ($K)	Tax Rev. at 50 ($K)
$2M	50%	$400K	$400K	$800K	$75K	$400K	19,051	9,525.5
$4M	33%	$400K	$400K	$400K	$150K	$400K	991,039	327,042.9
$2M	33%	$400K	$800K	$800K	$150K	$100K	20,429	6,741.6
$4M	50%	$400K	$800K	$400K	$75K	$100K	1,668	834
$4M	50%	$800K	$800K	$800K	$150K	$400K	23,044	11,522
$2M	33%	$800K	$800K	$400K	$75K	$400K	1,182	390
$2M	50%	$800K	$400K	$400K	$150K	$100K	167,241	83,620.5
$4M	33%	$800K	$400K	$800K	$75K	$100K	5,452	1,799.2

Next, we generated a normal probability plot of the estimated effects using Minitab. Figure 4 shows the results for the GDP response and Figure 5 shows the results for the tax revenues response. The conclusions were almost identical due to the relatively small effect of jaded senior workers sheltering money. Note that Minitab failed to identify any significant factors in either case. There is always a concern, however, with Type II errors, i.e., missing effects when applying standard analyses (Allen, Chantarat, and Taslim 2009). Based on informal experimentation and judgment, the author redrew the lines such that all factors are declared to be significant, i.e., there is no effect sparsity. This is supported by results from Section 3.3 in which there were three repeated center points with sample standard deviations: $2,269.7K and $1,134.8K for GDP and tax revenues respectively. All the effects are much larger than these numbers.

The main effects plot in Figure 6 summarizes the results from this (resolution III) regular fractional factorial application. The implications include that, within the confines of the magic number agent-based model, none of the factors has a dominant effect on the responses over the experimental region. The separate effects of all factors are comparably relevant to economic success. Also, the implications for GDP are almost the same as for tax revenues within the confines of the magic number model. This is not surprising since it is based on trickle down economic theory and the substitution effect.

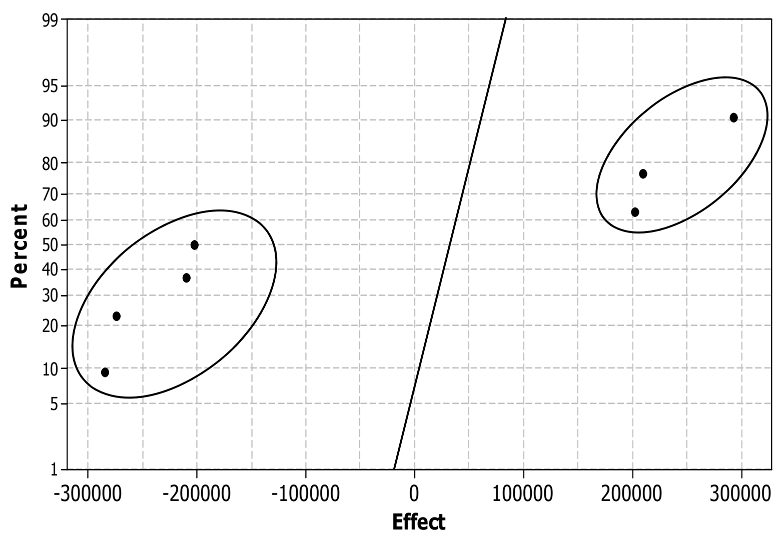

Figure 4: Normal probability plot of estimated effects for the GDP at 50 years response.

