

**OPTIMIZING LOCATIONS OF DECOYS FOR PROTECTING SURFACE-BASED RADAR
AGAINST ANTI-RADIATION MISSILE
WITH MULTI-OBJECTIVE RANKING AND SELECTION**

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ABSTRACT

This paper considers the decoy location problem, i.e., the problem of determining optimal locations for decoys that protect a surface-based radar against an anti-radiation missile. The objectives of the problem are to simultaneously maximize distances between the missile's detonation point and the radar as well as the decoys. The problem is solved using a stochastic simulation model providing the distances as well as a ranking and selection procedure called MOCBA-p. In the procedure, location combinations are evaluated through a multi-attribute utility function with incomplete preference information regarding weights related to the objectives. In addition, multi-objective computing budget allocation is used for allocating simulation replications such that the best combinations are selected correctly with high confidence. Numerical experiments presented in the paper illustrate the suitability of MOCBA-p for solving the decoy location problem. It provides computational advantages over an alternative procedure while also enabling ease of determining the weights.

1 INTRODUCTION

The surface-based radar is able to search and detect airborne targets from great distance in all kinds of weather conditions. It may be threatened by an anti-radiation missile (ARM) that operates passively by detecting the radiation transmitted by the radar and by guiding towards the radiation source. Although the radar is stationary, there exists a variety of measures to protect it from an attack such as (Fan, Ruilong, and Xiang 2001) the deployment of decoys to misguide the ARM, the shutting down of the transmitter of the radar upon detection of the launch of the ARM, and the use of means to neutralize the ARM or the platform carrying the missile. The surface-based radar is now assumed to be located on land although such a radar may also be deployed on, e.g., a ship. This paper considers the usage of the decoys by utilizing stochastic simulation and multi-objective ranking and selection (R&S).

The aim of using decoys is to lure an ARM to detonate in an area where it does not cause harm to the radar, and if possible, to the decoys either. The warhead of the missile is assumed to be small enough that the usage of decoys is practical, i.e., the missile is not able to destroy all the transmitters if they are not placed right next to each other. The effect of the fragments is omitted. The decoy itself is a transmitter that repeats the same waveform as the protected radar. The passive seeker of the ARM cannot separate the transmissions of the decoys from those of the radar based on, e.g., modulation, pulse width or carrier frequency. Therefore, the decoys can offer great protection against the threat posed by the ARM. A desirable feature of the decoys is that the radar can continue transmitting in order to provide surveillance

information. Another feature is that in the case of successful deceiving, the same decoys can be reused against a new ARM.

The effectiveness of decoys depends on their transmission power and location. If the transmission power is set too low compared to the side lobe level of the antenna of the radar, the decoys fail to lure the ARM. Setting the power too high makes the decoys vulnerable to the ARM. Similarly, regarding the locations, the decoys that are too far from the flight path of the ARM may not lure it whereas being too close makes the decoys vulnerable. Although it is better to sacrifice a single decoy instead of the radar, the best outcome is the one where both survive. In studies related to the usage of decoys (e.g., Emadi et al. 2008; Shi, Li, and Yuan 2006; Zhou et al. 2011), the locations of the decoys are usually assumed to be known or there are few possible locations but the decoys are not assigned to the locations in an optimal way. As the locations may considerably affect the outcome of survival, there is a need for an approach for determining the best possible locations. In reality, such planning is also affected by the geographical area which limits where the decoys can be placed.

