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OPTIMAL SWITCHING IN A DYNAMIC, STOCHASTIC, OPERATING ENVIRONMENT

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ABSTRACT

The value of flexibility in operations for hydroelectric power plants is apparent from their investment in multiple generators to allow variable intensity of operation. This flexibility supports rapid response to outside factors that impact their performance. It follows that hydroelectricity generation often competes for water flow with downstream water demands including irrigation, urban water use, or ecosystem services. Each of these follows stochastic process that implies asynchronous demand for water flow. The associated management problem is of interest encompassing multiple switching and motivating other extensions. This paper considers the dynamic stochastic problem of water allocation for hydroelectric generation and downstream use. Our main contribution is to present a novel numerical solution to this problem. Moreover, the method allows generalization not previously possible including: (*i*) multiple dynamic stochastic processes with jump diffusions, (*ii*) the number of contingent decisions with reversibility, and (*iii*) characterization of risk and uncertainty including dynamic constraints.

1 INTRODUCTION

Unanticipated stochastic variation in physical and economic operating environments often substantially impact the performance of management controls. *Ex poste* adaptation to such changes can incur substantial loss and motivates consideration of *a priori* strategies to mitigate loss. One such strategy is operational flexibility that supports as-soon-as-possible, real time adaptation by shifting operational modes to reduce loss. Such operational flexibility most often comes at both variable and sunk cost that may be substantial. Valuation of such flexibility for highly dimensioned operation management problems remains computationally challenging and has motivated substantial simplification of the representation of such problems. In practice, flexibility in operations may involve a high dimension of modes that over time may define a substantial number of optimal control paths for operations, changing paths may be path independent and may allow reversion to past modes of operation. In some cases, path dependence and an absence of reversion may restrict flexibility. Further, the timing of adaptation to new information with respect to state evolution may be continuous or over discrete time intervals.

An example of such an operations management problem is found in hydroelectric power generation. We pursue that problem as an illustrative example of methods we present as the main contribution of this paper. Here, we present an approach to quantify the value and control of operations where multiple switching is feasible across multiple modes and with the flexibility to revert to past mode configurations. Our approach is to specify an extended optimal switching problem that to our knowledge is new in the applied hydroelectric operations context. We present our model in a generalized specification to highlight its more general applicability to highly dimensioned problems with multiple, path independent switching.

Hydroelectric generation involves a control problem where the objective is impacted by dynamic stochastic processes. As dynamic paths of state variables such as realized weather and associated water

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inflow are uncertain, the management strategy will benefit by choosing a path that may involve multiple modes of operation and multiple switching opportunities. In this setting, the demand for electricity or supply of downstream water use are stochastic, dynamic processes. Forecasts of uncertain water inflow are critical to support rapid, optimal adaptation of hydroelectricity generation to short-term weather forecasts, see Ahmad and Hossain (2020). They note extensive literature on electricity demand and the role of renewable resources (Jacobson et al. 2017; O'connor et al. 2016; Vieira and Ramos 2009; Jacobson 2009; Jacobson et al. 2014; Jacobson et al. 2018; Jacobson and Delucchi 2011). Further, hydroelectricity generation incorporates considerable potential for adaptation through water and energy storage Sørensen (1981), rapid adjustment of water flow Egré and Milewski (2002) to meet multiple and potentially competing stochastic, dynamic demands, as well as power grid net demand as supply varies from other resource origins (Sørensen 1981; Kougias et al. 2016). Water flow demands considered in past literature have included irrigation for agriculture, urban water use, flood risk mitigation, or ecosystem services. Chen et al. (2020) highlight this complex water-energy-ecosystem nexus in the Mekong River Basin.

