Proceedings of the 2013 Winter Simulation Conference
R. Pasupathy, S.-H. Kim, A. Tolk, R. Hill, and M. E. Kuhl, eds.

TEMPORAL PERSPECTIVES IN CONSTRUCTION SIMULATION MODELING

Gunnar Lucko

Construction Engineering and Management
Department of Civil Engineering
Catholic University of America
Washington, DC 20064, USA

Amlan Mukherjee

Department of Civil and Environmental
Engineering, Michigan Technological University
Houghton, MI 49931, USA

ABSTRACT

Temporal perspectives play a vital role in shaping narratives. Such perspectives include models of time that support the practice of construction management. Although formal representations of time are rarely noticed, they strongly influence the variables and relationships that can be encoded in process models. The objective of this paper is to illustrate the distinct ways in which the time can be formalized and how they impact the understanding of project performance and productivity. It explores existing and new temporal representations on how they contribute to improving reasoning capabilities in construction processes. Existing models differ by whether they use time points or intervals to represent activities (e.g. activity-on-node networks versus Gantt bar charts) and how clearly they communicate changes during execution. While traditional approaches exhibit shortcomings, singularity functions have significant potential for further development and could benefit from conceptual integration with situational simulation toward a powerful and integrated temporal modeling scheme.

1 INTRODUCTION

Temporal perspectives shape construction management by imposing various assumptions and reasoning structures on decision-makers, including those in construction project management. These materialize most prominently in the various scheduling techniques, whose essence is addressing the question of "when" while planning "what will be done," "how will it be done," and "who will do it" (Hinze 2012).

"When" is of central importance, as temporal constraints define an overall model wherein other types of planning issues reside, because all activities carry a form of reference to the time of their execution. For example, the terms 'activity sequence,' 'float,' and 'cycle time' respectively are temporal issues that arise within a linear topology, acyclic, and cyclic network. Topologies can help organizing resources and calculate metrics of cost, productivity, and duration. Yet discussing time is incomplete without considering space. The spatial nature of projects (linear, repetitive, horizontal, or vertical) directly influences how time is formalized. Hence the time-space nexus will lead to new topologies. Linear and repetitive scheduling (LSM / RSM) research examines resources by spatial needs and operational continuity, different from network-based methods that reason solely based on early activity starts and project completion. These approaches therefore differ in data content (especially on work quantities); each is best suited for specific classes and aspects of construction projects. This paper therefore explores formal representations of time and space constraints. Paradigms discussed include time point and time interval representation. Time-space considerations of LSM / RSM are included to highlight interactions of time and space. This conceptual paper is reflective in nature, surveying the literature and recent research. It emphasizes the strengths and limitations of representations to reason about projects. The discussion takes a modeler' perspective, building on experiences by the construction modeling and simulation communities. It aims to sensitize the reader as to how the perception of time impacts planning techniques that are commonly used in practice.

2 TIME REPRESENTATIONS

2.1 Time Point Representations

This approach represents time as a sequence of discrete events and its passage as moving directly from one event to the next, which are not necessarily equally spaced or contiguous. Rather, the entire model is reduced to tracing temporal relationships between relevant events and omitting space. Each event uses one or more interacting resources, changing its state (from active to idle or *vice versa*). Simulations are motivated by the need to estimate the impact of uncertainty. Discrete event simulation (DES) languages like STROBOSCOPE or CYCLONE and modeling frameworks like SIMPHONY are general (Martínez and Ioannou 1999; Sawhney et al. 1998) or special purpose frameworks (AbouRizk and Mohamed 2000) to optimize time, cost, or production, or a weighted combination of these measures. They use the activity scanning (Puri and Martínez 2012) paradigm that treats events as time points. Models are a set of events in a network with directional arcs, from condition to activity to outcome, called activity cycle diagram (ACD). An earthmoving operation is thus represented by *PushLoad*, *Haul*, *DumpAndSpread*, and *Return* (Martínez and Ioannou 1999). Simulation systems use statistical distributions to characterize uncertainty in model parameters, e.g. cycle times, to repeatedly sample and aggregate results in the manner of a Monte Carlo simulation, yielding the distribution for outputs such as production rate of an entire operation. Further research added e.g. interruptions (Lu and Chan 2004). While DES can successfully model construction operations, it cannot represent occurrences *during* ongoing processes, nor examine multiple concurrent events. This requires another time traversal approach, not time points but time intervals. For example, *DumpAndSpread* should not only be an event of variable duration. Instead, it has sub-events to mark its start and finish, separated by its sampled duration. Admittedly, a DES approach can be extended to remove some limitations. But in doing so will lose the very elegance that allows it to effectively model construction operations. One must therefore review alternative continuous temporal representations.

2.2 Time Interval Representation

Discrete events were adequate to explore simple self-contained events and their interactions. Yet the need to model concurrent *activities* within projects shifts the focus from a task level to the overall schedule. Its formal emphasis consequently shifts from most likely task duration, e.g. of an earthwork operation, to seeking the most likely total project duration. Several assumptions add the necessary realism: Activities that occur concurrently are desirable to achieve a minimum project duration; they can be related by multiple temporal and resource constraints; uncertainty is difficult to represent, because it arises from several sources of (a) input quantities, e.g. productivity, cost, etc., (b) random external events, e.g. bad weather violations, which could even trigger one another; and the interruptions and delays will typically cascade across the remainder of the entire schedule. Time intervals represent concurrent events through temporal relationships for activities and events, beyond mere finish-to-start links, which may further have spatial implications such as congestion and conflicts. While they can handle rich project information, the computational effort of such algorithms is a major trade-off (Allen and Ferguson 1994). New modeling methods are therefore evolving to efficiently address questions based on possibly limited information and achieve more realism. These questions can be classified into (a) project dynamics and (b) project risk assessment.