In this paper, the decoy location problem of assigning a given number of decoys to a given number of alternative locations is considered. The objective of the problem is to select the location combinations that maximize distances from the ARM's detonation point to both the radar and the decoys. The larger the distance to a given decoy or radar, the greater is its likelihood of survival. The first priority is to protect the radar. In order to protect the radar successfully, some of the decoys may have to be sacrificed. The problem is solved by using a stochastic simulation model as well as a multi-objective R&S procedure called MOCBA-p presented in Mattila and Virtanen (2013). The simulation model describes the capability of the ARM's seeker to locate the radar. It provides the missile's detonation point that is used for calculating the distance objectives for a given location combination and a given initial state of the ARM. MOCBA-p is based on multi-objective computing budget allocation (MOCBA) (Lee et al. 2004; Lee et al. 2010; Chen and Lee 2010) which is designed for determining non-dominated designs of a multi-objective R&S problem. A given design is non-dominated if no other design has at least as good expected values for all objectives and better expected value for at least one objective. MOCBA maximizes the probability of correctly selecting non-dominated designs by allocating simulation replications, which are used for estimating the expected values of the objectives, among the designs through a set of allocation rules. In the MOCBA-p procedure, the distance objectives are aggregated with an additive multi-attribute utility function (Keeney and Raiffa 1976) describing a decision-maker's (DM) preference for the location combinations based on the values of the distance objectives. Weights of this function related to the relative importance of the objectives may be determined based on incomplete preference information elicited from the DM (e.g., Hannan 1981; White, Sage, and Dozono 1984; Kirkwood and Sarin 1985; Hazen 1986; Weber 1987). Thus, instead of unique weights, the DM gives constraints that imply a set of feasible weights. Location combinations referred to as pairwise non-dominated (Weber 1987) are determined based on the expected utilities of the combinations over the feasible weights. A combination is pairwise non-dominated when no other combination has at least as high expected utility for all feasible weights and higher for at least some weights. Because multiple different weights are considered, the R&S problem remains multi-objective despite the aggregation of the distance objectives. The conditions for dominance and pairwise dominance are similar. Therefore, MOCBA-p maximizes the probability of correctly selecting the pairwise non-dominated combinations by utilizing the allocating rules of MOCBA to allocate simulation replications among the location combinations. The pairwise non-dominated combinations returned by the procedure can be used by the DM when selecting the preferred combination that can be implemented in practice.

In multi-objective R&S procedures, objectives are typically aggregated using unique weights (e.g., Morrice, Butler, and Mullarkey 1998; Butler, Morrice, and Mullarkey 2001). Alternatively, in MOCBA, non-dominated designs based on the expected values of the objectives are determined. The advantage of using MOCBA-p in solving the decoy location problem is that it does not require unique weights; weighting the likelihood of survival for the radar against that for the decoys is a challenging task.

In numerical experiments presented in this paper, MOCBA-p is compared with MOCBA in solving the decoy location problem. Based on the results of these experiments, MOCBA-p has computational advantages because the number of pairwise non-dominated location combinations is generally smaller than the number of non-dominated combinations even with loose constraints on weights. The computational advantages also mean that the decoy location problem can be solved efficiently for a large number of alternative initial states of the ARM which facilitates the use of the simulation model and MOCBA-p for supporting decision-making on the deployment of the decoys.

The rest of the paper is organized as follows. Section 2 introduces the stochastic simulation model whose output is used for estimating the objectives of the decoy location problem. Section 3 describes how MOCBA-p is applied for solving the problem. In Section 4, numerical experiments illustrating the suitability of using MOCBA-p for the problem are presented. Concluding remarks are given in Section 5.

2 SIMULATION MODEL PROVIDING DISTANCE OBJECTIVES

The decoy location problem deals with a setting in which an ARM is launched towards a surface-based radar protected by decoys. The missile is equipped with a passive seeker that guides the missile towards the radar. In the region of interest, the seeker is assumed to make the decision of the radar's location at so called decision point after which it locks onto the radar and no longer updates the location. At the end of its flight, the missile has limited turning capability making large turns impossible. At the decision point, the seeker fixes the guidance according to the final line-of-sight (LOS) from the decision point to the estimated location of the radar. The location of the decision point is based on the seeker's capability to locate the radar as well as on the flight mechanical characteristics of the missile. The estimate for the final LOS is determined based on signals transmitted by the radar and the decoys. The missile detonates on the ground at the estimated target coordinates thus forming the required detonation coordinates for outcome evaluation.

In the stochastic simulation model used in the decoy location problem, it is assumed that a surface-based radar and decoys are placed on flat ground. Simulation inputs are the locations of the radar and the decoys as well as the location of the decision point. The model describes the capability of the seeker of an ARM to locate the radar at the decision point. The model does not take into account decisions made by the seeker prior to this point during its flight. Thus, the LOS is estimated at a specific moment at a certain distance from the radar. In the model, an estimate is calculated for the LOS from the decision point to the radar based on signals depending on the antenna gains and the transmission powers of the radar and the decoys. The output of the model, i.e., the location of the detonation point is determined solely based on the LOS estimate.