While hydroelectricity generation capacity incorporates these means of adaptation, the value of such flexibility has not been explicitly explored Block (2010). Rather than optimal adaptive management of flows, standard operations are managed by rule curves to maintain reservoir storage goals based on historic flows (Lee et al. 2009; Ficchì et al. 2016). The efficiency implications of such rules can be substantial. However, these studies have not considered the implications of competing stochastic, dynamic demands for water (stock and flow) use. While a variety of optimization approaches have been considered for managing water flow in power generation (see e.g. Wu and Chen (2013), Labadie (2004), Yeh (1985), Rani and Moreira (2010), Ahmad et al. (2014), Ahmadi et al. (2015)) they have typically considered single user benefits to establish optimal operating rules. Exceptions have used multiple objective method to consider competing use explicitly and within that context the role of forecasts to support decision-making. These studies either directly introduce subjectively weighted multiple objectives or define a focal objective and optimize with other goals as constraints, e.g. Ahmad and Hossain (2020). However, none of this literature, or other to our knowledge, has examined the value of adaptive management of power generators within a hydroelectricity plant to adapt to real time variation of water inflow as well as stochastic, dynamic demand processes. That is, rather than base management on rules derived from forecast-based optimization, we illustrate our methods within the context of hydroelectric generation with dynamic adaptation of water flow and power generation that is also faced with competing, stochastic demands dependent on downstream water flow.

The complexity of the hydroelectricity generation management problem within the context of joint allocation of water flow to hydroelectricity or irrigation lends itself to heuristic solution. Indeed, overtime, heuristic rules have been iteratively developed to guide the management of water flow allocation to power generation and irrigation, see e.g. Johnson et al. (1991). More recently, heuristic methods have been applied to derive optimal generation scheduling. Ahmad et al. (2014) noted this literature has considered optimization techniques developed and applied for managing water resources systems and reservoir operation and noted their focus on stable stochastic processes. In contrast, few studies have considered the problem in the context where the stochastic processes may be changing and uncertain allowing for anticipation that adaptation will occur in the future. Often these studies have focused on scenario analysis to illustrate optimal changes, see e.g. Ahmad et al. (2014) or Akbari-Alashti et al. (2018). Only a limited literature has considered adaptive reservoir operation rules under changing stochastic conditions, see e.g. Feng et al. (2017). We do not directly compare our solutions to particular heuristic rule-based management, though such applications are a natural applied extension of our work.

The relationship between the problem that we address and that of the exercise of American options is important to consider. The American option problem involves managing the timing of a one-time execution of a right to sell or buy an asset subject to a forward-looking evaluation of the stochastic value of the underlying asset. The exercise timing is the control and reversion after execution is not possible. Thus, the American option problem is an optimal switching problem with path dependence and a binary choice of time to execute prior to expiry, see e.g. Pham (2009). Variations in option exercise have been studied including interdependent (compound) options and swing options (see e.g. Calvo-Garrido et al. (2019)); however, these are representable as sequences of standard options with multiple exercise intervals spaced by prescribed time (refraction) intervals (see e.g. Grüne and Kloeden (2006). It follows they can be posed as multiple switching problems, see Gassiat et al. (2012) and Martyr (2016) in the absence of jump diffusion processes, Calvo-Garrido et al. (2019).

The problem we take up in this paper can be viewed as a specification that is fundamentally different from the financial option problem. First, our problem incorporates multiple modes that are not time-ordered, allowing for relaxation of path dependence. Financial option problems involve binary modes, exercise or not, that are time dependent and not reversible. Second, our problem involves multiple stochastic processes and incorporates jump diffusion. Third, our problem requires the agent at each time period to choose a path conditional on expectations of future outcomes conditioned by multiple stochastic processes. In our problem, at each time, choice is not path dependent and it is feasible for the agent to revert to mode configurations chosen in the past. This setting requires a solution approach that implements methods at each current and future time period in a backwardation process.

Our approach builds on an extensive literature that has considered optimal switching, for a recent review see Zervos et al. (2018). Typical of interest has focused on sequential switching with recent work extending to multiple switches across a limited number of modes, e.g. Carmona and Ludkovski (2010). Our methods contribute by extending the well-known Longstaff and Schwartz Monte Carlo (LSMC) approach for implementation in the multiple switching problem. We present a specific approach to derive optimal paths for sequential adaptation posed as switching across modes of operation and allowing for recourse to revert to previous configurations of modes. While the LSMC method has been widely used in financial options literature to evaluate the exercise choice at the current time, our application requires an algorithm that at each time embeds LSMC for each future and the current time period. We present a specific approach to derive optimal paths for sequential adaptation posed as switching across modes of operation and allowing for recourse to revert to previous configurations of modes. Our problem differs from that of the decision to exercise swing options that can be characterized as a multiple switching problem with fixed refraction time intervals, see Calvo-Garrido et al. (2019), Haarbrücker and Kuhn (2009), Jaillet et al. (2004). Further, in financial options problems, it is not technically feasible for the agents holding an option to sell excess amounts of the underlying asset that they do not own or could produce. In our problem and its application to hydroelectric power generation, it is feasible to sell more than the amount produced through transactions in an outside spot market (e.g. wholesale electricity market).