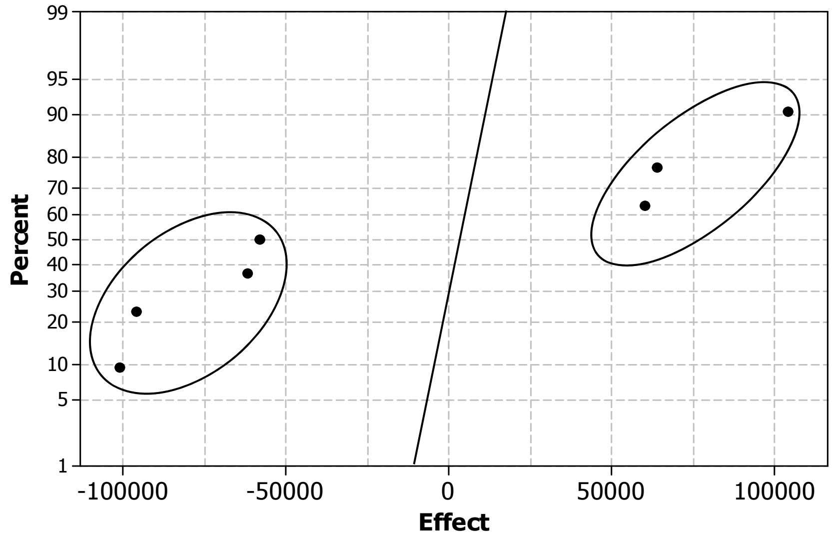

Figure 5: Normal probability plot of estimated effects for the tax revenues at 50 years response.

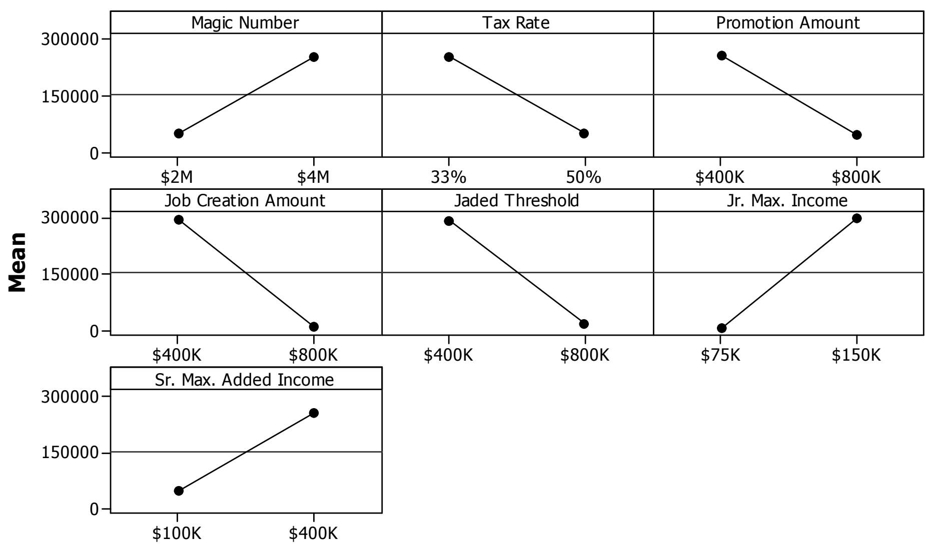

Figure 6: The main effects plots for the GDP plot.

3.2 Investigation of 2nd Order Interactions Using a Resolution V Fractional Factorial

This application is based on what might be called an intermediate method. Courses in experimental design tend to focus on so-called resolution III fractional factorials designed to support fitting an accurate first order model form and responses surface methods designed to support fitting an accurate second order model form. The resolution V fractional factorial experiment in Table 2 is designed to support fitting a first order model plus second order interactions. The responses are shown on the right-hand side of the table from a single replicate of the magic number agent-based model.

Table 2: Resolution V fractional factorial experimental design and response values.

Magic Number	Tax Rate	Promotion Amount	Jr. Max Income	Sr. Max Income	GDP at 50 ($K)	Tax Revenue at 50 ($K)
$2M	50%	$400K	$75K	$100K	3179	1589.5
$4M	50%	$800K	$150K	$400K	61660	30830.0
$4M	33%	$400K	$75K	$100K	4064	1341.1
$2M	50%	$800K	$150K	$100K	32691	16345.5
$2M	50%	$800K	$75K	$400K	2577	1288.5
$4M	33%	$800K	$75K	$400K	1600	528.0
$2M	50%	$400K	$150K	$400K	176522	88261.0
$2M	33%	$400K	$75K	$400K	8953	2954.5
$4M	50%	$400K	$75K	$400K	11189	5584.5
$4M	50%	$800K	$75K	$100K	1508	754.0
$2M	33%	$400K	$150K	$100K	62144	20507.5
$4M	33%	$800K	$150K	$100K	39364	12990.1
$4M	33%	$400K	$150K	$400K	229680	75794.4
$2M	33%	$800K	$150K	$400K	46838	15456.5
$2M	33%	$800K	$75K	$100K	1932	637.6
$4M	50%	$400K	$150K	$100K	63151	31575.5

The normal probability plots of the main effects and interaction effects are shown in Figure 7 and Figure 8 for the two responses. Again, Minitab found that many effects are not significant that expert judgment suggests are significant. All effects are much larger than the standard deviation of repeated observations so all are declared to be significant.