A final LOS estimate expressed in azimuth and elevation directions is determined in the simulation model via auxiliary LOS values from the decision point to each transmitter, i.e., either radar or decoy. A signal-to-noise ratio of the transmitter i , denoted by SNR^i , depends on the power of received transmissions S and noise N in the seeker. It is calculated through (Hannen 2013, Skolnik 2008)

$$SNR^i = \frac{P_i G_i (az_m, el_m, \beta^i) G_r \lambda^2 I}{(4\pi)^2 R^2 k T B F L}. \quad (1)$$

Here, $R = \sqrt{(x_i - x_{dec})^2 + (y_i - y_{dec})^2 + z_{dec}^2}$ is the Euclidean distance between the transmitter and the decision point where $(x_{dec}, y_{dec}, z_{dec})$ refers to the location of the decision point and (x_i, y_i) to the location of transmitter i . Furthermore, $\lambda = 0.1m$ is the wavelength, $I = 1$ (0dB) is the integration gain, $T = 290K$ is the noise temperature, $L = 1$ (0dB) defines the total losses, and k is the Boltzmann constant. The numerical values listed here are used in this paper. The parameters of the seeker are the antenna gain $G_r = 3$ (4.8dB), the effective bandwidth $B = 1MHz$ obtained through signal processing, and the noise figure $F = 30$ (14.8dB). The parameters of the transmitter, which describe a generic surface-based radar, include the peak power P_i and the antenna gain G_i . The peak power is 50kW for the radar and 4kW for the decoys, i.e., the decoys

are assumed to be identical. The antenna gain depends on the antenna bearing of the transmitter β^i as well as on the seeker's azimuth az_m and elevation el_m defined in the radar's spherical coordinates according to the model of a transmitter pattern (Schelkunoff 1943). The main lobe gains of the radar in azimuth and elevation direction both equal 1585 (32dB). The gains for the decoys are 2 (3.0dB) and 1.5 (1.8dB). The decoys have an omnidirectional antenna so their bearings and gains remain constant. The bearing of the radar is a random value drawn from the uniform distribution in the range $[0, 2\pi]$. Thus, the bearing is the first source of uncertainty in the simulation model.

For transmitter i , the total error of an auxiliary LOS value is characterized by a random measurement error dependent on SNR^i , a fixed random error, and a bias error. The SNR dependent error is the most notable (Curry 2005) and the only one considered in the simulation model. Then, the noisy LOS value for transmitter i , denoted by LOS^i , is

$$LOS^i = LOS_{true}^i + n, \quad (2)$$

where LOS_{true}^i is the noiseless LOS value, which is based on geometry and orientation of the missile, from the decision point to the location of transmitter i and n is a random value drawn from the normal distribution with zero mean and standard deviation of the measurement error s_n^i which is the second source of uncertainty in the simulation model. The measurement error depending on SNR^i is (Curry 2005)

$$s_n^i = \frac{\theta}{k_M \sqrt{2SNR^i}}, \quad (3)$$

where $\theta = 36deg$ is the angular resolution (-3 dB beamwidth), $k_M = 1.6$ is a constant factor, and SNR^i is calculated with Equation (1).

The LOS estimate denoted by LOS_{est} representing the result of the estimation process of the seeker is the sum of all noisy LOS values weighted by the signal-to-noise ratios, i.e.,

$$LOS_{est} = \frac{\sum_i LOS^i \cdot SNR^i}{\sum_i SNR^i}. \quad (4)$$

Therefore, the more the radar provides power toward the seeker, the more it contributes to the LOS estimate.

The missile is assumed to be able to hit to the detonation point with no deviation. Therefore, the resulting LOS estimate is converted into the location of the detonation point, denoted by (x_{det}, y_{det}) , by projecting its azimuth and elevation to the ground. Thus, the output of the simulation model in terms of the location of the detonation point is obtained.

The logic of the simulation model is summarized with the following steps:

1. Set the locations of the radar and the decoys as well as of the decision point based on knowledge of selected missile.
2. Calculate the noiseless LOS values from the decision point to each transmitter.
3. Determine the noisy LOS values corresponding to each transmitter using Equations (1-3).
4. Calculate the LOS estimate through Equation (4).
5. Project the LOS estimate to the ground.

The simulation model offers a straightforward and easy to understand way to simulate the capability of a passive seeker to locate a transmitting surface-based radar. Although simple, the simulation model is a transparent representation for estimating the LOS based on models presented earlier in the literature. When solving the decoy location problem with MOCBA-p, estimates for the distance objectives of the problem are required. These estimates are obtained by performing multiple replications of the above simulation for each location combination that is allocated replications by MOCBA-p.

