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# A SIMHEURISTIC ALGORITHM FOR THE LOCATION ROUTING PROBLEM WITH FACILITY SIZING DECISIONS AND STOCHASTIC DEMANDS

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## ABSTRACT

Location routing is a well known problem in which decisions about facility location and vehicle routing must be made. Traditionally, a fixed size or capacity is assigned to an open facility as the input parameter to the problem. However, real-world cases show that decision-makers usually have a set of size options. If this size is selected accurately according to the demand of allocated customers, then location decisions and routing activities would raise smaller cost. Nevertheless, choosing this size implies additional variables that make an already *NP-hard* problem even more challenging. In addition, considering stochastic demands contributes to making the optimization problem more difficult to solve. Hence, a simheuristic algorithm is proposed in this work. It combines the efficiency of metaheuristics and the capabilities of simulation to deal with uncertainty. A series of computational experiments show that our approach can efficiently deal with medium-large instances.

## 1 INTRODUCTION

The Location Routing Problem (LRP) is one of the most complete problems in logistics optimization, since it includes all decision levels, i.e., strategic, tactical, and operational. From an Operational Research perspective, it can be seen as the combination of the Facility Location Problem (FLP) and the Vehicle Routing Problem (VRP), which are both *NP-hard* problems. Hence, the LRP is also *NP-Hard*, and heuristic approaches are required for solving medium- and large-sized instances. Due to its complexity, the first reported studies on the LRP tackled it by separating the corresponding sub-problems (Salhi and Rand 1989; Nagy and Salhi 2007). As expected, this approach led to sub-optimal solutions. More recently, given the increase in computational power and the development of non-exact approaches, such as heuristic and metaheuristic algorithms, the LRP has been studied in an integrated way, which has clearly improved the obtained results (Prodhon and Prins 2014). The LRP has been used to support decision-making processes related to supply chain network design (Lashine et al. 2006), humanitarian logistics (Ukkusuri and Yushimito 2008), horizontal cooperation (Quintero-Araujo et al. 2019), and city logistics (Nataraj et al. 2019), among others. One of the most studied versions of the LRP is the Capacitated LRP, in which both depot and vehicle capacity constraints must be satisfied (the acronym LRP will henceforth refer to this problem). However, all previous works consider the depot capacity as a fixed value. This could not be a suitable approach when dealing with realistic problems, since it is usual that decision-makers can select the size of a facility from a discrete set of known available sizes, or even freely. For real-world problems, this set is usually associated with investment activities, such as building facilities (Zhou et al. 2019), purchasing equipment (Tordecilla-Madera et al. 2017), or qualifying workforce (Correia and Melo 2016). From an academic point of view, the consideration of flexible sizes in the facilities has been rarely addressed in the literature.

Nevertheless, real-life examples from both LRP (Zhou et al. 2019; Hemmelmayr et al. 2017; Tunalıoğlu et al. 2016) and non-LRP (Tordecilla-Madera et al. 2017; Correia and Melo 2016) contexts show the relevance of considering a variety of facility sizes to select from – instead of just a single predefined size, as it is the case in most LRP studies. These problems consider that parameters are deterministic, i.e., they assume that inputs are known in advance. This assumption is far from reality in many applications, such as waste collection or humanitarian logistics. The LRP literature addressing stochastic parameters is still scarce. Most found research hybridizes simulation with a heuristic or metaheuristic to tackle efficiently both uncertainty and *NP-hardness*. For example, Quintero-Araujo et al. (2020) propose a simheuristic to solve an LRP with stochastic demands. They hybridize Monte Carlo simulation with an iterated local search metaheuristic. A set of benchmark instances are used to test the proposed approach. Rabbani et al. (2019) also propose a simheuristic approach that combines a non-dominated sorting genetic algorithm-II (NSGA-II) and Monte Carlo simulation. They address a multi-objective multi-period LRP in the context of the hazardous waste management industry. Both generated waste and number of people at risk are stochastic. Inventory decisions are also taken into account.

The customers' demand is one of the most addressed stochastic parameters. For instance, Sun et al. (2019) address a real-world case from an express delivery company in Shanghai. Authors tackle an LRP in which demand can be split for self-pickup. Then, a simulation-based optimization model is proposed and two heuristics' results are compared. Mehrjerdi and Nadizadeh (2013) present a fuzzy-chance-constrained programming model where demands are modeled as fuzzy numbers in the context of an LRP. A four-phase method called "greedy clustering" is proposed. In their method, both an ant colony system metaheuristic and stochastic simulation are included. Additionally, the emergence of new technologies introduces new challenges. This is the case in the work by Zhang et al. (2019), who address the problem of locating battery swap stations and routing electric vehicles with stochastic demands. This problem is solved by a hybrid approach combining a variable neighborhood search with a binary particle swarm optimization algorithm. The problem's complexity increases when considering the low autonomy of this type of vehicles, since route failures can frequently be present when demands are not known in an accurate manner.

