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APPLYING A SPLITTING TECHNIQUE TO ESTIMATE ELECTRICAL GRID RELIABILITY

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ABSTRACT

As intermittent renewable energy penetrates electrical power grids more and more, assessing grid reliability is of increasing concern for grid operators. Monte Carlo simulation is a robust and popular technique to estimate indices for grid reliability, but the involved computational intensity may be too high for typical reliability analyses. We show that various reliability indices can be expressed as expectations depending on the rare event probability of a so-called power curtailment and explain how to extend a Crude Monte Carlo grid reliability analysis with an existing rare event splitting technique. The squared relative error of index estimators can be controlled, whereas orders of magnitude less workload is required than when an equivalent Crude Monte Carlo method is used.

1 MOTIVATION

Many modern societies have grown accustomed to a very reliable electricity supply by power transmission grids. However, substantial implementation of intermittent renewable generation, such as photovoltaic power or wind power, may threaten grid reliability. Power imbalances caused by generation intermittency may force grid operators to curtail power to ensure grid stability. As they are obliged to keep reliability at a prescribed level, grid operators must be able to perform a quantitative reliability analysis of the grid.

2 COMPUTATIONAL INTENSITY OF CRUDE MONTE CARLO

For quantitative assessment of grid reliability various reliability indices exist (Billinton and Li 1994). Most of them depend on the probability $\mathbb{P}(C)$, where C denotes the event of a power curtailment during the time interval $[0,T]$ of interest. We model the uncertain energy sources as stochastic processes, discretized in time. At each time step the mapping of these sources to the occurrence of a curtailment C requires solving a nonlinear algebraic system known as the power flow equations. Since this mapping is only implicitly defined, we can not derive $\mathbb{P}(C)$ directly, and we estimate it by a Monte Carlo simulation. That is, for each time step we sample the stochastic processes and solve the power flow equations. Then we check if nodal voltage values are between acceptable bounds and connection current values do not exceed an allowed maximum. We assume that a power curtailment will occur if at least one of these constraints is violated. Repeating this for all time steps yields one realization of C, and the average over many such realizations constitutes a Monte Carlo estimate for $P(C)$.

However, as power curtailments are undesirable, we may expect their occurrence to be rare. When T is equal to one week, values of $\mathbb{P}(C) \approx 10^{-4}$ or even much smaller are not uncommon (CEER 2011). Crude Monte Carlo (CMC) estimation for rare event probabilities requires a large number of samples to achieve a fixed accuracy level (Rubino and Tuffin 2009). Since one CMC sample already involves solving a large number of nonlinear systems, CMC estimation is computationally too intensive for general grid reliability analyses.

3 SOLUTION BY APPLYING A SPLITTING TECHNIQUE

To reduce the computational burden, we extend the CMC method with a rare event simulation technique called splitting (Garvels 2000, L'Ecuyer et al. 2006). Many reliability indices (e.g. probability, duration, number and severity of power curtailments) can be written as an expectation $\mathbb{E}[I]$. We decompose this expectation into

$$\mathbb{E}[I] = \mathbb{P}(C)\mathbb{E}\left[I|C\right],$$

a product of the rare event probability and the *conditional index*. The product $\hat{I} := \hat{P}\hat{I}_C$ of estimators for these two factors serves as an estimator for $\mathbb{E}[I]$. Using an appropriate splitting technique called Fixed Number of Successes (Amrein and Künsch 2011) we find an unbiased estimator $\hat{P}$ for $\mathbb{P}(C)$. Secondly, we perform one additional splitting stage by resampling from the rare event entrance state, yielding the unbiased estimator $\hat{I}_C$ for $\mathbb{E}\left[I|C\right]$. As $\hat{P}$ and $\hat{I}_C$ are independent, their product is an unbiased estimator for $\mathbb{E}[I]$.

To control the accuracy of the estimate one has to control the relative variance of $\hat{I}$, given by

$$\frac{\mathrm{Var}(\hat{I})}{\mathbb{E}^2[I]} = \frac{\mathrm{Var}(\hat{P})}{\mathbb{P}(C)^2} + \frac{\mathrm{Var}(\hat{I}_C)}{\mathbb{E}^2\left[I|C\right]} + \frac{\mathrm{Var}(\hat{P})}{\mathbb{P}(C)^2}\frac{\mathrm{Var}(\hat{I}_C)}{\mathbb{E}^2\left[I|C\right]}. \tag{1}$$

The first term on the right-hand side in (1) denotes the relative variance of the rare event probability estimate $\hat{P}$, which we control by fixing a sufficient number of hits per splitting level. The second term on the right-hand side in (1) denotes the relative variance of the conditional index estimator, which we control by splitting a sufficient number of times from the rare event entrance state. In this way, we control the left-hand side of (1), or in other words, we control the accuracy of the estimate.

4 RESULTS

We test the splitting technique on a small transmission network. We choose parameter values such that the network is indeed reliable ($\mathbb{P}(C) \approx 10^{-4}$). Wind energy is generated at one node of the network and is modeled by an Ornstein-Uhlenbeck process. We propose an importance function for the splitting technique that attains increasing values whenever the network state moves closer towards the constraint boundary. Fixing the accuracy such that we obtain 95% confidence intervals, we estimate $\mathbb{P}(C)$ as well as the expected duration, number and size of power curtailments during a week.

In total 3075 MC samples were required using the splitting technique. A CMC confidence interval for $\mathbb{P}(C)$ of comparable width is expected to require as much as 250 000 MC samples. This workload decrease of a factor 79 illustrates the computational gain of the splitting technique compared to the CMC method. For smaller values of $\mathbb{P}(C)$, this gain is expected to be even larger.

REFERENCES

Amrein, M., and H. R. Künsch. 2011. "A Variant of Importance Splitting for Rare Event Estimation: Fixed Number of Successes". *ACM Transactions on Modeling and Computer Simulation* 21 (2): 13.

Billinton, R., and W. Li. 1994. *Reliability Assessment of Electrical Power Systems using Monte Carlo Methods*. Springer.

CEER 2011. "5th CEER Benchmarking Report on the Quality of Electricity Supply 2011". Technical report, Council of European Energy Regulators. Also available at http://www.energy-regulators.eu/portal/page/portal/EER_HOME/CEER_5thBenchmarking_Report.pdf.

Garvels, M. 2000. *The Splitting Method in Rare Event Simulation*. Ph. D. thesis, Universiteit Twente.

L'Ecuyer, P., V. Demers, and B. Tuffin. 2006. "Splitting for Rare-Event Simulation". In *Proceedings of the Winter Simulation Conference 2006*, 137–148. IEEE.

Rubino, G., and B. Tuffin. 2009. *Rare Event Simulation using Monte Carlo Methods*. John Wiley & Sons.