3 APPLYING MOCBA-P TO THE DECOY LOCATION PROBLEM

In this section, the application of the MOCBA-p procedure (Mattila and Virtanen 2013) in solving the decoy location problem is described. First, let us formally state the problem at hand. Here, the general case of $n - 1$ decoys is discussed. Thus, the number of location combinations for decoys, denoted with K , can be large. In addition, let $\mathbf{X}_k = (X_{k1}, \dots, X_{kn})$ denote random variables representing distances from the detonation point to the radar and the decoys for location combination k . The distances are calculated using the locations of the detonation point, the radar and the decoys available from the output and input of the simulation model. Here, X_{k1} corresponds to the distance to the radar. Furthermore, X_{k2}, \dots, X_{kn} are the distances to the $n - 1$ decoys sorted in ascending order. The sorted distances are considered in order to make the weighting of the objectives meaningful when applying MOCBA-p. Because the decoys are identical, there would be no reason to give non-identical weights to the distances related to given decoys. However, weights related to the sorted distances may, e.g., be interpreted such that the decoy least likely to survive is given more weight relative to the other decoys.

The multi-objective R&S problem of determining the optimal locations of the decoys involves the simultaneous maximization of the expected distances $E[X_{ki}]$, i.e.,

$$\max_{k \in \{1, \dots, K\}} (E[X_{k1}], \dots, E[X_{kn}]). \quad (5)$$

Moreover, the nature of the R&S problem is that the computing budget in terms of the number of simulation replications available for estimating the expected distances is limited. Thus, the replications must be allocated to the location combinations such that the best combinations are selected correctly with the highest possible confidence.

With MOCBA, the best ones refer to the non-dominated combinations. When using MOCBA-p, the distance objectives are aggregated with an additive multi-attribute utility (MAU) function that incorporates incomplete preference information through a set of feasible weights related to the objectives. Then, non-dominated location combinations with respect to the expected utilities of the combinations over the feasible weights are selected, i.e., pairwise non-dominated combinations are obtained.

3.1 MAU Function for Aggregating Distance Objectives

With an additive MAU function, the utility reflecting the preference of a DM for a given location combination is calculated as the weighted sum of single-attribute utility functions describing the DM's preference for the values of distances X_{k1}, \dots, X_{kn} . The additive MAU function is of the form (Keeney and Raiffa 1976)

$$U(\mathbf{X}_k) = \sum_{i=1}^n w_i u_i(X_{ki}). \quad (6)$$

Here, the single-attribute utility functions are denoted with $u_i, i = 1, \dots, n$ and they take on values in the range $[0, 1]$. Weights $w_i \in [0, 1], i = 1, \dots, n, \sum_{i=1}^n w_i = 1$, reflect the relative importance of the objectives by indicating the contribution of each distance into the utility. The additive MAU function is an appropriate way to describe the DM's preferences when a condition called additive independence of the distances X_{k1}, \dots, X_{kn} holds. This condition relates to preferential interaction between the attributes of a decision situation which, if present, must be handled by using an MAU function of another form. Different MAU functions are discussed in detail in, e.g., von Winterfeldt and Edwards (1986), but their description is omitted here for brevity. In the decoy location problem, it suffices to note that there is no apparent reason that the DM's preference for a given distance would depend on other distances. Therefore, additive independence is assumed to hold and an additive MAU function is used.

In the decoy location problem, the same single-attribute utility function, denoted by u , is used for all distances because they essentially reflect the likelihood that the radar or the decoy is destructed by the missile. For the construction, the DM must provide preference statements regarding different values

of the distance. The least desired value is $x_* = 0$ for obvious reasons. The most desired value x^* can be determined based on a distance beyond which the missile can be assumed to have no impact on the radar or the decoy. These values are assigned single-attribute utilities of 0 and 1, i.e., $u(x_{i*}) = 0$ and $u(x_i^*) = 1$. For values between x_* and x^* , the radius around the detonation point in which the missile is able to destruct the radar or the decoy can be utilized. Here, it is assumed that u increases slowly between the least desired value x_* and an approximate lower bound of the radius denoted with r_l because the likelihood of survival within the radius remains small. Then, u increases sharply between r_l and an approximate upper bound r_u for the radius because leaving the radius increases the likelihood of survival. Finally, u is assumed to increase slowly between r_u and the most desired value x^* . The values of r_l and r_u depend on the type of missile under consideration. In this paper, values determined by the authors are used for illustrative purposes. Whereas the construction of u now relies on curve drawing, formal techniques for the construction are also available. They are discussed in, e.g., von Winterfeldt and Edwards (1986). In addition, damage functions (Jaiswal 1997) representing the areas of impact for different weapons could be utilized when constructing single-attribute utility functions.