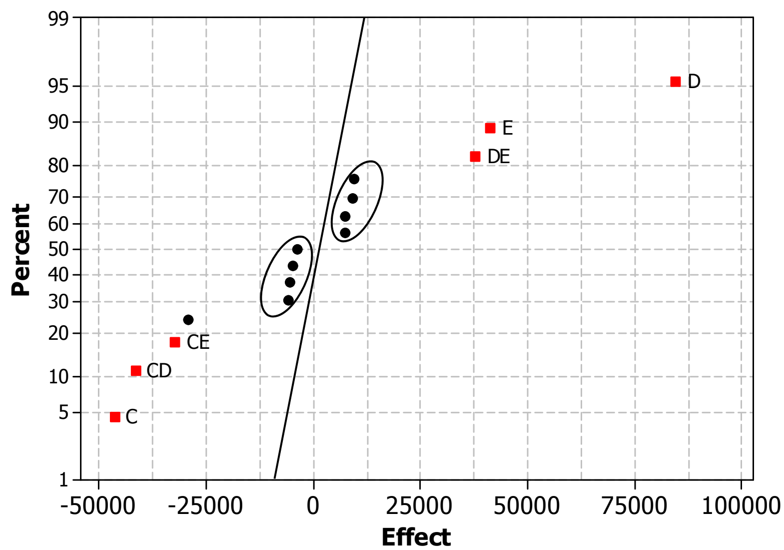

Figure 7: Normal probability plot of estimated effects for the GDP at 50 years response.

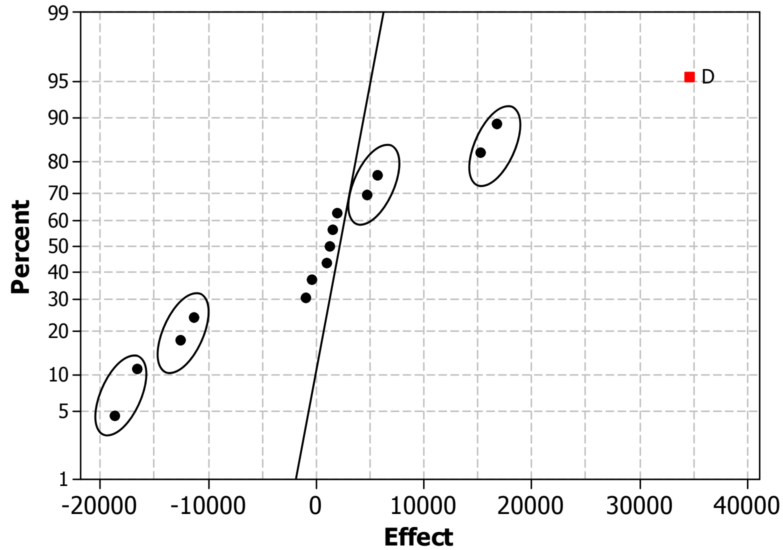

Figure 8: Normal probability plot of estimated effects for the tax revenues at 50 years response.

Figure 9 provides an interaction plot for the gross domestic product (GDP) at 50 years response and Figure 10 is an interaction plot for the tax revenues at 50 years response. The points are average model predictions when each pair of factors is varied while holding other factors constant and mid-values. The interaction between the tax rate and the magic number are interesting. The model suggests that raising tax rates will not increase revenue rates if the magic numbers are high (greedy population). However, if the magic numbers are low, then increasing rates has the economically beneficial effect of restraining senior workers who are contributing from retiring. Also, the non-parallelism between the junior maximum income (s_1) and the promotion amount/threshold indicates a significant interaction. If the promotion amount/threshold was high ($800K) then the effect of s_1 increasing was small. For a low promotion threshold ($400K), the effect of increasing the junior income maximum is large. This might occur because of the importance of promoting workers so that they become high earners as quickly as possible.