2 MODEL

Hydroelectric power plants use the energy of flowing water to generate electricity. The energy of water flow is transformed by a generator into electricity. The extent of transformation can be adjusted by controlling the volume of water flow through the number of control gates operated. The rate of hydroelectricity production with m number of control gates operated can be written as:

$$Y_t(m_t) = E \cdot R_t = E \cdot R(m_t, H_t) \tag{1}$$

where Y denotes the flow rate of electricity produced (MWh), R is the rate of water release from the reservoir and depends on the hydrologic head H, the vertical distance of the reservoir water level to the generator turbines, and E is a constant based on the overall efficiency of the generation plant. Given the relationship in (1), storing water behind the dam as a stock increases the head heights and increases of the subsequent flow of generated electricity.

We assume the hydroelectricity suppliers must supply sufficient electricity to meet the demand of its customers. If the current amount of hydroelectricity generated is insufficient, they must either generate additional electricity or purchase it in wholesale markets for other electricity sources. Let electricity demand facing the supplier be $D_t(P_t)$ with $\partial D_t / \partial P_t < 0$ where P_t is the electricity price. While the price could be considered a competitive price set by markets, we consider the case where it is set by the hydroelectric generator. We note time dependence with the subscript t. We assume the wholesale electricity market is perfectly competitive and that the supplier is able to buy or to sell any volume of electricity at a constant price M_t that only depends on time. This specification extends our problem beyond a variation of swing options where the option to buy supply from the wholesale market is not feasible. This wholesale market price represents the opportunity cost to the supplier of electricity sold to its retail customers. The instantaneous return from electricity production and retail can then be expressed as the following static problem,

$$\begin{aligned}\pi_t \equiv \pi(m_t, H_t|M_t) &= \max_{P_t}(P_t - M_t)(D_t(P_t) - Y_t(m_t)) + (P_t - c)Y_t(m_t) \\ &= \max_{P_t}(P_t - M_t)D_t(P_t) + (M_t - c)Y_t(m_t)\end{aligned} \tag{2}$$

where Y_t, the electricity generated at time t through the supplier's hydroelectric facilities, is defined in (1), and c is the variable cost of hydroelectric generation. The solution to (2) produces the following mark-up rule for P relative to the wholesale market price M,

$$P_t = \frac{\eta_t}{\eta_t - 1} \cdot M_t. \tag{3}$$

where $\eta_t = -(\partial D_t/\partial P_t)(P_t/D_t)$ is the elasticity of electricity demand faced by the supplier. From (3), P_t depends only on the wholesale market price and the demand elasticity, indicating that the static electricity pricing problem can be separated from the problem of determining optimal water release. The fact that generated electricity is hard to store for future arbitrage also supports that separation. The pricing solution in (3) yields the maximum revenues from production and sale of any amount of electricity generated through the hydroelectricity production function. The resulting profit function, $\pi_t(m_t, H_t|M_t)$, is incorporated in the objective functional in the control problem to determine at each time period the optimal path of operations.

To proceed, we extend the problem to incorporate a second source of revenue from operations and consider the supply of irrigation water as an input into farm production. Our methods allow general consideration of the number and variety of downstream water flow uses. The downstream water supply is constrained by the rate of water release from the reservoir, i.e. R in (1). This implies that the operation of control gates in the hydroelectric plant influences the irrigation water supply. Further, gate control affects the dynamic path of electricity generation. Static agricultural production models have ignored the importance of the timing of the water release (and the application) as a determine of the growth rate of crop production. In order to incorporate this feature, let G_t denote the aggregate stock of crop grown by the farmers at time t during the production season. The return from the crop growth is realized only at maturity (harvest) defined as the terminal period T. However, harvest is defined as path dependent and conditional on water release and application of other inputs, X_t. Crop growth then depends on the water release R_t and other inputs X_t and can be represented by the dynamic production function (maybe with risk).

$$dG_t = g(G_t, R_t, X_t).$$

Revenues from farm production are the product of the market price, F_T, and the total production, G_T, at the harvest or terminal period T. Farmers purchase X_t freely through the market at price c_1, assumed for simplicity to be constant over the growing season. We note input prices could be introduced as stochastic and uncertain. Costs of surface water application are also assumed to be constant, i.e. c_2 per unit of water released. Revenues from the sale of both electricity and crops are functions of the magnitude and the timing of the release of water downstream. The optimal dynamic allocation of this resource requires balancing its value in current use with its value for storage and subsequent release.