2.2.1 Project Dynamics

While DES employs activity cycle diagrams, system dynamics (SD) asks how relationships that define a complete system may impact its behavior (Forrester 1991). General purpose tools include VENSIM and STELLA. System dynamics derives from control theory, with feedback to maintain a specified level of output in a productive process. In construction research, SD enabled considering inherent complexities (Sterman 1992) and the influence of contextual variables (Ashley and Avots 1984). It models continuous time to study changes in inventory and interactions in scenarios, particularly tipping-point behaviors (Taylor and Ford 2008), wherein causal feedback loops reinforce flows. SD approaches typically use top-down control strategies, given a set of constraints and feedback loops that define a project. Yet no formal methods exist to explore project-specific factors that would initiate dynamic feedback loops, balance, or reinforce them when control measures are executed. SD enables pre-project and post-project simulations (Lyneis and Ford 2007), fueling research that investigated cause-effect relationships and contingency plans (Motawa et al. 2007), and leading to new efforts to integrate it with DES (Alvanchi et al. 2009).

2.2.2 Project Risk Assessment

Unlike uncertainty at the task level, uncertainty at the project level arises from a combination of factors, among them the ‘schedule design’ and site layout. While some factors are almost impossible to predict, others can be predicted, albeit with difficulty. Such uncertainties are *epistemic* – arising from reasons pertaining to the underlying structure of the system, i.e. here a construction project to create a built facility. Examples are congestion and reduced productivity due to spatial conflicts of crews resulting from delayed predecessor activity. Uncertainty in such scenarios is combinatorial in nature, often resulting from constraint violations combined with predictable external events. Risk in this context arises from a lack of predictability in planning about structure, outcomes, or consequences (Hertz and Thomas 1983); and epistemic risk from uncertainty in said structure of the project plan with its constraints. This calls for representing relationships at the project level, while allowing for significant detail in the activities. Simulating such a model thus must monitor changing relationships and estimate corresponding risks. In addition, the decision-maker should be enabled to exploring alternative macro-level strategies to mitigate such risk.

ICDMA (Interactive Construction Decision Making Aid) is a construction situational simulator developed by one of the co-authors to investigate the impacts of alternative project decisions at the project (macro) level (Tebo et al. 2010; Rojas and Mukherjee 2003) and assess contingencies (Anderson et al. 2007). It resembles a first-person strategy game that exposes participants to diverse project management situations that rapidly unfold in simulated time (Mukherjee and Rojas 2003). Participants react by making strategic decisions about resources and activities to complete the project on schedule and within budget. Its continuous time advancement uses a state-based representation (Rojas and Mukherjee 2005). Instead of moving among events as in DES, simulated time in ICDMA “advances from time point to the next contiguous time point” (Tebo et al. 2010, p. 3124). At each one a decision-maker can interactively change some or all state variables of the project. Thus each time point provides a control point in the simulation.

Inputs to the simulation platform are twofold: A resource-loaded (constrained) project schedule and a set of events that could disrupt the execution of the project. These are coded in a *temporal constraint network* (Anderson et al. 2009) that handles activity and event relationships like “earthwork cannot start until pavement removal is complete,” plus traditional schedule relationships. An event E is an external disruptions (different from DES) that affects the project, modeled as conditions that enable an event, its effects, and its probability. For example, a drop in labor productivity due to bad weather is represented as: “When the activity is outdoors and it snows, there is 50% chance that labor productivity will drop to 75%.” Each ‘day’ in the simulation is a time point T that corresponds to a simulation state S_T. The algorithms generate the next state S_{T+1} from the current S_T in temporal constraint network with generating events probabilistically and interactive inputs by a decision-maker. Hence, systems are encoded as sets of temporal assertions that declare the *state* of a project at any given time. Interactive decisions change this state when advancing it from one time step to the next. This is significantly different from the DES paradigm, where states define the condition of entities in an operation, and events change the state of one or more entity. A querying algorithm simulates and queries the combinatorial space of future outcomes (all possible realizations of S_{T+i} , $i = 1, ...,$ given S_T and decision inputs) with Monte Carlo sampling at the end of each time point and classifies the results by impact and probability. Figure 1(a) shows how the querying algorithm works (Anderson et al. 2009). It calculates risk distributions at the end of each simulation time point *T,* showing the sensitivity of the simulation to decisions and the dynamic evolution of risk (Figure 1(b)).

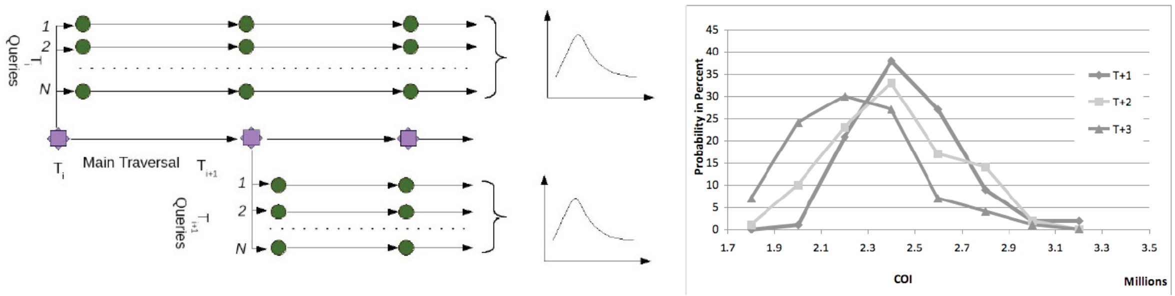

Figure 1: (a) Querying schematic diagram; (b) Query distribution at end of 3 time points.

3 TIME-SPACE NEXUS

Construction project planning in North America is traditionally dominated by network-based methods (Galloway 2006), primarily activity-on-node, to represent the schedules. Activities are reduced to nodes with durations; this representation emphasizes their relationships. The sequential algorithm of the critical path method (CPM) is a simplified linear programming; solving a set of *'start plus duration equals finish'* and *'predecessor finish becomes successor start'* equations. It adds durations to the start, using maxima at merges (forward pass); then it subtracts durations from the finish, using minima at branches (backward pass) to derive flexibility to shift (float) and its absence, criticality. The longest continuous path is considered critical and yields the project duration. It is an artifact that emerges in a network; a sequence of activities that is important to the project success because of its sensitivity to delaying it. Precedence diagrams (Fondahl 1964), an extension of CPM, allow four link types (finish-to-start, start-to-start, finish-to-finish, and a rare start-to-finish), not just finish-to-start, that can carry lag or lead (positive or negative) durations. Figure 2 shows them in a small example. It is almost impossible to understand the reasons for its structure, the critical path (marked by thick lines), why several activities are split into partial durations, which have higher or lower productivity throughout their execution, or where they are located spatially.