Other parameters are also considered as uncertain. For instance, Herazo-Padilla et al. (2015) hybridize an ant colony optimization metaheuristic with discrete event simulation to solve an LRP in which both transportation cost and vehicle travel speeds are considered stochastic. Authors demonstrate that their proposed approach is not only efficient, but able to find statistical interactions among the different parameters. Zhang et al. (2018) present an approach that hybridizes a genetic algorithm with simulation to solve a sustainable multi-objective LRP in the context of emergency logistics. The authors consider the travel distance, the demand, and the cost of opening a depot as uncertain variables. All these stochastic-LRP papers assume that the capacity to install is not flexible, but just a predefined quantity. To the best of our knowledge, there are no publications addressing a stochastic LRP considering facility-sizing decisions. Hence, the main contributions of this work are: *(i)* a new variant known as the Location Routing Problem with Facility Sizing Decisions and Stochastic Demands and, *(ii)* a competitive simulation-optimization approach to solve the aforementioned problem. The remainder of this paper is organized as follows: Section 2 presents a description of the problem; Section 3 explains the proposed simheuristic approach; Section 4 describes a series of computational experiments and analyzes the obtained results; and, finally, Section 5 draws conclusions and gives future research perspectives.

## 2 PROBLEM DESCRIPTION

The Location Routing Problem with Facility Sizing Decisions and Stochastic Demands consists in: *(i)* opening one or more depots (facilities) of different sizes; and *(ii)* designing, for each open depot, a number of routes whose aggregated customers' demand does not exceed its capacity. Each route must start and finish at the same depot. As demands are stochastic, a percentage of the vehicles' capacity is reserved as a safety stock (*SS*), in case the demand is higher than expected. Therefore, the main decision variables of this problem are related to the number of facilities to open, their locations, the size to be installed for each facility, the allocation of customers to each open depot, the number of vehicles to be used, and how to design the associated routes. This problem is *NP-hard* since it contains, as special cases, the Capacitated Vehicle Routing Problem (CVRP), the Multi-Depot VRP (MDVRP), and the Facility Location Problem (LRP), all of them known to be computationally hard. Figure 1 provides an example of a complete LRP solution where facilities are represented by squares and customers by circles. The grey squares correspond to non-open facilities, while black squares are the open ones. For each open depot, a set of routes starting and finishing at the corresponding depot location is designed to serve all customers' demands.

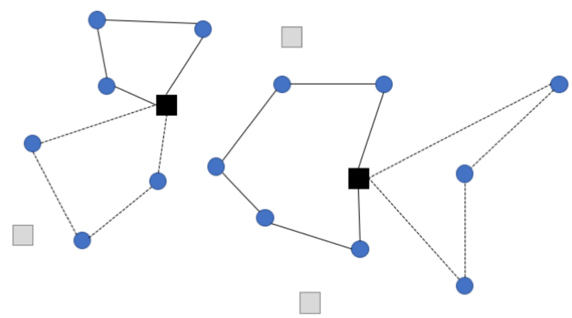


Figure 1: Graphical representation of an LRP solution.

Formally speaking, the LRP can be defined on a complete, weighted, and undirected graph $G(V,E,C)$, in which $V$ is the set of nodes (comprising the subset $J$ of potential depot locations and subset $I$ of customers), $E$ is the set of edges, and $C$ is the cost matrix of traversing each edge. A set $K$ of unlimited homogeneous vehicles with capacity constraints is available to perform the routes. Moreover, it is assumed that all vehicles are shared by all depots (i.e., no depot has a specific fleet) and each edge $e \in E$ satisfies the triangle inequality. The customers' demands are stochastic and follow a known probability distribution. The version studied in this paper considers that the size of the facilities to open is a decision to be made. To achieve this, a set of alternative sizes for each depot and associated opening cost are provided as inputs. Depots might have equal or different capacities among them. Each customer must be serviced from the depot to which it has been allocated by a single vehicle, i.e., split deliveries are not allowed.

The objective is to minimize the expected total cost ($TC$), which include the cost of opening depots ($OC$), the routing cost ($RC$), and the stochastic cost ($SC$), i.e., $TC = OC + RC + SC$. The latter cost incur each time a route fails, i.e., each time that the realized demand of a route is greater than the vehicle capacity. In this case, two different types of cost are calculated, depending on the corrective action considered: *(i)* a reactive strategy with cost $c_{reac}$, in which a vehicle must perform a round-trip to the depot for a replenishment in the occurrence of a route failure; and *(ii)* a preventive strategy with cost $c_{prev}$, in which a vehicle performs a detour to the depot before visiting the next customer, if the expected current non-served demand is higher than the vehicle's current load. Then, the expected stochastic cost are computed as $SC = \min\{c_{reac}, c_{prev}\}$.