The dynamic problem of operating a hydroelectric power plant involves control through control gate adjustment. This flow management can be posed as a dynamic, stochastic problem that determines the optimal release of water, electricity generated, and provision of downstream water flow for irrigation. Relative values of the multiple outputs of the dam operation are easily added to our notation and will depend on applications. The complexity of this control problem and its possible high dimensionality are illustrated in its problem statement. At every time period, the manager is assumed to choose dynamic controls to optimize the expected return:

$$V_0(m_0,M_0,K_0) = \sup_{u_0\in\mathscr{U}_0} \mathbb{E}\left[\int_0^T [\pi_t(m_t,M_t,K_t)]dt - \sum_{l\geq 1} C_{m_{l-1},m_l}\right] \tag{4}$$

where $u_t = ((\tau_l)_{l\geq 0},(m_l)_{l\geq 0})$ is the dynamic control from time t such that current control u_0 is given. The path of this control over time defines the control strategy. That is, define τ_l as the timing for the l-th switching of modes, and m_l as the mode to be changed at l-th switching. $\mathscr{U}_t$ is the set of all available operating controls. If we follow the hydroelectric model introduced in Section 2, m and τ can be interpreted as the number of water gates operated and the timing of switches in their configuration for operation. Next, we define the dynamics of M_t as:

$$d\log M_t = \lambda(\bar{M}_t - M_{t-1})dt + \sigma_1 dW_1 + \xi dN$$

where we incorporate the trend and seasonal dynamics as: $\bar{M}_t = 10(0.95+0.05\cos(-2\pi t)$. dt is the time differential, σ is the volatility measure, and W_t is a Wiener process and N is a Poisson process. We define the stochastic, dynamics of the wholesale price for procurement of electricity, K_t, as:

$$d\log K_t = (\kappa - K_{t-1})dt + \sigma_2 dW_2$$

Next, we consider the case for 5 operating modes and characterize the profit function associated with each mode. Let $\pi_t(m_t,H_t)$ be the profit available at time t with the choice of mode m_t given two state variables M_t and K_t. We specify mode specific return as a quadratic function, generally defined as:

$$\pi_t(m_t,M_t,K_t) = \alpha_0(M_t - \alpha_1 K_t)^\gamma$$

3 METHODS

Our method involves a generalized approach to valuation of flexibility in the presence of high dimensionality of our underlying extended optimal switching problem. Relying on purely probabilistic tools used in financial and real options such as the Snell envelope of processes and other regularity conditions, the problem in (4) can be transformed into a sequence of multiple but simple optimal stopping problems. This transformation opens the opportunity to implement a numerical simulation approach based on Monte-Carlo regressions, as inspired by the work of Longstaff and Schwartz (2001), to solve the optimal switching model. Importantly, this transformation and numerical approach allows substantial generalization of the dimensions of optimal switching problems including multi-dimensional stochastic state processes, path independence or specific interdependence of intertemporal decisions, and different types of risk (with a possibility of discontinuities).

In order to transform the problem described in (4), consider first a restricted situation in which there is a fixed upper bound on the total number of switching opportunities on the specified time horizon. That is, let $\mathscr{U}_t^k$ be the set of all available controls on the interval $[t,T]$ with at most $k\geq 0$ number of switching opportunities allowed:

$$\mathscr{U}_t^k \equiv \{u\in\mathscr{U}_t : \tau_{k+1} = T\}.$$

Note that k does not depend on or follow from the system design, but is added for the numerical scheme following the feature of almost surely (a.s.) finite variation of available controls. See Section 1.2.2 for the details. In this setting, $\mathscr{U}_t^k \subset \mathscr{U}_t$ where $\mathscr{U}_t$ is the set of all available controls not constrained by the switching opportunity parameter k. So, $\mathscr{U}_t$ is the same set of available controls defined in the original stochastic control problem in (4). For example, consider the case in which there is no remaining opportunity to switch the mode allowed, i.e., $k = 0$, at any time t. Then, the set of all available controls with $k = 0$ is denoted by $\mathscr{U}_t^0$ and the set $\mathscr{U}_t^0$ will consist of a single control in which the current mode is retained over time.