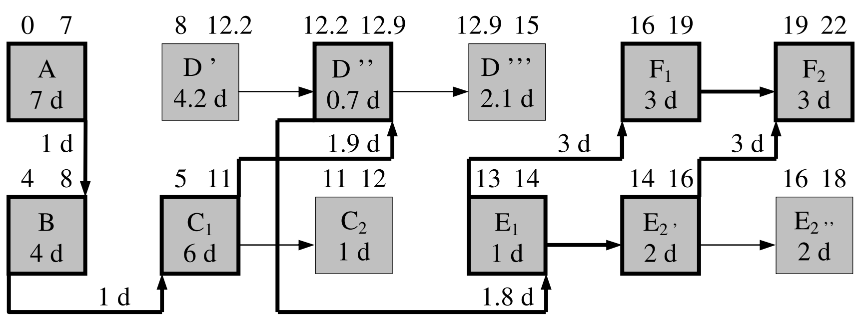

Figure 2: Network schedule with precedence diagramming (adapted from Lucko (2009)).

Converting it into a linear schedule will add much-needed clarity. The linear or repetitive nature of many construction activities creates temporal and spatial constraints, yet networks cannot represent the latter (Hegazy and Kamarah 2008). Thus various approaches to explicitly model time-space-relationships, productivity, and workflow developed, notably line-of-balance (Office of Naval Material 1962) of manufacturing, and the linear repetitive scheduling methods (LSM and RSM) (Harmelink and Rowings 1998, Harris and Ioannou 1998), which allow time, space, and resource constraints, preserve resource continuity, and facilitate multi-objective optimization (Ipsilandis 2007). They directly link work and time, plot location and direction of workfaces, reveal delays and interruptions, and show relative activity productivities. Connections also exist to other areas, e.g. lean construction and its focus on eliminating waste within the productive process. Yet linear scheduling is barely known and hardly used in North America, where CPM networks are near-ubiquitous (Galloway 2006) and constitute the dominant mental model.

3.1 Criteria for New Applications in Project Management

Traditional construction planning and scheduling has self-limited to a single time resolution, typically workdays. Conceptually, it is an output of combining the more important inputs of work quantity and productivity. Yet they remain hidden, as activities are defined with work breakdown structures (Jung and Woo 2004) that ignore productivity. Rather, they require only a 'reasonable' level of detail, i.e. that activities do not exceed 15 workdays (O'Brien and Plotnick 2007). Productivity, in turn, depends on technological means and methods and crew composition. While it is prudent to employ realistic duration estimates so that a schedule is reasonably resilient to uncertainty and risk, putting time at the center of a schedule may provide opportunities for manipulation in favor of its creator. Based on the aforementioned limitations of network-based approaches, construction project management is in need of a novel approach that should fulfill several criteria: (a) It should focus explicitly on productivity as a central underlying factor based on which companies compete in a marketplace, but also allow integrating other factors; (b) it should function at any user-selected resolution of time and support more than one unit within the same model; depending on available data and desired results; (c) it should be compatible with existing techniques or allow conversion into their schedules, provided that matches or at least analogies exist between their elements; (d) it should facilitate an intuitive communication of its mathematical results to support its use in teaching future project managers and also facilitate its eventual adoption in construction practice.

3.2 Definitions and Uses of Singularity Functions

Singularity functions are an ideal mathematical model toward this vision. They extend signal functions of electronics, but are more powerful in their capabilities (Terry and Lucko 2012): The *Dirac delta* is a case distinction that is zero at $\{x < 0, x > 0\}$, but a peak of plus infinity at $x = 0$, this operator is a single discontinuity, but remains rather abstract; The *Kronecker delta* simplifies Dirac to $y(x) = 1$ at $x = 0$ with the idea of a customized 'switch' that can be applied within any other functions; and the *Heaviside operator is* a triple case distinction of $y(x) = 0$ for $x < 0$, $y(x) = 0.5$ at $x = 0$, and $y(x) = 1$ for $x > 0$; forming a step function, which can be integrated to a ramp function. Yet it is neither left-continuous nor right-continuous at $x = 0$. Singularity functions use the operator of Equation (1) (Lucko 2007). Besides calculating criticality and float in linear schedules (Lucko and Peña Orozco 2008), they have also been successfully applied to modeling resource use (Lucko and Peña Orozco 2009) and cash flows (Lucko and Thompson 2010).

They generalize regular functions $y(x)$ by activating them at a cutoff $x = a$. Their historical origin (Clebsch 1862) lies in structural engineering, to model variable loads along beams to derive shear and moment reactions by integration. *Singularity functions express a behavior* y(x) *if* x *is mapped unambiguously to* y(x) *for all arguments and ranges of* x *and* y(x). This approach opens unique potential for applications in project management (Lucko 2009) for several reasons: The meaning of x and $y(x)$ can be freely defined by users; $y(x)$ can incorporate functions of any behavior; ranges can overlap and are theoretically unlimited in number and length; the concept can equally be applied to y and $z(y)$ andsoforth for a seamless conversion in a multi-dimensional model of the variable of interest; and $y(x)$ can be integrated and differentiated like regular functions. They can contain u segments, each performing a case distinction when they are evaluated for a given x to determine if it is already active (i.e. yields a value not equal to zero) or not. In them, y is an output and x an input, i a counting index from 1 to the number of segments u, and s the strength, starting at cutoff a with a shape n. Note that ranges of different behavior can freely overlap:

Superposition models any complex behavior $y(x)$ – complex in the sense that multiple aspects change concurrently – from simpler ones per Equation (1) by additively overlaying them. Basic terms are a step

$s_0 \cdot \langle x - a\rangle^0$ with height s_0, and a ramp $s_1 \cdot \langle x - a\rangle^1$ with slope s_1. For many applications these two suffice. They could be expanded by incorporating other functions, but this is left to be explored in future research.