## 3 SOLVING APPROACH

We propose a simheuristic approach (Juan et al. 2018) for minimizing the expected total cost. Simheuristics have been recently used to solve optimization problems with stochastic components, such as Arc Routing Problems with Stochastic Demands (Gonzalez-Martin et al. 2018) or Stochastic Waste Collection Problems (Gruler et al. 2017). In particular, our methodology combines an Iterated Local Search (ILS) metaheuristic with Monte Carlo simulation to deal with the stochastic nature of the problem. As discussed in Grasas et al. (2016) and Ferone et al. (2019), the ILS and GRASP metaheuristic frameworks offer a well-balanced combination of efficiency and relative simplicity, and can be easily extended to a simheuristic. Figure 2 depicts the main characteristics of our approach, composed of three stages. During the first stage, a set of promising facility-location maps are generated using a constructive heuristic, which employs biased-randomization techniques (Ferrer et al. 2016). The main input parameters of our model are the number of customers and potential depots, their locations in Cartesian coordinates, the vehicle capacity, the available sizes for each potential depot, the customers' demands with known mean and standard deviation, and the fixed and variable cost of opening a depot. The algorithm starts computing the minimum and maximum number of required depots, based on both the total demand and the maximum and minimum available sizes. Next, it randomly selects both the number of depots to be opened – which is a value between the range of values previously computed – and the size of each depot. This size is selected from a discrete set of known available sizes. Subsequently, the customers are assigned to a specific depot. This allocation is performed using a marginal-savings criterion proposed by Juan et al. (2015). Broadly speaking, this criterion computes the savings of assigning a customer $i \in I$ to an open facility $j \in J$ with respect to assigning $i$ to the best alternative facility $j^* \in J$. Finally, when all customers have been allocated to a given facility, a modified version of the savings heuristic described by Belloso et al. (2019) is applied to generate an initial routing plan. This heuristic is based on biased-randomization techniques. As discussed in Dominguez et al. (2016), these techniques refer to the use of skewed probability distributions to induce an 'oriented' (non-uniform) random behavior. This process permits transforming a deterministic heuristic into a probabilistic algorithm, while still preserving the logic behind the heuristic. This complete procedure is repeated until the maximum computation time for this phase is reached. Finally, all maps are evaluated according to the total deterministic cost, formed by the opening cost and the routing cost. Then, the most "promising" maps obtained in this first stage are sent to the second one. The number of most "promising" maps depends on how much time is available to perform a broader exploration. Our algorithm stores the two maps with the lowest deterministic cost.

During the second stage, an initial short number of Monte Carlo simulation runs is carried out to estimate a safety stock. This decision variable refers to a percentage of the vehicle capacity that is reserved to respond more effectively to the random demand. Then, the ILS metaheuristic improves the set of "promising" maps by iteratively exploring the search space and conducting a second process of simulation runs. This procedure is based on: *(i)* perturbing the current solution to obtain a new starting point; and *(ii)* exploring the neighborhood of this new solution using a local search. As perturbation methods, we have used two different strategies. In the first one, the algorithm randomly selects a set of customers and tries to reassign them in a random way to another facility without violating its capacity. Regarding the second strategy, the algorithm randomly exchanges the allocation of a percentage of customers among facilities. This starts with 50 %, and it is successively increased in each new iteration of the algorithm to explore different neighborhood sizes. The strategy to be used in each iteration of the algorithm is randomly selected. Following this perturbation process, the algorithm starts a local search around the newly generated solution in order to improve it. As local search we have used a *two-opt inter-route* operator, which interchanges two chains of randomly selected customers between different facilities. This operator is applied until it cannot be further improved. After this local search, we quickly assess the obtained solution under stochastic conditions by employing a short number of Monte Carlo simulation runs. This allows us to generate rough estimates of the solution performance under stochastic conditions, which also enables for identifying a pool of 'elite' solutions. Whenever a new solution outperforms the current base solution of the iterated local search, the latter is updated with the former and added to the pool of elite solutions. Notice that the Monte Carlo simulation does not only provide estimates to the expected cost associated with the solutions generated by our approach, but it also reports feedback to the metaheuristic search process. In order to further diversify the search, the algorithm might occasionally accept non-improving solutions following an acceptance criterion. The process is repeated until the stopping criterion of this stage is met. Finally, in the third stage a refinement procedure using a larger number of simulation runs is applied to the elite solutions. This enables to obtain a more accurate estimation of the expected total cost as well as other statistics, e.g., solution reliability, variance, etc. Particularly, the main output variables in our experiments