Now, let W^k be the value achieved consistent with the restricted set of available controls $\mathscr{U}^k$. In other words, the stochastic control problem is to optimize the control subject to $\mathscr{U}^k$. Then, the value function W^k is represented as

$$\begin{aligned} W_t^0(m_t, M_t, K_t) &= \mathbb{E}\left[\int_t^T \pi(m_s, M_s, K_s)ds\right] \\ W_t^k(m_t, M_t, K_t) &= \sup_{u \in \mathscr{U}_t^k} \mathbb{E}\left[\int_t^T \pi_s(m_s, M_s, K_s)ds - \sum_{t \le \tau_l < T} C_{m_{l-1}, m_l}\right]. \end{aligned} \tag{5}$$

for $k \ge 1$. Again, when $k = 0$, there are no switching opportunities allowed so that the value function W^k has only a single control in the set $\mathscr{U}_t^0$: current mode i is retained until the maturity of the problem. However, when $k \ge 1$, the value function W^k may reflect increased expected profit due to changing current mode at most k times during the specified period $[t, t]$ at the expense of the switching cost.

Importantly, observe that two optimizations over $\mathscr{U}^k$ and over $\mathscr{U}^{k-1}$ are able to be related. In fact, the Bellman's principle of optimality states that the restricted problem with k number of switching opportunities is equivalent to finding the first optimal switching time τ, which maximizes the initial return until τ, plus another value function at τ corresponding to optimal switching with $k-1$ number of switching opportunities over the remaining time horizon. This formal argument suggests that we should be able to solve the original problem in (4) recursively through multiple but (relatively) simple optimal stopping problems.

For a given stopping time t, let $\mathscr{S}_t$ be the set of all available stopping times after t, i.e., $\mathscr{S}_t \equiv \{\tau \le T : \mathbb{F}_t - stopping \text{ time s.t. } \tau \ge t \text{ a.s.}\}$. Also, define a functional operator $\mathscr{L}$ on an arbitrary function V by

$$\mathscr{L}V(i, M, K) \equiv \max_{m \ne i}\{-C_{ij} + V(m, M, K)\}.$$

By using the definitions on the set of all available stopping times $\mathscr{S}_t$ and the functional operator $\mathscr{L}$, the value function $V_t^k(m_t, H_t)$ corresponding to the sequence of multiple optimal stopping problems can be iteratively defined for $t \in [0, T]$, $i \in \{0, \cdots, M-1\}$:

$$\begin{aligned} V_t^0(m_t, M_t, K_t) &= \mathbb{E}\left[\int_t^T \pi(m_s, M_s, K_s)ds\right] \\ V_t^k(m_t, M_t, K_t) &= \sup_{\tau \in \mathscr{S}_t} \mathbb{E}\left[\int_t^\tau \pi(m_s, M_s, K_s)ds + \mathscr{L}V_\tau^{k-1}(m_\tau, M_\tau, K_\tau)\right], \qquad k \ge 1. \end{aligned}$$

Note that the iterative value function V^0 has only a single control in the set of stopping times because of $k = 0$ as in the restricted value function defined in (5). In the case of $k \ge 1$, however, if the iterative value function V_t^k shows that it is optimal to stop at time τ, the subsequent value function V_t^{k-1} needs to be considered. Moreover, observe that the defined functional operator $\mathscr{L}$ reflects the recursive structure of the value functions and specifies the best value which can be achieved by making an immediate switch from the current mode i at time τ given k number of switching opportunities remaining. If there are only two operating modes, for example, the maximum of the operator $\mathscr{L}$ will not needed as there is only one remaining mode to switch into other than the current mode.

As has been noted by Andersen and Broadie (2004), solution methods such as ours cannot guarantee the optimality of results based on simulation. They suggested a primal-dual approach to estimate the confidence interval around the (unknown) optimal value rather than the point estimate provided by the simulation method. Their approach has, however, only been applied to relatively simple option problems with a single exercise opportunity and low-dimensional states. Thus, an important direction for future research is to consider the dual problem of the model to improve the optimality gap even though the primal-dual transformation is complex with the existence of substantial dimensionality.