$$y(x)=\sum_{i=1}^{u}\left(s_i \cdot \langle x-a_i\rangle^{n_i}\right) \text{ and } s\cdot\langle x-a\rangle^n = \begin{cases} 0 & \text{if } x<a \\ s\cdot(x-a)^n & \text{if } x\geq a \end{cases}. \qquad (1)$$

3.3 Linear and Repetitive Scheduling

Figure 3 illustrates how to represent temporal and spatial relationships. Its activities {A to F} have varying segment productivities (Harmelink and Rowings 1998). Rotating it by 90° can switch work quantity x and time $y(x)$ if needed. Time buffers, marked in gray, are minimum durations between activities (Kallantzis and Lambropoulos 2004). Equivalent work buffers for a distance in work units are omitted, but would act analogously, measured along the x-axis. Buffers prevent workflow problems from interference between activities, but should not be overly large, which would create built-in inefficiencies in a schedule. White triangular areas within a linear schedule constitute float (Lucko and Peña Orozco 2009). This schedule is already optimized toward its minimum duration by 'stacking' and 'consolidating' activities (Lucko 2009). Equation (2) exemplifies modeling an activity, here C, as a singularity function. Its first term is its *intercept* at $x = 0$ on the y-axis, second a *slope* that starts at $x = 0$, and third a *slope change* by subtracting the previous slope and adding the subsequent one at the coordinate of the 'bend' in Figure 3.

$$y(x)_C = 5\cdot\langle x-0\rangle^0 + 6/35\cdot\langle x-0\rangle^1 + (-6/35+1/15)\cdot\langle x-35\rangle^1 . \qquad (2)$$

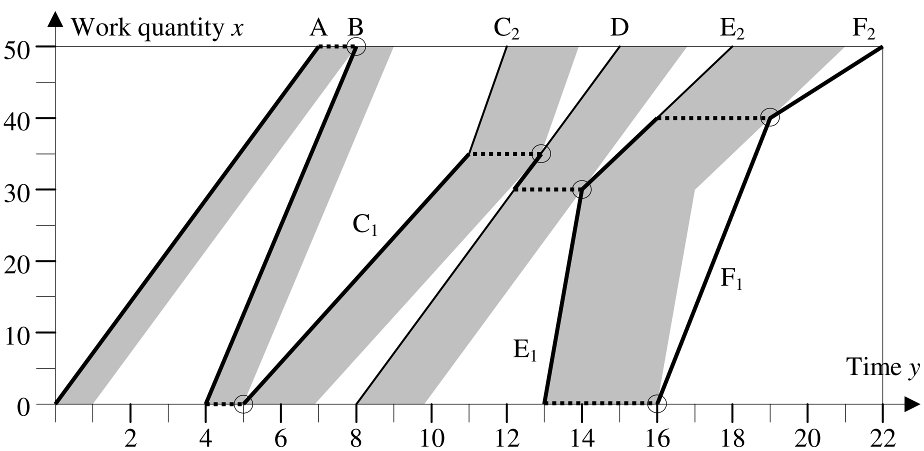

Figure 3: Linear schedule example (adapted from Lucko 2009).

4 OPPORTUNITIES FOR MANAGING PRODUCTIVE TIME

Recent research has newly applied singularity functions to time, cost, and resource optimization (Lucko and Peña Orozco 2009, Lucko and Thompson 2010). Advantages of these range-based expressions are their productivity focus and that they can integrate various metrics of work quantity; compare as-planned and as-built progress akin to earned value; identify opportunities from concurrency and float; and customize resolutions to available planned or updated data. They open new avenues to manage productive time.

4.1 Modeling Stationary versus Moving Activities

Consider different activity types in Figure 4 – bar or block (A), continuous partial-span (B), and continuous full span (D_2) (Harmelink and Rowings 1998). A is stationary at a work area for its duration. Equation (3) adds its constant step and subtracts it later. B moves; physically if it is location-based, e.g.

paving a road, or quantitatively if it creates units of a work product, e.g. m^3 of concrete. Equation (4) adds a slope and later subtracts it. C is performed by two parallel crews in units 4 and 5. Equation (5) models two different activities by using a possible shift s in its start and finish cutoffs. D has two crews in different directions, e.g. two tunnel-boring machines. Equation (6) is similar to (4), but the downward slope of D_1 requires either a negative slope or changing its direction as right to left. D_2 resembles Equation (4).

$$y(x)_A = 3.5 \cdot \left[\langle x-1 \rangle^0 - \langle x-1.5 \rangle^0 \right] . \tag{3}$$

$$y(x)_B = 1.5 \cdot \langle x-0 \rangle^0 + 3.5/2 \cdot \left[\langle x-2 \rangle^1 - \langle x-4 \rangle^1 \right] . \tag{4}$$

$$y(x)_C = 3/1 \cdot \left[\langle x-a_S \rangle^1 - \langle x-a_F \rangle^1 \right] \text{ where } a_s = 4 + s,\ s = \{0, 1\} \text{ for } \{C_1, C_2\},\ a_F = a_S + 1. \tag{5}$$

$$y(x)_{D_1} = 4 \cdot \langle x-7.5 \rangle^0 - 4/1 \cdot \langle x-7.5 \rangle^1 \text{ equivalent to } y(x)_{D_1} = 4/1 \cdot \left[\langle 8.5-x \rangle^1 - \langle 7.5-x \rangle^1 \right] . \tag{6}$$

4.2 Modeling Interruptability versus Continuity

Activities A, B, and C in Figure 5 differ as A is interrupted from time 2 to 3, B skips work units 4 to 5, and C has multiple segments of different planned or measured productivities. The step in A may be caused by weekends, holidays, or sudden breakdowns; a skip in B by accessibility constraints, sequencing rules, or lack of work (e.g. if one apartment of several need not be renovated). Note that the productivity of A is reduced from 1.0 to 0.5 work units per day when restarting. B, however, continues at at its productivity, as its work crew merely relocates. They are modeled as follows: Equation (7) for A has a step of 1 at $x = 2$ and a slope change term in round brackets. Equation (8) for B skips by first subtracting and then re-adding its slope. Equation (9) for C has several slope changes, written unsimplified in round brackets for clarity.