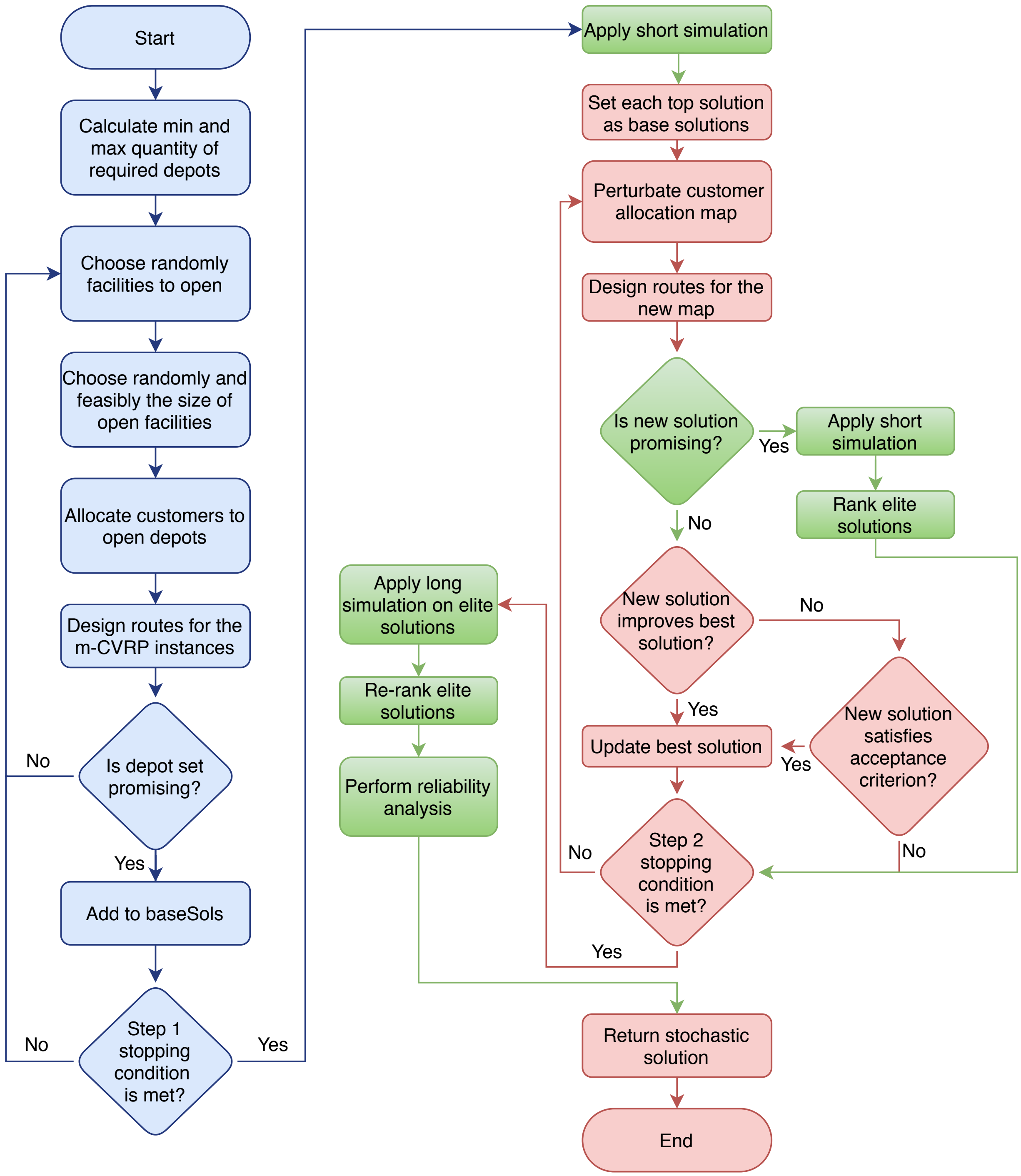



Figure 2: Flowchart of our simheuristic approach.

are the total opening and routing cost, the total stochastic cost incurred whenever a route fails, the solution reliability, and the safety stock.

## 4 COMPUTATIONAL EXPERIMENTS

All our experiments were carried out using Akca's instances (Akca et al. 2009). Such instances are designed for a deterministic and non-flexible LRP. Therefore, they were adapted to our stochastic and flexible case. Three main modifications were made:

1. We assume that the demand proposed in these instances is the mean of a log-normal probability distribution. If $D_i$ is the random variable representing the demand of the customer $i \in I$, and $d_i$ is the deterministic demand in Akca's set, then $E[D_i] = d_i$. Also, three different values of variance are considered: low, medium, and high, i.e., for $\lambda \in \{0.05, 0.10, 0.20\}$, $Var[D_i] = \lambda \cdot d_i$. It is worth to notice that our approach allows for using any other probability distributions.
2. As the original Akca's instances consider that a fixed size is available to be assigned to open depots, we provide a total of five alternative sizes, from which our algorithm may select for each open depot. If $s_j$ is the size proposed by the original instance for each potential depot $j \in J$, and $L$ is the set of available sizes, the alternative sizes in our approach are $s_{jl} \in \{(1-2r)s_j, (1-r)s_j, s_j, (1+r)s_j, (1+2r)s_j\}$, where $l \in L$ and $0.0 < r < 0.5$. This parameter represents the difference between available sizes. When $r = 0$, the case is the same as Akca's. Our experiments consider a fixed value of $r = 0.25$.
3. Akca's instances consider fixed cost ($f_j$) to incur whenever a depot $j \in J$ is open. Our experiments also consider this. Additionally, we introduce a variable cost term ($o_{jl}$) depending on $f_j$ and $s_{jl}$, as given in Equation (1). This formula preserves $o_{jl}$ in the same order as $f_j$ for each depot $j \in J$. Besides, it yields negative cost whenever $s_{jl} < s_j$, positive cost whenever $s_{jl} > s_j$, and no cost when $s_{jl} = s_j$. Thus, our results can be compared with those found in the LRP literature.