4 APPLICATION

Consistent with our intent to illustrate use of our generalized method, we provide results for a hypothetical specification for a hydroelectricity generation problem with consideration of irrigation use of downstream water flow. Specific applications can be readily founded on specifications and parameterizations that are consistent with a wide range of applications. To proceed, we illustrate our method for a hypothetical hydropower generation problem. We specify draws from stochastic processes in our model to generate 5k dynamic paths. At each time along each path, our approach is to generate controls to optimize the simulated expectation of profits associated with operation of each mode. In sum, our illustrative case involves a large number of modes and long time period during which operating conditions change stochastically.

We posit a dynamic programming approach for this application and solve it using an adaptation of the widely popular Longstaff Schwartz, Monte Carlo approach. The least-squares Monte-Carlo approach of Longstaff and Schwartz (2001) allows estimation of V_t^k in (13). In a first step, Ω number of independent simulated paths are generated for the discretized processes of $(M,K) = (M_t,K_t)_{t\in[0,T]}$, with a fixed initial condition (m_0,M_0,K_0). Then, the pathwise values of $V_t^k(m_t,M_t,K_t)$ are computed in a backward manner to time t and a forward manner in switching opportunity k, starting with $V_T^k(m_T,M_T,K_T) = 0$ for all k. With these simulated paths in hand, the expectation is approximated in $V_0^k(m_0,M_0,K_0)$ by $(1/\Omega)\sum_{\omega\in\Omega} V_0^k(m_0^{(\omega)},M_0^{(\omega)},K_0^{(\omega)})$. Note that the computation is simultaneously implemented within both backward and forward recursion in the parameters at time t and switching opportunities k, respectively. Thus, the current time values with $k-1$ number of switching opportunities, $V^{k-1}(m_t^{(\omega)},M_t^{(\omega)},K_t^{(\omega)})$, are fully known for each mode $j \neq i$ (for every j indeed) when the simulation is in the step of computing $V^k(m_t^{(\omega)},M_t^{(\omega)},K_t^{(\omega)})$.

Using the optimal decision at time t_1 along the ω-path, the pathwise optimal switching times can be updated. Let $\tau^k(m_t^{(\omega)},M_t^{(\omega)},K_t^{(\omega)})\cdot\Delta t$ be the optimal switching time for the pathwise gain $V_t^k(m_t^{(\omega)},M_t^{(\omega)},K_t^{(\omega)})$. Recall the computation is recursively implemented within a backward fashion in time t and forward recursion in switching opportunity k. Thus, the optimal switching times at $t+dt$, i.e. $\tau^k(m_{t+dt}^{(\omega)},M_{t+dt}^{(\omega)},K_{t+dt}^{(\omega)})$, is known for all k when the simulation is in the step of computing $\tau^k(m_t^{(\omega)},M_t^{(\omega)},K_t^{(\omega)})$. Therefore, the optimal decision τ^k is updated recursively following the equation:

$$\tau^k(m_t^{(\omega)},M_t^{(\omega)},K_t^{(\omega)}) = \begin{cases} \tau^k(m_{t+dt}^{(\omega)},M_{t+dt}^{(\omega)},K_{t+dt}^{(\omega)}), & \text{if} \quad m_t^{(\omega)} = m_{t+1}^{(\omega)}, \\ t, & \text{otherwise,} \end{cases}$$

To provide an illustration of the numerical approach, next it is applied to the water flow already defined and using the parameterization presented in Table 1. Within this setting, it is demonstrated that the numerical approach presented allows for generalization of the number of modes, the types of stochastic processes (Wiener and Poisson), and the extent of multiple switching.

5 RESULTS

Figure 1 presents results for expected profit (6) for one control path (w=56) of the 5000 simulated paths generated. As indicated in (6), we presume only one mode is operated at a time and that the choice of control path involves choosing to implement the mode that is estimated to render the highest expected profits. Recall, each mode was defined as a particular combination of generators. Thus, choice of mode is a discrete choice that attempts to choose the most profitable operation mode to implement at any time.

Table 1: Parameterization.