Weights of the MAU function are also determined based on preference statements given by the DM concerning, e.g., the ratios $w_i/w_j, i \neq j$. Techniques for this task are discussed in, e.g., Keeney and Raiffa (1976), von Winterfeldt and Edwards (1986). In this paper, it is assumed that information regarding the weights is incomplete. Then, the DM gives preference statements that imply linear constraints on the weights. For instance, the DM may determine intervals for the ratios of the weights. The constraints and the requirement that the weights sum up to one define the set of feasible weights. Detailed discussion concerning how to determine the weights based on incomplete preference information appears in, e.g., Park and Kim (1997).

3.2 Determining Pairwise Non-Dominated Location Combinations

In MOCBA-p, location combinations are compared based on expected utilities over feasible weights. Clearly, a combination is preferred to another one, if the former has higher expected utility with all feasible weights (Weber 1987). Such a relation is referred to as pairwise dominance. Formally, combination k dominates combination l according to the pairwise dominance, if $E[U_k(\mathbf{w})] \geq E[U_l(\mathbf{w})] \forall \mathbf{w} \in W$ and if the inequality is strict for at least one \mathbf{w} . Here, $E[U_i(\mathbf{w})]$ denotes the expected utility of location combination i with weights included in vector \mathbf{w} and W denotes the set of feasible weights. When applying MOCBA-p, the expected utilities of the location combinations are estimated based on the output of the simulation model discussed in Section 2.

Pairwise non-dominated location combinations are obtained in MOCBA-p using a set of allocation rules that provide the optimal share of the computing budget for each combination based on estimates for expected utilities as well as for variances of the utilities. The rules are applied in each iteration of the procedure for allocating a fraction of the computing budget, i.e., a given number of simulation replications, to the combinations. Then, the allocated replications are performed with the simulation model and the estimates are updated. Iterations are carried out until the entire computing budget has been consumed. Ultimately, the pairwise non-dominated combinations are selected according to the final estimates of the expected utilities. Following the procedure, the probability of correctly selecting these combinations is maximized. Detailed description of the procedure is given in Mattila and Virtanen (2013).

MOCBA-p applies, as the name of the procedure suggests, the allocation rules of the MOCBA procedure (Chen and Lee 2010) when allocating simulation replications in each iteration. The number of allocated replications for a given location combination depends on the degree of uncertainty about the pairwise dominance relations in which this combination is involved. For instance, for a combination regarded as pairwise non-dominated, the number of replications depends on the numbers for the combinations that the non-dominated one dominates most likely. Allocating the simulation replications in this manner allows to reduce the uncertainty about the dominance relations which leads to the maximization of the probability of correct selection. In MOCBA-p, the allocation rules of MOCBA can be applied because of the similarity of

the conditions for pairwise dominance and dominance considered through expected values of the objectives in MOCBA. That is, combination k dominates combination l if $E[X_{ki}] \geq E[X_{li}] \forall i = 1, \dots, n$ and at least one of the inequalities is strict. Details of the justification for the use of the rules as well as the description of the rules are omitted in this paper for brevity but they are available in Mattila and Virtanen (2013).

4 NUMERICAL EXPERIMENTS

This section discusses numerical experiments conducted to assess the suitability of MOCBA-p for solving the decoy location problem. First, a problem with 2 decoys and a fixed decision point of an ARM is solved in order to illustrate that MOCBA-p gives the true pairwise non-dominated location combinations and provides computational advantages over an alternative procedure, i.e., MOCBA. Second, problems with 3, 4, and 5 decoys are solved. In these problems, the decision point is randomly determined from a sector describing the anticipated direction of approach of the ARM. These problems represent a more versatile setting in which the number of location combinations is also larger.