$$o_{jl} = \frac{(s_{jl} - s_j)}{2s_j} \frac{\sum_j f_j}{|J|} \tag{1}$$

Our algorithm uses the following parameters to run the experiments: *(i)* a total of 350 iterations for map perturbations; *(ii)* a total of 150 iterations for the biased-randomized savings heuristic; *(iii)* a total of 150 iterations for splitting; *(iv)* a random value between 0.05 and 0.80 for $\beta_1$, the parameter of the geometric distribution associated with the biased-randomized selection during the allocation map process; *(v)* a random value between 0.07 and 0.23 for $\beta_2$, the parameter of the geometric distribution associated with the biased-randomized heuristic for routing; *(vi)* a total of $n = 100$ runs for the initial simulation stage; and *(vii)* a total of $N = 5,000$ runs for the intensive simulation stage. The safety stock is estimated through a fast simulation process of 100 iterations, testing only discrete values of *SS* between 0 % and 10 %. Our proposed algorithm was coded as a Java application. All experiments were executed on a standard Windows PC with a Core *i*5 processor and 6 GB RAM. A total of ten different random seeds were used for each instance. Our results are compared in terms of both types of cost and reliability ($R$) with those obtained by Quintero-Araujo et al. (2020), who do not consider flexibility in facility sizes. If $M$ is the set of routes in a solution, the reliability of a single route $m \in M$ is defined as the probability that it does not fail, i.e.,

$$R_m = \left(1 - \frac{b_m}{N}\right) \cdot 100 \ \%$$

where $b_m$ is the total number of simulation runs in which the route $m \in M$ fails. Routes within a solution are considered independent components in a series system. Thus, the estimated reliability of a solution with $|M|$ routes is computed as:

$$R = \prod_{m=1}^{M} R_m$$

Tables 1, 2, and 3 show our best-found stochastic solution (OBS) considering low-, medium-, and high-variance scenarios, respectively. Regardless of the variance level, and due to the flexibility in the selection of the facility size, our approach is able to outperform the results provided in Quintero-Araujo et al. (2020) in terms of expected total cost ($TC$) for 11 out of 12 instances. Hence, the flexibility in the facility size enables to reach savings up to 6.72 % in a single instance. On the average, our solutions are also better when comparing just opening (*OC*) or just routing cost (*RC*), which is a direct consequence of increasing the flexibility in depot-sizing decisions. Most instances show a reduction in opening cost, although routing cost increase. Except for instance *Cr30x5a-3*, savings in opening cost are always higher than increments in routing cost.

Table 1: Results with a low-variance level.

| Instance | Quintero-Araujo et al. (2020) | | | | | Our Best Stochastic | | | | | | TC | Reliability |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *OC* | *RC* | *SC* | $TC$ | *R* | *OC* | *RC* | *SC* | $TC$ | *R* | *SS* | Gap | difference |
| Cr30x5a-1 | 200.00 | 619.51 | 2.10 | 821.61 | 94 % | 200.00 | 575.14 | 2.37 | 777.51 | 94 % | 0 % | -5.37 % | 0 % |
| Cr30x5a-2 | 200.00 | 621.45 | 0.00 | 821.45 | 100 % | 200.00 | 607.28 | 0.04 | 807.32 | 100 % | 3 % | -1.72 % | 0 % |
| Cr30x5a-3 | 200.00 | 502.29 | 4.53 | 706.82 | 85 % | 187.50 | 509.25 | 10.99 | 707.74 | 83 % | 3 % | 0.13 % | -2 % |
| Cr30x5b-1 | 200.00 | 680.03 | 1.11 | 881.14 | 98 % | 225.00 | 623.22 | 9.37 | 857.59 | 85 % | 0 % | -2.67 % | -13 % |
| Cr30x5b-2 | 200.00 | 625.32 | 0.00 | 825.32 | 100 % | 187.50 | 625.32 | 0.00 | 812.82 | 100 % | 2 % | -1.51 % | 0 % |
| Cr30x5b-3 | 200.00 | 684.58 | 1.74 | 886.32 | 96 % | 187.50 | 684.58 | 2.25 | 874.33 | 97 % | 1 % | -1.35 % | 1 % |
| Cr40x5a-1 | 200.00 | 729.12 | 0.04 | 929.16 | 100 % | 162.50 | 731.84 | 0.03 | 894.37 | 100 % | 1 % | -3.74 % | 0 % |
| Cr40x5a-2 | 200.00 | 688.80 | 0.00 | 888.80 | 100 % | 225.00 | 639.02 | 0.10 | 864.12 | 100 % | 0 % | -2.78 % | 0 % |
| Cr40x5a-3 | 200.00 | 748.64 | 4.67 | 953.31 | 93 % | 162.50 | 752.88 | 0.97 | 916.35 | 98 % | 0 % | -3.88 % | 5 % |
| Cr40x5b-1 | 200.00 | 852.05 | 5.24 | 1057.29 | 86 % | 162.50 | 852.04 | 6.90 | 1021.45 | 87 % | 1 % | -3.39 % | 1 % |
| Cr40x5b-2 | 200.00 | 781.54 | 0.05 | 981.59 | 100 % | 225.00 | 690.57 | 0.08 | 915.65 | 100 % | 1 % | -6.72 % | 0 % |
| Cr40x5b-3 | 200.00 | 769.76 | 2.34 | 972.10 | 92 % | 175.00 | 772.87 | 0.07 | 947.93 | 100 % | 2 % | -2.49 % | 8 % |
| **Average** | 200.00 | 691.92 | 1.82 | 893.74 | 95 % | 191.67 | 672.00 | 2.76 | 866.43 | 95 % | 1 % | -2.96 % | 0.00 % |

Table 2: Results with a medium-variance level.