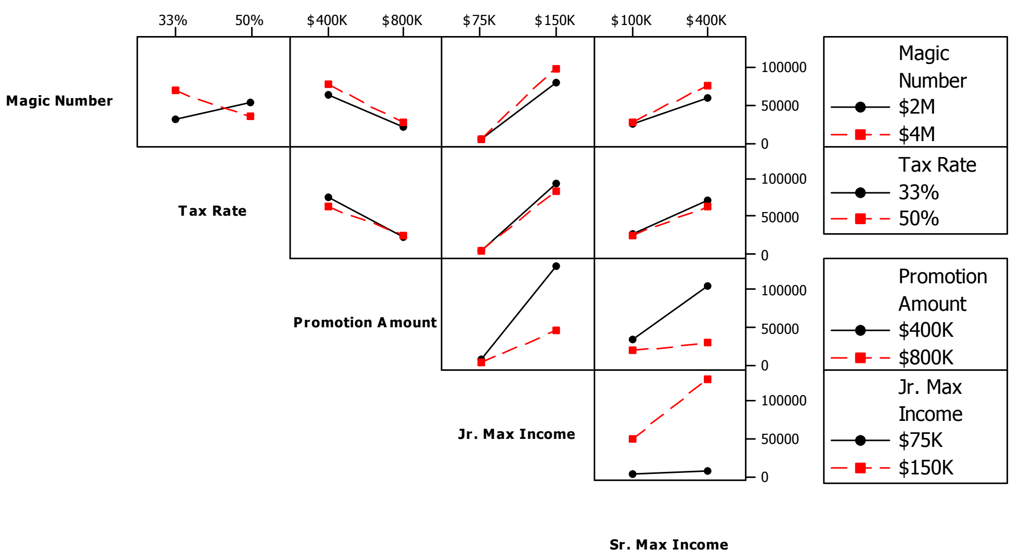

Figure 9: Interaction plot for the GDP at 50 years response.

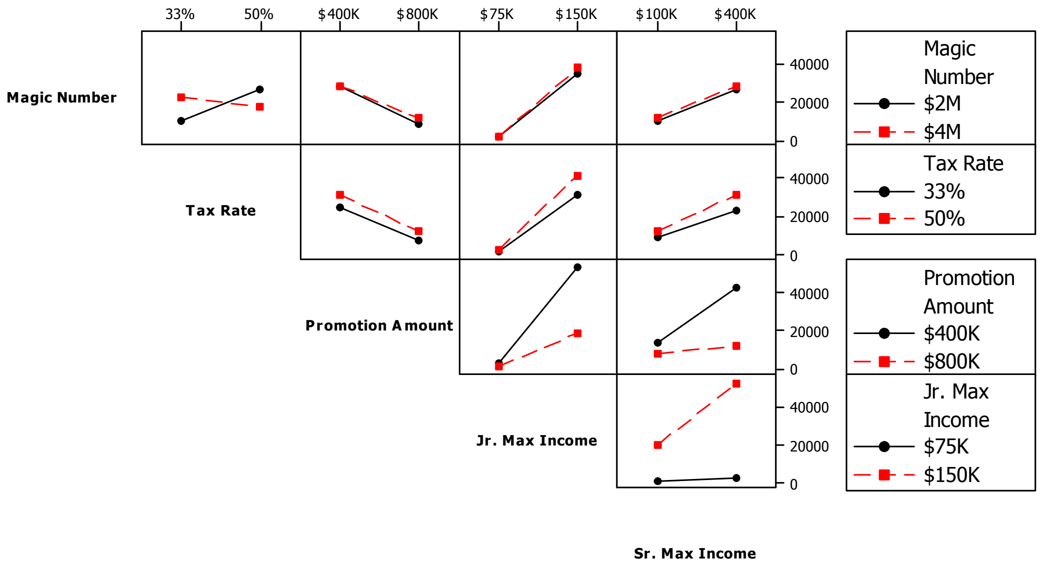

Figure 10: Interaction plot for the tax revenues at 50 years response.

3.3 Response Surface Method Application Using a Box Behnken Experimental Design

This section documents the application of a standard response surface method (RSM) approach based on Box Behnken experimental design. The experimental design and response values are shown in Table 3. As mentioned previously, this experimental design includes three repeated (center point) observations.

Table 3: Box Behnken experimental design and response values.