In all problems, the radar is placed in the origin, i.e., its coordinates in x- and y-axis are (0,0). There are 10 possible locations where the decoys can be placed. With 10 locations, there are a sufficient number of location combinations for the illustration of the MOCBA-p procedure. Each of these locations is determined by drawing its coordinates in x- and y-axis from uniform distributions on the intervals $[-500, 500]$ and $[0, 500]$, respectively. The unit of measurement for the coordinates is meters. The coordinates of the fixed decision point of the ARM in the problem with 2 decoys are (0,800,200). In the other problems, the distance of the decision point from the radar is unchanged but its direction from the radar in terms of the azimuth angle from the x-axis is drawn from the normal distribution with mean $\pi/2$ and standard deviation $\pi/8$. The possible locations for the decoys are presented in Figure 1. In addition, Figure 1 depicts the radar, the fixed decision point as well as example detonation points in 100 replications of the simulation model with a given location combination for 2 decoys.

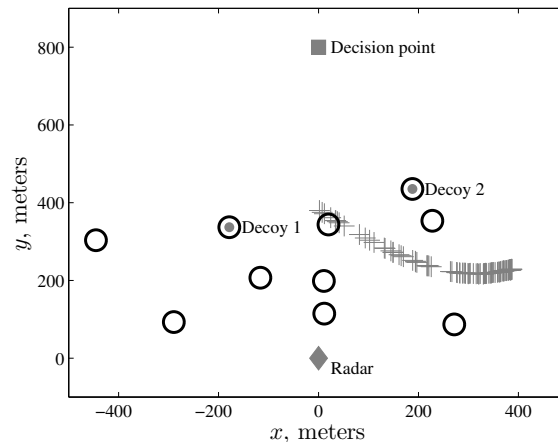


Figure 1: The possible locations for the decoys, the radar, and the decision point are marked with circles, a diamond and a square. Filled circles represent the locations where a decoy is placed. The detonation point corresponding to one simulation replication is presented with a plus sign.

In the solution of all decoy location problems considered in this section, the single-attribute utility function described in Section 3.1 is used. Weights of the MAU function are discussed with each problem separately. Parameters of MOCBA-p include the number of replications allocated per iteration and the maximum number of replications allocated to an individual location combination per iteration. These are both set to 100. The number of replications allocated initially to each combination is also set to 100. The computing budget in all problems, i.e., with 2, 3, 4, and 5 decoys, is 100,000 simulation replications which

appeared to provide consistently the same pairwise non-dominated combinations in the tests performed by the authors.

4.1 Optimal Location Combinations for 2 Decoys

With 2 decoys and 10 possible locations, the number of location combinations is 45. The set of feasible weights of the MAU function is determined based on the assumption that the radar is regarded equally or more important than any of the decoys. The weight related to the distance between the detonation point of the ARM and the radar is greater than or equal to all other weights, i.e., $w_1 \geq w_i, i = 2, 3$. In addition, the smallest distance between the detonation point and the decoys is considered at least as important as the second smallest distance. Thus, the feasible weights must also satisfy $w_2 \geq w_3$. No further information regarding the weights is given.

The performance of MOCBA-p is evaluated by utilizing the true set of pairwise non-dominated location combinations as well as MOCBA as an alternative reference procedure. The true pairwise non-dominated set can be determined by simulating each location combination with a large number of replications because the number of combinations is relatively small. Here, 100,000 replications were performed for each combination. The pairwise non-dominated combinations selected based on the resulting estimates for the expected utilities of the combinations over the feasible weights are treated as the true pairwise non-dominated set. MOCBA selects non-dominated location combinations. In order to compare it with MOCBA-p, the MAU function applied in MOCBA-p is also utilized for the posterior comparison of the non-dominated location combinations.

Figure 2 depicts the probabilities of correct selection for MOCBA-p and MOCBA as a function of the computing budget. These probabilities include the ones for pairwise non-dominated location combinations when applying MOCBA-p as well as for pairwise dominance and dominance when applying MOCBA. The probabilities are estimated by solving the problem at hand 100 times and by calculating the fraction of cases in which the correct selection of the pairwise non-dominated combinations and the non-dominated combinations are made.

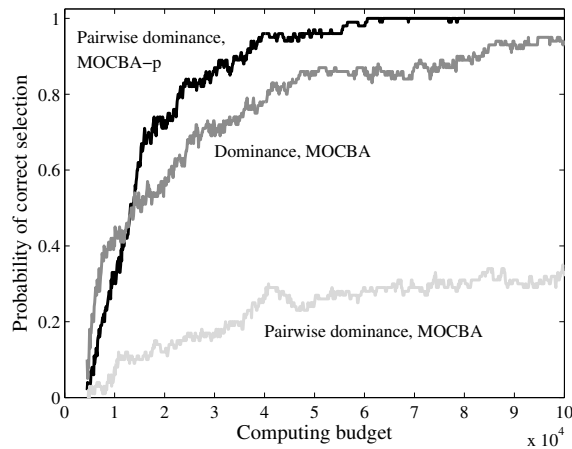


Figure 2: The probabilities of correct selection as a function of the computing budget. The black line depicts the probabilities for pairwise dominance using MOCBA-p and the dark gray line for the same probabilities using first MOCBA and then the same MAU function as the one in MOCBA-p. The light gray line depicts the probabilities for dominance with MOCBA.