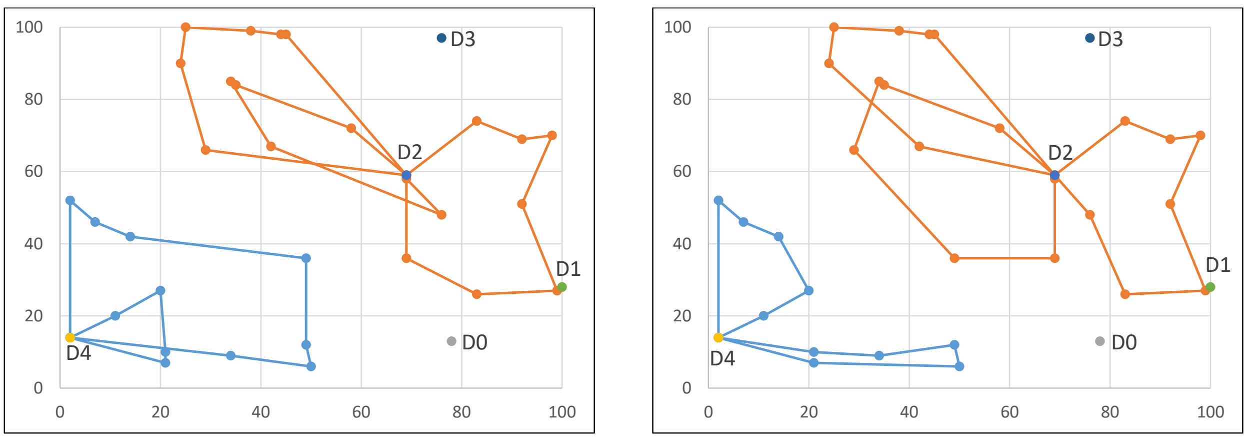
| Instance | Quintero-Araujo et al. (2020) | | | | | Our Best Stochastic | | | | | | TC | Reliability |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *OC* | *RC* | *SC* | $TC$ | *R* | *OC* | *RC* | *SC* | $TC$ | *R* | *SS* | Gap | difference |
| Cr30x5a-1 | 200.00 | 619.51 | 6.41 | 825.92 | 84 % | 200.00 | 575.14 | 7.63 | 782.77 | 83 % | 2 % | -5.22 % | -1 % |
| Cr30x5a-2 | 200.00 | 621.46 | 0.27 | 821.73 | 100 % | 200.00 | 607.28 | 0.46 | 807.74 | 99 % | 3 % | -1.70 % | -1 % |
| Cr30x5a-3 | 200.00 | 502.29 | 7.44 | 709.73 | 76 % | 187.50 | 509.25 | 18.50 | 715.25 | 72 % | 3 % | 0.78 % | -4 % |
| Cr30x5b-1 | 200.00 | 682.97 | 2.61 | 885.58 | 95 % | 225.00 | 623.22 | 14.63 | 862.85 | 77 % | 0 % | -2.57 % | -18 % |
| Cr30x5b-2 | 200.00 | 625.32 | 0.00 | 825.32 | 95 % | 187.50 | 625.32 | 0.00 | 812.82 | 100 % | 2 % | -1.51 % | 5 % |
| Cr30x5b-3 | 200.00 | 684.58 | 6.49 | 891.07 | 86 % | 187.50 | 684.58 | 10.21 | 882.28 | 86 % | 0 % | -0.99 % | 0 % |
| Cr40x5a-1 | 200.00 | 731.84 | 0.70 | 932.54 | 98 % | 162.50 | 739.24 | 0.01 | 901.75 | 100 % | 3 % | -3.30 % | 2 % |
| Cr40x5a-2 | 200.00 | 688.81 | 0.01 | 888.82 | 100 % | 225.00 | 643.52 | 3.07 | 871.59 | 93 % | 1 % | -1.94 % | -7 % |
| Cr40x5a-3 | 200.00 | 752.89 | 4.25 | 957.14 | 93 % | 162.50 | 752.88 | 4.46 | 919.85 | 93 % | 1 % | -3.90 % | 0 % |
| Cr40x5b-1 | 200.00 | 862.57 | 1.53 | 1064.10 | 98 % | 162.50 | 858.58 | 4.54 | 1025.62 | 91 % | 2 % | -3.62 % | -7 % |
| Cr40x5b-2 | 200.00 | 781.54 | 2.06 | 983.60 | 98 % | 225.00 | 690.57 | 2.06 | 917.63 | 98 % | 1 % | -6.71 % | 0 % |
| Cr40x5b-3 | 200.00 | 769.76 | 5.18 | 974.94 | 82 % | 175.00 | 772.87 | 1.42 | 949.29 | 96 % | 2 % | -2.63 % | 14 % |
| **Average** | 200.00 | 693.63 | 3.08 | 896.71 | 92 % | 191.67 | 673.54 | 5.58 | 870.79 | 91 % | 2 % | -2.78 % | -1.42 % |

In contrast, less costly routes can be designed when either increasing or keeping the same total available capacity. The latter case yields the same opening cost but it re-configures the open facilities size. Figure 3 shows an example of this situation for the instance *Cr30x5a-2* in the high-variance scenario. Figure 3(a) displays the routes obtained by Quintero-Araujo et al. (2020) while Figure 3(b) shows our best stochastic solution. Both solutions open depots *D2* and *D4*. However, while the approach by Quintero-Araujo et al. (2020) opens two depots with a size of $1,000$ each, we open one depot with a size of 750 (*D4*) and one depot with a size of $1,250$ (*D2*). This slight change induced by the flexibility in sizing decisions allows for reassigning one customer and reducing routing cost. A similar analysis can be carried out for the remaining instances.