Magic Number ($M)	Tax Rate (%)	Jr. Max Income ($K)	GDP at 50 ($K)	Tax Revenue at 50 ($K)
6	33	150	76206	25148.0
6	50	112.5	19253	9626.5
6	67	150	64111	42954.4
6	33	75	2688	887.0
2	50	150	51390	25695.0
2	50	75	2826	1413.0
10	33	112.5	17697	5840.0
10	50	75	3624	1812.0
6	50	112.5	17708	8854.0
10	67	112.5	18145	12157.2
10	50	150	64557	32278.5
6	67	75	3014	2019.4
2	33	112.5	14656	4836.5
2	67	112.5	17253	11559.5
6	50	112.5	14784	7392.0

The predictions from the fitted model are shown in Figure 11 and in Figure 12. Figure 11 shows the diminished effect of the magic number if the tax rates are very high. This likely occurred in the model because the simulated individuals did not reach the magic number any time in the 50 years when the tax rates are very high. Figure 12 indicates that the junior person maximum salary was much more influential than the magic number and having only a limited interaction with the magic number. After collecting the data, a second order model was fitted to both responses. Because the responses were very similar in their results, we focus somewhat arbitrarily on the tax revenue response. In both cases, the adjusted R-squared values were high: 95.8% and 97.0% for the two responses. The normal probability plots of residuals (not shown) line up with no outliers for both responses. The variance inflation factors are clearly acceptable for both applications because the Box Behnken experimental design was applied. The model for the tax revenue responses is indicated in Table 4.

Table 4: Fitted second order model for the tax revenue at 50 years response.

Term	Coefficients	SE Coefficients	T	P
Constant	8624.2	1269.9	6.791	0.001
Magic Number	1073	777.6	1.38	0.226
Tax Rate	3997.4	777.6	5.14	0.004
Jr. Max Income	14993.1	777.6	19.28	0
Magic Number*Magic Number	-1239.2	1144.6	-1.083	0.328
Tax Rate*Tax Rate	1213.3	1144.6	1.06	0.338
Jr. Max Income*Jr. Max Income	7914.7	1144.6	6.915	0.001
Magic Number*Tax Rate	-101.5	1099.7	-0.092	0.93
Magic Number*Jr. Max Income	1546.1	1099.7	1.406	0.219
Tax Rate*Jr. Max Income	4168.5	1099.7	3.79	0.013

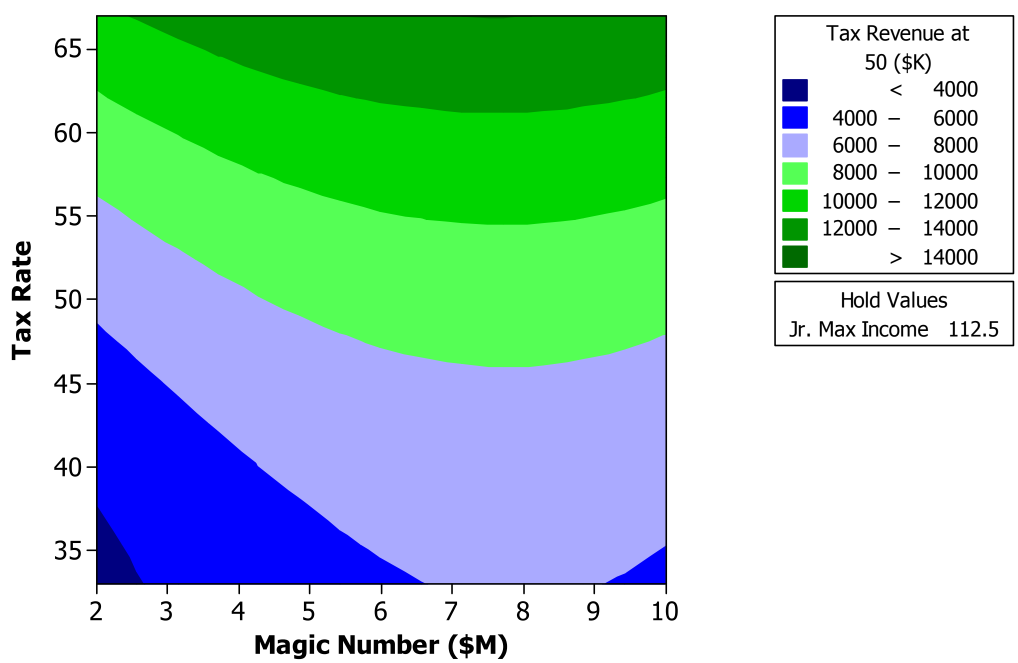

Figure 11: Contour plot (A versus B) for the tax revenues at 50 years response.