The probabilities for pairwise dominance with MOCBA-p are mostly higher than the ones for dominance with MOCBA. Thus, with MOCBA-p, the location combinations of interest are selected correctly with a higher level of confidence. In addition, the advantage of MOCBA-p is that the number of pairwise

non-dominated combinations obtained is 4 whereas MOCBA returns 10 non-dominated combinations. It is obvious that the DM's task of comparing the remaining combinations posterior to the simulations is considerably easier when using MOCBA-p. Compared with MOCBA, utilizing preference information in MOCBA-p allows the DM to reduce the number of remaining combinations by over a half with the same computing budget.

Next, the probability of correct selection for pairwise dominance with MOCBA is considered. This probability reflects the feasibility of the approach where preference information is incorporated posterior to the simulations when determining non-dominated location combinations. As indicated by Figure 2, these probabilities remain low even with the increasing computing budget. Because MOCBA attempts to make sure that the non-dominated combinations are selected correctly, some non-dominated combinations for which dominance is evident are allocated only few replications. Thus, these combinations may not be compared with a high level of confidence with each other. Therefore, besides selecting fewer combinations, MOCBA-p selects them correctly with a higher probability.

The average number of simulation replications allocated to each location combination with MOCBA-p and MOCBA are depicted in Figure 3. In terms of the pairwise non-dominated combinations, i.e., 23, 34, 41, and 45, the largest number of replications is allocated to combination 34 in MOCBA-p. This is due to uncertainty related to the pairwise dominance of the combination. For the other pairwise non-dominated combinations, uncertainties and thus the average number of replications are smaller. In addition, several replications are allocated to some of the pairwise dominated combinations in order to maximize the probability of correct selection.

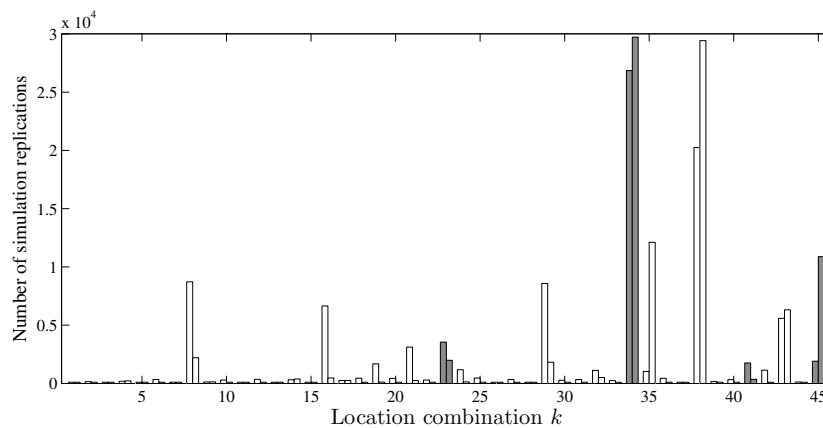


Figure 3: The average numbers of replications allocated to each location combination with MOCBA-p and MOCBA. The bars on the left for each combination represent the replications for MOCBA-p and the ones on the right for MOCBA. The pairwise non-dominated combinations are depicted in gray and the pairwise dominated combinations in white.

With MOCBA, the average number of simulation replications allocated to the pairwise non-dominated combinations turns out to be fairly similar as the one with MOCBA-p. Moreover, combinations 34 and 45 are allocated more replications on average compared with MOCBA-p. However, with MOCBA, some of the pairwise dominated combinations are allocated only few replications. These combinations cannot be compared with the pairwise non-dominated ones with a high degree of confidence which is also implied by the low probabilities of correct selection for pairwise dominance using MOCBA. Therefore, MOCBA-p facilitates the posterior comparison of the combinations better than MOCBA.