Our best-found deterministic solution (OBD) is also tested in a stochastic environment, using 0 % of safety stock protection against uncertainty. Table 4 compares this solution's results with our best

Table 3: Results with a high-variance level.

| Instance | Quintero-Araujo et al. (2020) | | | | | Our Best Stochastic | | | | | | TC Gap | Reliability difference |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *OC* | *RC* | *SC* | *TC* | *R* | *OC* | *RC* | *SC* | *TC* | *R* | *SS* | | |
| Cr30x5a-1 | 200.00 | 619.51 | 14.99 | 834.50 | 67% | 200.00 | 575.14 | 19.66 | 794.80 | 66% | 2% | -4.76% | -1% |
| Cr30x5a-2 | 200.00 | 621.46 | 3.34 | 824.80 | 96% | 200.00 | 607.74 | 0.02 | 807.76 | 100% | 5% | -2.07% | 4% |
| Cr30x5a-3 | 200.00 | 502.29 | 11.31 | 713.60 | 66% | 187.50 | 509.25 | 27.86 | 724.61 | 57% | 2% | 1.54% | -9% |
| Cr30x5b-1 | 200.00 | 682.97 | 8.00 | 890.97 | 86% | 225.00 | 623.22 | 19.99 | 868.21 | 68% | 10% | -2.55% | -18% |
| Cr30x5b-2 | 200.00 | 625.32 | 0.04 | 825.36 | 100% | 187.50 | 625.32 | 0.10 | 812.92 | 100% | 3% | -1.51% | 0% |
| Cr30x5b-3 | 200.00 | 684.57 | 16.64 | 901.21 | 68% | 187.50 | 684.58 | 24.93 | 897.00 | 68% | 1% | -0.47% | 0% |
| Cr40x5a-1 | 200.00 | 729.13 | 4.75 | 933.88 | 87% | 162.50 | 737.20 | 2.85 | 902.55 | 92% | 2% | -3.35% | 5% |
| Cr40x5a-2 | 200.00 | 688.80 | 0.80 | 889.60 | 99% | 225.00 | 642.02 | 1.79 | 868.82 | 95% | 3% | -2.34% | -4% |
| Cr40x5a-3 | 200.00 | 760.10 | 2.53 | 962.63 | 92% | 162.50 | 763.69 | 5.78 | 931.97 | 87% | 2% | -3.19% | -5% |
| Cr40x5b-1 | 200.00 | 863.91 | 7.98 | 1071.89 | 89% | 237.50 | 786.00 | 4.65 | 1028.14 | 91% | 3% | -4.08% | 2% |
| Cr40x5b-2 | 200.00 | 781.54 | 9.55 | 991.09 | 88% | 225.00 | 690.57 | 9.35 | 924.91 | 89% | 2% | -6.68% | 1% |
| Cr40x5b-3 | 200.00 | 769.76 | 10.97 | 980.73 | 69% | 175.00 | 780.62 | 4.14 | 959.76 | 92% | 3% | -2.14% | 23% |
| **Average** | 200.00 | 694.11 | 7.58 | 901.69 | 84% | 197.92 | 668.78 | 10.09 | 876.79 | 84% | 3% | -2.63% | -0.17% |




(a) (b)

Figure 3: Best-found solution by the non-flexible LRP (a) and our approach (b) for the instance *Cr30x5a-2*.

stochastic solution (OBS) in terms of cost and reliability. Comparisons are drawn in terms of both cost gaps and reliability differences. On average, results show a slight improvement in the obtained cost, yielding a decrease up to 0.47 % in the high-variance scenario. Nevertheless, the real contribution of using a stochastic approach in our problem is the increase in reliability, which reaches a maximum of 48 % in a single instance, and an average of 12.5 % in the high-variance scenario.

## 5 CONCLUSIONS

This work has presented a Location Routing Problem in which the size of the facilities to open is an additional variable. This size is usually selected from a discrete set related to diverse investment activities. However, the literature addressing this variable is still scarce, despite the relevance of the problem in real-world applications. Moreover, we have considered that customers' demands are stochastic. To the best of our knowledge, it is the first time that the Stochastic LRP with Facility Sizing Decisions has been studied. As this problem is *NP-hard*, we have proposed a simheuristic algorithm as solving approach. Medium-sized benchmark instances with different variability levels were used. On the one hand, the obtained results show that cost savings are attained due to the considered flexibility in facility sizing. These savings may be yielded by *(i)* a reduction in opening cost, given the installation of smaller-size facilities; or *(ii)* a reduction in routing cost, given the installation of higher-size facilities, which enables to reconfigure the designed

Table 4: Best deterministic vs. best stochastic solutions under stochastic scenarios.