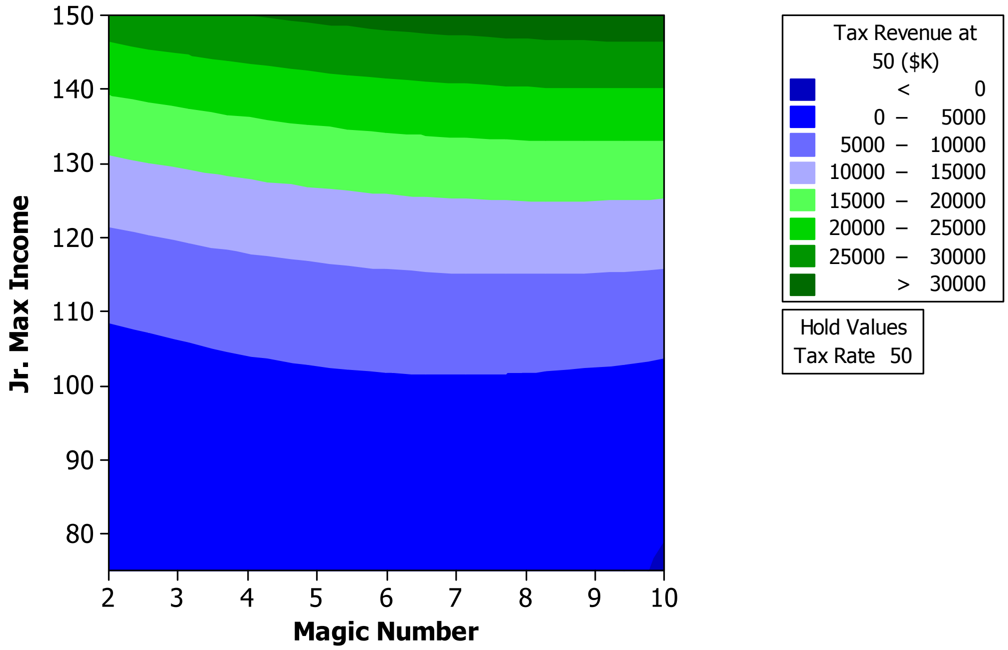

Figure 12: Contour plot (A versus C) for the tax revenues at 50 years response.

4 CONCLUSION

This paper has proposed a new model designed to investigate the interactions of usual supply-side economic theories with each other and with the magic number assumed phenomenon. Three types of design of experiments techniques are also illustrated. The plots generated by these methods provide insights into the predictions from the magic number model. They also provide an indication that no verification errors or bugs exist in the code because all the predictions seem explainable in terms of the underlying assumptions within the model. The results surprised us in that, in the ranges we studied, the role of lower taxes and job creation was not as offset by the magic numbers as we had initially expected, even when the numbers were fairly low.

Given that we admit that the model is too simplistic to draw wide conclusions about actual economies, what is the conclusion? The conclusion is that the two forces of job creation and retirement do predictably clash. However, somewhat surprisingly the force of retirement plays a relatively weak role considering that new jobs are only created when sufficient capital is accumulated. One issue that could be explored further, perhaps, is the role of demand and other sources of seed capital besides the successful operators already within the industry. Other sources could change the growth picture such that the local job creators are less critical. It is hoped that additional theories much more grounded in real-world data that incorporate the magic number modeling assumptions will be explored by others.

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AUTHOR BIOGRAPHIES

SHIH-HSIEN TSENG is an Assistant Professor in the Department of Business and Entrepreneurial Management at Kainan University. His primary interests relate to new methods for optimal experimental design and novel statistical methods for quality management and decision-making. He has published in academic journals and worked at ACE Geosynthetics for about 3 years applying Six Sigma and related techniques to improve the manufacturing process. His email address is tsengsh@mail.knu.edu.tw.

THEODORE T. ALLEN is an Associate Professor in the Department of Integrated Systems Engineering, College of Engineering at The Ohio State University. His interests focus on the overlap of operations research and quality improvement with emphasis on applied statistics and design of experiments (DOE). Dr. Allen has worked with numerous companies applying related techniques including as a full-time intern at The Ford Motor Company. Dr. is a fellow of the American Society For Quality and the author of over 40 peer reviewed papers. His two textbooks are *Introduction to Engineering Statistics and Lean Sigma* and *Introduction to Discrete Event Simulation*. His email address is allen.515@osu.edu.