$$y(x)_A = 1/1 \cdot \left[\langle x-0 \rangle^1 - \langle x-3 \rangle^1 \right] + 1 \cdot \langle x-2 \rangle^0 + (-1/1 + 1/0.5) \cdot \langle x-2 \rangle^1 . \tag{7}$$

$$y(x)_B = 2.5/1 \cdot \left[\langle x-3 \rangle^1 - \langle x-4 \rangle^1 + \langle x-5 \rangle^1 - \langle x-6 \rangle^1 \right] . \tag{8}$$

$$y(x)_C = 1/1 \cdot \langle x-6 \rangle^1 + (-1/1 + 2/0.5) \cdot \langle x-8 \rangle^1 + (-2/0.5 + 1/1) \cdot \left[\langle x-8.5 \rangle^1 - \langle x-10 \rangle^1 \right] . \tag{9}$$

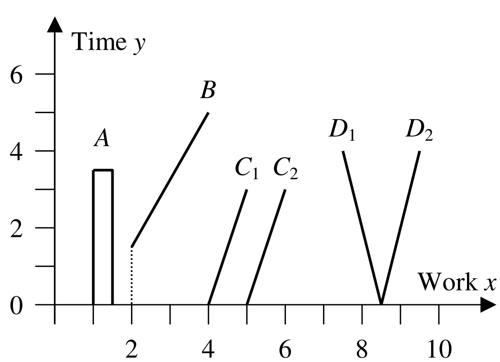

Figure 4: Stationary versus moving.

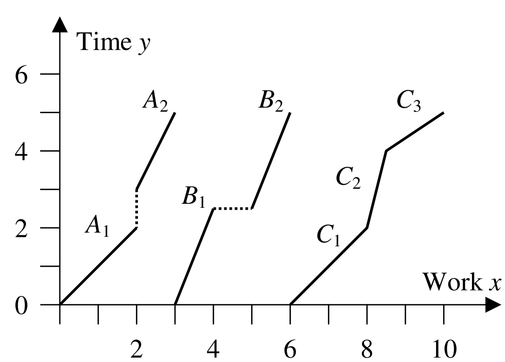

Figure 5: Interruptible versus continuous.

4.3 Modeling Worktime versus Calendar Time

Converting workdays into actual calendar days is straightforward with singularity functions. Its curves are interrupted at non-work periods by inserting breaks. Relating productive time and calendar dates is subject to calendar constraints that may apply to some or all activities or their resources. Figures 6(a) and 6(b) show how they impact activities. *A* and *B* differ in their worktime. Both take a weekend, marked in gray, but *A* does not work on Friday due to illness or vacation. Thus *A'* finishes only one day before *B'*, not two as in Figure 6(a). Equations (10) and (11) express worktime for *A* and *B* and their calendarized version *A'* and *B'*, respectively. It is possible to derive calendarization formulas, e.g. for cyclical weekdays and weekends as multiples of their durations. In general, the correct *x* where work is interrupted by a weekend is found by setting *y*(*x*) equal to a singularity function $y(x)=i\cdot 5\cdot\langle x-0\rangle^{0}$, where intercept *i* is in multiples {5, 10, 15, ...}, evaluating for the valid range of *y*, and solving for the unknown *x* where a 2-day-long step is inserted. Such calendarization can identify weekdays by mapping the continuous time *y*(*x*) onto multiples of {1 to 7}. The singularity function converts any continuously growing inputs into a stepped growth pattern $c_1(y)=\lfloor y\rfloor\cdot\langle y-0\rangle^{1}$ with a rounddown operator $\lfloor\ \rfloor$ and resets to 1 whenever 8 (i.e. start of next week) is reached by subtracting excess multiples of 7 with $c_2(y)=7\cdot\langle\lfloor(y-1)/7\rfloor-0\rangle^{1}$.

$$y(x)_A=2/3\cdot\langle x-2\rangle^{1} \text{ and } y(x)_{A'}=y(x)_A+3\cdot\langle x-3.5\rangle^{1}. \tag{10}$$

$$y(x)_B=4/5\cdot\langle x-0\rangle^{1} \text{ and } y(x)_{B'}=y(x)_B+2\cdot\langle x-2.5\rangle^{1}. \tag{11}$$

4.4 Modeling Proximity as Time and Work Buffers

Buffers are defined by and act parallel to axes. Their axis orientation defines their intended meaning (time buffer or work buffer), which could overlap and act concurrently. Accordingly, Figures 7(a) and 7(b) show a time buffer that succeeds *A* in gray. Figure 7(a) is a minimum constraint, i.e. at least 1 day must pass before *B* may occur; it remains outside the buffer but should be placed as close as possible. Figure 7(b) shows a maximum buffer. *B'* must occur at most 2.5 days after its predecessor *A'* to remain inside the gray area. An example of the former is concrete curing (hardens before other work can be done) and of the latter is tunneling (supports must be installed no more than a critical distance behind tunnel-boring machine to avoid cave-ins). Constraints could be extended to e.g. a buffer preceding its activity or whose shape need not mirror it. *B* in Figure 7(a) was first 'stacked' with a tentative intercept $y_{S,\,B}$ that conservatively is set equal to $y_{F,\text{ buffer A}}$ = 5, and 'consolidated' by subtracting $y(x)_{\text{tentative B}} - y(x)_{\text{buffer A}}$, finding its minimum and subtracting it to gain $y(x)_{\text{final B}}$, lowering said intercept to its final place. The maximum of buffer *A* in Equation (12) yields the minimum possible finish $y_{F,\,B}$ = 5 (and intercept $y_{S,\,B}$ = 3) to just touch buffer *A*, guaranteeing the minimum possible schedule duration (Lucko 2009). A finish-to-finish link from *A* to *B* emerges solely from their relative productivities. Conversely, Equation (13) obeys the maximum buffer in Figure 7(b); tentatively assigning an intercept $y_{S,\,B'}$ = 0, subtracting $y(x)_{\text{buffer A'}} - y(x)_{\text{tentative B'}}$, finding its minimum, and adding that distance to gain $y(x)_{\text{final B'}}$, plus checking that $y(x)_{\text{final B'}} \geq y(x)_A$ is true for all $x \in \{0, 5\}$ to guarantee that *B'* remains within the buffer. Analogous maximum constraints for the buffers (Kallantzis and Lambropoulos 2004) are possible, which would be useful to model work that must remain in close temporal or spatial proximity for safety reasons. Examples of activities to model with a maximum work constraint are unsupported sections of a newly excavated tunnel (Kallantzis and Lambropoulos 2004), or cantilevering sections of a newly erected bridge. A maximum time constraint would be found at a detailed time resolution e.g. in the permissible duration to haul concrete from a batch plant, avoiding 'cold joints' between placing multiple lifts of concrete, or using a roller in asphalt paving.