| Instance | OBD | | OBS | | TC | Reliability |
|---|---|---|---|---|---|---|
| | *TC* | *R* | *TC* | *R* | gap | difference |
| **Low Variance** | | | | | | |
| Cr30x5a-1 | 778.25 | 94 % | 777.51 | 94 % | -0.10 % | 0 % |
| Cr30x5a-2 | 807.34 | 100 % | 807.32 | 100 % | 0.00 % | 0 % |
| Cr30x5a-3 | 707.44 | 84 % | 707.74 | 83 % | 0.04 % | -1 % |
| Cr30x5b-1 | 855.49 | 86 % | 857.59 | 85 % | 0.25 % | -1 % |
| Cr30x5b-2 | 812.82 | 100 % | 812.82 | 100 % | 0.00 % | 0 % |
| Cr30x5b-3 | 874.61 | 96 % | 874.33 | 97 % | -0.03 % | 1 % |
| Cr40x5a-1 | 894.40 | 100 % | 894.37 | 100 % | 0.00 % | 0 % |
| Cr40x5a-2 | 873.29 | 82 % | 864.12 | 100 % | -1.05 % | 18 % |
| Cr40x5a-3 | 916.47 | 98 % | 916.35 | 98 % | -0.01 % | 0 % |
| Cr40x5b-1 | 1022.24 | 86 % | 1021.45 | 87 % | -0.08 % | 1 % |
| Cr40x5b-2 | 915.68 | 100 % | 915.65 | 100 % | 0.00 % | 0 % |
| Cr40x5b-3 | 953.44 | 69 % | 947.93 | 100 % | -0.58 % | 31 % |
| **Average** | 867.62 | 91 % | 866.43 | 95 % | -0.13 % | 4.08 % |
| **Medium Variance** | | | | | | |
| Cr30x5a-1 | 785.26 | 84 % | 782.77 | 83 % | -0.32 % | -1 % |
| Cr30x5a-2 | 808.15 | 99 % | 807.74 | 99 % | -0.05 % | 0 % |
| Cr30x5a-3 | 716.16 | 71 % | 715.25 | 72 % | -0.13 % | 1 % |
| Cr30x5b-1 | 860.37 | 76 % | 862.85 | 77 % | 0.29 % | 1 % |
| Cr30x5b-2 | 812.82 | 100 % | 812.82 | 100 % | 0.00 % | 0 % |
| Cr30x5b-3 | 882.73 | 85 % | 882.28 | 86 % | -0.05 % | 1 % |
| Cr40x5a-1 | 895.16 | 98 % | 901.75 | 100 % | 0.74 % | 2 % |
| Cr40x5a-2 | 881.88 | 68 % | 871.59 | 93 % | -1.17 % | 25 % |
| Cr40x5a-3 | 919.86 | 92 % | 919.85 | 93 % | 0.00 % | 1 % |
| Cr40x5b-1 | 1034.90 | 66 % | 1025.62 | 91 % | -0.90 % | 25 % |
| Cr40x5b-2 | 917.95 | 98 % | 917.63 | 98 % | -0.03 % | 0 % |
| Cr40x5b-3 | 959.70 | 56 % | 949.29 | 96 % | -1.08 % | 40 % |
| **Average** | 872.91 | 83 % | 870.79 | 91 % | -0.23 % | 7.92 % |
| **High Variance** | | | | | | |
| Cr30x5a-1 | 799.56 | 67 % | 794.80 | 66 % | -0.60 % | -1 % |
| Cr30x5a-2 | 811.14 | 95 % | 807.76 | 100 % | -0.42 % | 5 % |
| Cr30x5a-3 | 726.17 | 57 % | 724.61 | 57 % | -0.21 % | 0 % |
| Cr30x5b-1 | 864.96 | 67 % | 868.21 | 68 % | 0.38 % | 1 % |
| Cr30x5b-2 | 812.94 | 100 % | 812.92 | 100 % | 0.00 % | 0 % |
| Cr30x5b-3 | 897.49 | 67 % | 897.00 | 68 % | -0.05 % | 1 % |
| Cr40x5a-1 | 898.12 | 90 % | 902.55 | 92 % | 0.49 % | 2 % |
| Cr40x5a-2 | 892.21 | 53 % | 868.82 | 95 % | -2.62 % | 42 % |
| Cr40x5a-3 | 926.90 | 82 % | 931.97 | 87 % | 0.55 % | 5 % |
| Cr40x5b-1 | 1052.68 | 45 % | 1028.14 | 91 % | -2.33 % | 46 % |
| Cr40x5b-2 | 925.67 | 88 % | 924.91 | 89 % | -0.08 % | 1 % |
| Cr40x5b-3 | 967.39 | 44 % | 959.76 | 92 % | -0.79 % | 48 % |
| **Average** | 881.27 | 71 % | 876.79 | 84 % | -0.47 % | 12.50 % |

routes to make them shorter. On the other hand, our approach increases the reliability of solutions when compared against the best deterministic solution tested in a stochastic setting. All in all, these results illustrate the benefits of using a simheuristic approach.

Future work includes the design of a more guided heuristic to select a suitable size for each facility, since the current procedure is purely random. Additionally, a multi-objective approach can be implemented in order to increase the solutions' reliabilities without deteriorating cost savings. Finally, bigger instances can be tested to evaluate the influence of the number of nodes on our methodology's performance.

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