4.2 Optimal Location Combinations for 3, 4, and 5 Decoys

With 3, 4, and 5 decoys and 10 possible locations, the number of location combinations are 120, 210, and 252. Weights of the MAU function are determined similarly to the problem with 2 decoys. The radar is considered most important and smaller distances to the decoys are assigned larger weights. Thus, the sets of feasible weights are given by the constraints $w_1 \geq w_2 \geq \dots \geq w_n$.

Pairwise non-dominated as well as non-dominated location combinations for the problems are determined using MOCBA-p and MOCBA. MOCBA-p obtains 7, 9, and 9 pairwise non-dominated combinations with 3, 4, and 5 decoys, respectively. MOCBA, in turn, gives 42, 99, and 164 non-dominated combinations which clearly illustrates the advantage of incorporating preference information with MOCBA-p. There are sufficiently few pairwise non-dominated combinations such that the DM may compare them intuitively whereas the number of the non-dominated combinations does not allow such comparison without screening out some of the combinations.

Figure 4 depicts the number of times that each location appears in a pairwise non-dominated location combination for the problems. The locations with the largest number of appearances can be regarded as critical ones and the decoys should at least be assigned to them. For instance, the location with the largest y-coordinate is assigned a decoy in all pairwise non-dominated combinations in the problems with 4 and 5 decoys and in all but one pairwise non-dominated combination in the problem with 3 decoys. Thus, this location should at least be selected. In turn, the locations with few or none appearances should obviously not be assigned a decoy. Overall, the locations furthest away from the radar, both in the direction of x- and y-axis, appear in several pairwise non-dominated combinations. This information as well as, e.g., the values of the distance objectives for the pairwise non-dominated combinations can be utilized by the DM for selecting the preferred combination that is used in practice.

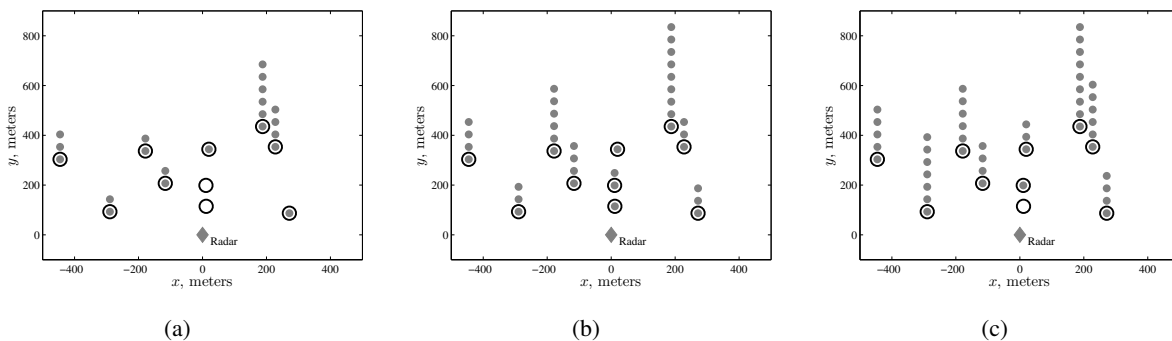


Figure 4: The number of times each location appears in a pairwise non-dominated location combination in the problems with (a) 3, (b) 4, and (c) 5 decoys. The appearances are depicted with the filled circles. The black circles denote the possible locations.

5 CONCLUSIONS

This paper considered the decoy location problem in which the locations of decoys are selected in order to protect a surface-based radar from the threat of an ARM. The problem was solved using a stochastic simulation model and an R&S procedure called MOCBA-p. Besides the analyses presented in this paper, the simulation model and MOCBA-p can be utilized in other ways as well. For instance, location combinations can be evaluated with respect to the threat of several ARMs such that the performance of decoys with some of them eliminated is taken into account. Another possibility is to analyze the decoy location problem inversely such that feasible decision points of an ARM are established with respect to the given geometry of the radar and the decoys.

There are extensions that would allow a more elaborate consideration of the decoy location problem. Possible locations for decoys could be determined based on characteristics of terrain. Then, certain directions may fall in shadow region for some decoys because the terrain blocks transmissions. The transmission power of individual decoys can also be taken into account in an R&S setting. Then, alternative designs represented by location combinations in this paper would include the combinations of both location and transmission power. Finally, the simulation model could, e.g., describe in more detail an ARM whose operation depends on several technical features. These extensions work towards increasing the computational requirements of solving the decoy location problem. However, MOCBA-p represents a viable choice for handling these requirements.

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