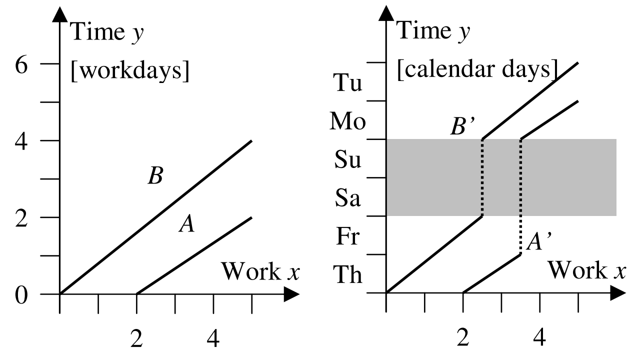

Figure 6: (a)Workdays; (b)Calendar days.

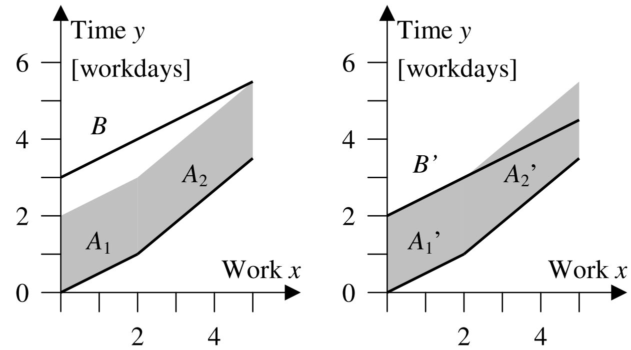

Figure 7:(a)Minimum buffers; (b)Maximum buffers.

$$\max y(x)_{buffer\,A} = \max\left\{2\cdot\langle x-0\rangle^0 + 0.5/1\cdot\langle x-0\rangle^1 + 2/3\cdot\langle x-2\rangle^1\right\} = y(5) = 5 \Rightarrow y_{F,B} = 5. \tag{12}$$

$$\min y(x)_{buffer\,A} = \min\left\{2\cdot\langle x-0\rangle^0 + 0.5/1\cdot\langle x-0\rangle^1 + 2/3\cdot\langle x-2\rangle^1\right\} = y(0) = 2 \Rightarrow y_{S,B} = 2. \tag{13}$$

4.5 Modeling Different Time Resolution within Schedules

It is important to reveal the level of detail (as-planned or as-built) of numerical values. Figure 8 shows weekly progress by connecting the start and finish of *A*. Yet if progress is measured daily, *A'* shows individual work quantities. If completing work packages matters, the step *A''* can help, e.g. in billing. Multiple resolutions can be used, provided no higher-than-known accuracy is pretended. Equation (14) converts $y(x)_A$ per Equation (4) from continuous time into a stepped growth with the rounddown operator $\lfloor\ \rfloor$ for integer days. Note that its slope of 2 work units per day in *A'* yields an infinite series of steps every 2 work units. Dividing inside $\lfloor x\rfloor$ by any desired frequency, e.g. 5 workdays per workweek in *A''* per Equation (15), modifies the step duration if desired. Importantly, linear and repetitive schedules thus can have a time resolution as detailed as the available input data allow. Revisiting *C* in Figure 5, its bends between segments may indeed indicate that its progress was updated and added between its start and finish.

$$y(x)_{A'} = 5/10\cdot\langle\lfloor x\rfloor - 0\rangle^1 \text{ equivalent to } y(x)_{A'} = 1\cdot\left[\langle x-2\rangle^0 + \langle x-4\rangle^0 + \langle x-6\rangle^0 + \ldots\right]. \tag{14}$$

$$y(x)_{A''} = 5/10\cdot\langle\lfloor x/5\rfloor - 0\rangle^1 \text{ equivalent to } y(x)_{A''} = 5\cdot\left[\langle x-10\rangle^0 + \langle x-20\rangle^0 + \langle x-30\rangle^0 + \ldots\right]. \tag{15}$$

4.6 Comparing As-Planned versus As-Built Progress

Figure 9 shows planned progress of *A* as a solid line compared to the achieved progress of a partially completed *A**. Note that the vertical time axis means that high productivity here is graphically represented in an inverse manner with shallow slopes and *vice versa*. To bring underperforming *A** on track, one must calculate a remedial accelerated *A°*. While vectorized subtraction works, singularity functions are even better suited for as-planned versus as-built progress. In Equation (16), the as-built finish $y_{F,\,A*}$ becomes an intercept, i.e. start coordinate $y_{S,\,A°}$ of the as-needed progress, and the slope contains their differences in the respective x and y coordinates of the start and finish of *A°*. This not only yields the direction, but also its correct start and finish coordinates. Corrective action for underperforming activities can thus literally be calculated as the difference between these singularity functions for as-planned and as-built progress.

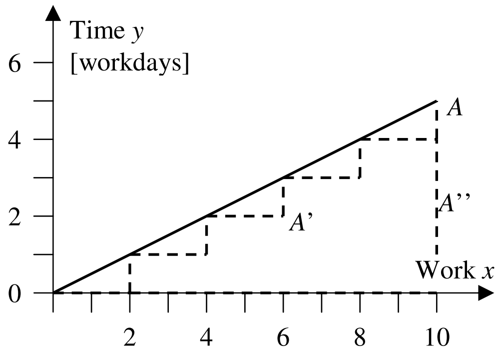

Figure 8: Different time resolutions.

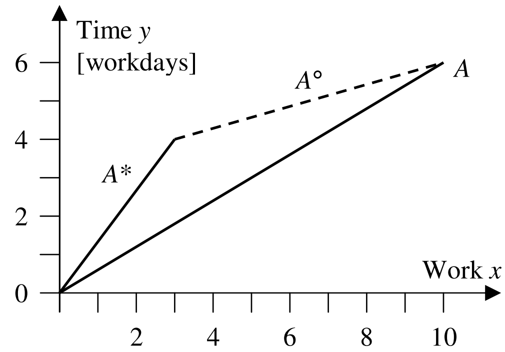

Figure 9: As-planned, as-built, as-needed progress.

$$y(3)_{A*} = 1/1 \cdot \langle 3-0 \rangle^{1} = 4 \text{ so that } y(x)_{A°} = 4 \cdot \langle x-3 \rangle^{0} + \left((6-4)/(10-3)\right) \cdot \langle x-3 \rangle^{1}. \quad (16)$$

5 CONCLUSION

This paper has reviewed time point and time interval representations, including their role in DES, SD, a recent temporal constraint network-based situational simulator, and traditional network scheduling. It has been found that treating time as a continuum creates several advantages toward better understanding and optimization for the rich interplay of temporal, spatial, and resource constraints. Modeling capabilities of singularity functions that are newly applied to linear schedules have been reviewed, including expressing stationary and moving activities, worktime and calendar time, work proximity, and different resolutions of time. This temporal representation – which also considers spatial aspects if the work axis is expressed in a space-related unit of measurement, e.g. miles of road, floors of building, or whose amount implies a specific location within the construction site – provides a new paradigm that can significantly change the ways in which temporal reasoning can be performed in construction management. Such novel modeling approach can overcome the current conceptual limitations of temporal representation in scheduling, which is primarily based on time points and dominates construction project management, to open new avenues for managing productive time in support of a efficient, robust, and multi-objective project management.

Further research is necessary on several aspects of adapting suitable temporal representations for construction management. This includes identifying and solving informational challenges of integrating the situational simulations and singularity functions into a single modeling paradigm. Their terminology, and input and output elements must be mapped for compatibility, assumptions should be understood in detail, and limitations need to be acknowledged. Analogies between control points in simulations and the change points in singularity functions can then be derived and integrated into a flexible new framework, which will include algorithms to process the queries of construction managers reasoning about their projects.

REFERENCES

AbouRizk, S. M., and Y. Mohamed. 2000. "Simphony – An Integrated Environment for Construction Simulation." In *Proceedings of the 2000 Winter Simulation Conference*, Edited by P. A. Fishwick, K. Kang, J. A. Joines, and R. R. Barton, 2, 1907-1914. Piscataway, NJ: IEEE.

Allen, J. F., and G. Ferguson. 1994. "Actions and Events in Interval Temporal Logic." *Journal of Logic and Computation* 4(5): 531-579.

Alvanchi, A., S. Lee, and S. M. AbouRizk. 2009. "Modeling Architecture for Hybrid System Dynamics and Discrete Event Simulations." In *Proceedings of the 2009 Construction Research Congress*, Edited by S. T. Ariaratnam and E. M. Rojas, 2, 1290-1299. Reston, VA: ASCE.

Anderson, G. R., A. Mukherjee, and N. Onder. 2007. "Expecting the Unexpected: Representing, Reasoning about, and Assessing Construction Project Contingencies." In *Proceedings of the 2007 Winter*

Simulation Conference, Edited by S. G. Henderson, B. Biller, M.-H. Hsieh, J. Shortle, J. D. Tew, and R. R. Barton, 2041-2050. Piscataway, NJ: IEEE.

Anderson, G. R., A. Mukherjee, and N. Onder. 2009. "Traversing and Querying Constraint Driven Temporal Networks to Estimate Construction Contingencies." *Automation in Construction* 18(6): 798-813.

Ashley, D. B., and I. Avots. 1984. "Influence Diagramming for Analysis of Project Risks." *Project Management Journal* 15(1): 56-62.

Clebsch, R. F. A. 1862. *Theorie der Elasticität fester Körper*, Leipzig, Germany: B. G. Teubner.

Fondahl, J. W. 1964. *Methods for Extending the Range of Non-Computer Critical Path Applications.* Report NBy-45818, Stanford, CA: Stanford University.

Forrester, J. W. 1991. "System Dynamics and the Lessons of 35 Years." Report D-4224-4, Cambridge, MA: Sloan School of Business, Massachusetts Institute of Technology.

Galloway, P. D. 2006. "Survey of the Construction Industry Relative to the Use of CPM Scheduling for Construction Projects." *Journal of Construction Engineering and Management* 132(7): 697-711.

Harmelink, D. J., and J. E. Rowings. 1998. "Linear Scheduling Model: Development of Controlling Activity path." *Journal of Construction Engineering and Management* 124(4): 263-268.

Harris, R. B. and P. G. Ioannou. 1998. "Scheduling Projects with Repeating Activities." *Journal of Construction Engineering and Management* 124(4): 269-278.

Hegazy, T., and E. Kamarah. 2008. "Efficient Repetitive Scheduling for High-Rise Construction." *Journal of Construction Engineering and Management* 134(4): 253–264.

Hertz, D. B., and H. Thomas. 1983. *Risk Analysis and its Applications.* New York, NY: Wiley.

Hinze, J. W. 2012. *Construction Planning and Scheduling.* 4th ed., Boston, MA: Prentice Hall.

Ipsilandis, P. G. 2007. "Multi-Objective Linear Programming Model for Scheduling Linear Repetitive Projects." *Journal of Construction Engineering and Management* 133(6): 417-424.

Jung, Y., and S. Woo. 2004. "Flexible Work Breakdown Structure for Integrated Cost and Schedule Control." *Journal of Construction Engineering and Management* 130(5): 616-625.

Kallantzis, A., and S. Lambropoulos. 2004. "Critical Path Determination by Incorporating Minimum and Maximum Time and Distance Constraints into Linear Scheduling." *Engineering, Construction and Architectural Management* 11(3): 211-222.

Lu, M., and W.-H. Chan. 2004. "Modeling Concurrent Operational Interruptions in Construction Activities with Simplified Discrete Event Simulation Approach (SDESA)." In *Proceedings of the 2004 Winter Simulation Conference*, Edited by R. G. Ingalls, M. D. Rosetti, J. Smith, and B. Peters, 2: 1260-1267, Piscataway, NJ: IEEE.

Lucko, G. 2007. "Flexible Modeling of Linear Schedules for Integrated Mathematical Analysis." *Proceedings of the 2007 Winter Simulation Conference*, Edited by S. G. Henderson, B. Biller, M.-H. Hsieh, J. Shortle, J. D. Tew, and R. R. Barton, 2159-2167, Piscataway, NJ: IEEE.

Lucko, G., and A. A. Peña Orozco. 2008. "Calculating Float in Linear Schedules with Singularity Functions." In *Proceedings of the 2008 Winter Simulation Conference*, Edited by S. J. Mason, R. Hill, L. Mönch, and O. Rose, 2512-2518, Piscataway, NJ: IEEE.

Lucko, G. 2009. "Productivity Scheduling Method: Linear Schedule Analysis with Singularity Functions." *Journal of Construction Engineering and Management* 135(4): 246-253.

Lucko, G., and A. A. Peña Orozco. 2009. "Resource Leveling of Linear Schedules with Singularity Functions." In *Proceedings of the 2009 Winter Simulation Conference*, Edited by M. D. Rossetti, R. R. Hill, B. Johansson, A. Dunkin, and R. G. Ingalls, 2571-2582, Piscataway, NJ: IEEE.

Lucko, G., and R. C. Thompson. 2010. "Derivation and Assessment of Interest in Cash Flow Calculations for Time-Cost Optimizations in Construction Project Management." In *Proceedings of the 2010 Winter Simulation Conference*, Edited by B. Johansson, S. Jain, J. Montoya-Torres, J. Hugan, and E. Yücesan, 3037-3048, Piscataway, NJ: IEEE.

Lyneis, J.M., and D. N. Ford. 2007. "System Dynamics Applied to Project Management: A Survey, Assessment, and Directions for Future Research." *System Dynamics Review* 23(2/3): 157-189.

Martínez, J. C., and P. G. Ioannou. 1999. "General-Purpose System for Effective Construction Simulation." *Journal of Construction Engineering and Management* 125(4): 265-276.

Motawa, I. A., C. J. Anumba, S. Lee, and F. Peña-Mora. 2007. "An Integrated System for Change Management in Construction." *Automation in Construction* 16(3): 368-377.

Mukherjee, A., and E. M. Rojas. 2003. "Reasoning about Actions and Events in Situational Simulations." In *Proceedings of the 2003 Winter Simulation Conference*, Edited by S. Chick, P. J. Sánchez, D. Ferrin, and D. J. Morrice, 2, 1552-1560, Piscataway, NJ: IEEE.

O'Brien, J. J., and F. L. Plotnick. 2009. *CPM in Construction Management.* 7th ed., New York, NY: McGraw-Hill.

Office of Naval Material. 1962. "Line of Balance Technology: A Graphic Method of Industrial Programming." Report NAVEXOS P1851 (Rev. 4-62), Washington, DC: Department of the Navy.

Puri, V., and J. C. Martínez. 2012. "Combined Discrete and Continuous Modeling of Construction Activities Using Three Phase Activity Scanning Approach." In *Proceedings of the 2012 Construction Research Congress*, Edited by H. Cai, A. A. Kandil, M. Hastak, and P. D. Dunston, 1192-1201, Reston, VA: ASCE.

Rojas, E. M., and A. Mukherjee. 2003. "Modeling the Construction Management Process to Support Situational Simulations." *Journal of Computing in Civil Engineering* 17(4): 272-280.

Rojas, E. M., and A. Mukherjee. 2005. "Interval Temporal Logic in General Purpose Situational Simulations." *Journal of Computing in Civil Engineering* 19(1): 83-93.

Sawhney, A., S. M. AbouRizk, and D. W. Halpin. 1998. "Construction Project Simulation Using CYCLONE." *Canadian Journal of Civil Engineering* 25(1): 16-25.

Sterman, J. D. 1992. "Systems Dynamics Modeling for Project Management." Whitepaper, Cambridge, MA: Sloan School of Business, Massachusetts Institute of Technology.

Taylor, T. R. B., and D. N. Ford. 2008. "Managing Tipping Point Dynamics in Complex Construction Projects." *Journal of Construction Engineering and Management* 134(6): 421-423.

Tebo, C., A. Mukherjee, and N. Onder. 2010. "A Multipurpose Simulation Platform for Decision-Making in Construction Management." In *Proceedings of the 2010 Winter Simulation Conference*, Edited by B. Johansson, S. Jain, J. Montoya-Torres, J. Hugan, and E. Yücesan, 3123-3131, Piscataway, NJ: IEEE.

Terry, S. B., and G. Lucko. 2012. "Algorithm for Time-Cost Tradeoff Analysis in Construction Projects by Aggregating Activity-Level Singularity Functions." In *Proceedings of the 2012 Construction Research Congress*, Edited by H. Cai, A. A. Kandil, M. Hastak, and P. D. Dunston, 226-235, Reston, VA: ASCE.

AUTHOR BIOGRAPHIES

GUNNAR LUCKO is Associate Professor of Civil Engineering and Director of the Construction Engineering and Management Program at Catholic University of America. His research interests are mathematical representation and analysis of schedules and their relation with simulation models, constructability, construction equipment operations and economics, optimization methods, and engineering education. His email is lucko@cua.edu and his website can be found at http://faculty.cua.edu/lucko.

AMLAN MUKHERJEE is Associate Professor in the Department of Civil and Environmental Engineering at Michigan Technological University. His research interests are artificial intelligence technologies, such as agent based modeling and temporal logic, construction engineering and management, interactive simulations, construction engineering education, and cognitive modeling of expert decision-making. His email is amlan@mtu.edu and his website can be found at http://www.cee.mtu.edu/~amlan.