η	σ_1	ξ	N	κ	$m=1$		$m=2$		$m=3$		$m=4$		γ
					α_0	α_1	α_0	α_1	α_0	α_1	α_0	α_1	
7	0.5	Exp(0.1)	Poi($0.3dt$)	10	1	0.2	1.2	0.35	1.4	0.4	1.45	0.45	2/3

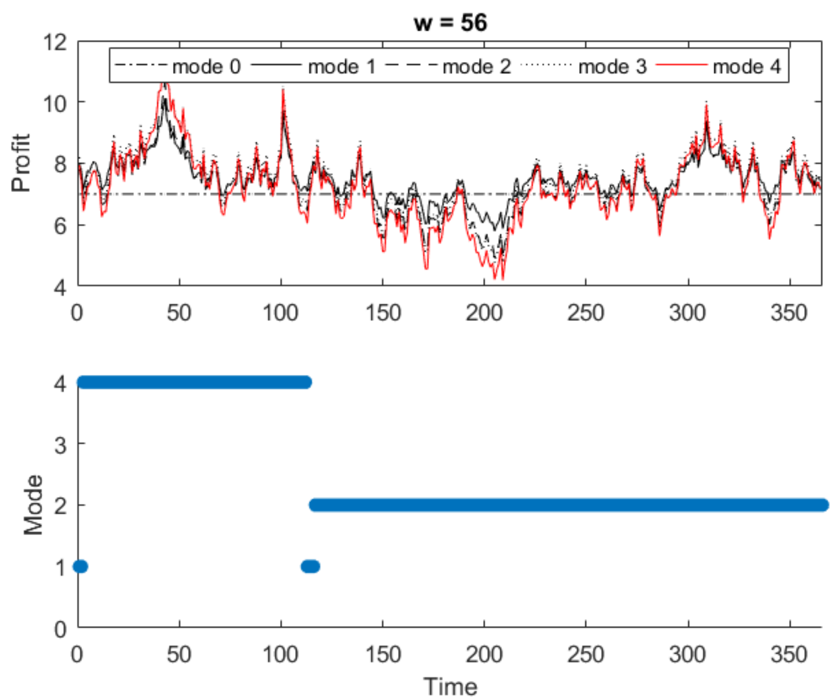

Figure 1: Operation.

Optimal switching is illustrated in Figure 1 by the variation in mode for which its profit path dominates other modes at each time. From Figure 1, we see that at any time t, one mode dominates and we assume the firm switches to that mode. Further, it is apparent from Figure 1 that a particular realization of the stochastic processes up to a decision point t will induce an initial position for reconsideration of the optimal mode to operate in the following period, given expectations over profits associated with the underlying stochastic processes of the state variables.

To value the flexibility in operations implicit in the capacity to switch, note that a frontier encompassing the profits streams associated with operation of the set of modes can be defined as the expected profits of optimal operation. That is, such a frontier would trace the expected profits of the dominant modes over time. From Figure 1, we can see such a frontier as the supremum of expected profits from each mode and compare it to the expected profit stream associated with any particular mode that might be presumed to be locked-in or rigid due to an absence of flexibility. That is, the frontier of profits or the suprema defined in problem (6) can be compared to any one of the streams of expected profits associated with a particular mode. From Figure 1, we see difference between such a frontier and any particular expected profit stream is nonzero. This deviation measures the value of flexibility to switch modes as stochastic operating conditions change over time.

6 CONCLUSIONS

In this paper, we introduce an approach to computation of a time path of optimal operational controls across a set of alternative modes of operation. This provides an approach for choice of optimal controls in a context where the operating environment evolves stochastically and choice of control paths are dynamically optimal within the context of the stochastic operating environment. We derive optimal conditions for multiple switching across multiple modes and the choice of dynamic control paths. We illustrate this method within the context of operation of hydroelectric power generation that also impacts downstream water flow and use. There, we consider irrigation of agricultural crops, the growth of which is dependent on the water flow. We show our approach generates solutions that can direct management to switch modes of operation through time as new information about the current or future stochastic environment changes. This stands in contrast with rule-based water flow management or simple static optimization based on forecasts and secondary objectives specified as constraints. Our approach allows for considerable generalization of the problem's dimensions as compared to past studies of optimal switching. Approaches based on lattices (binomial or trinomial tree) are limited in their capacity expand dimensionality with in dynamic optimal control problems given their focus on element-wise evolution of states which implies the computational complexity increases exponentially with dimensionality. IN contrast, the approach we present assumes pathwise evolution of the stochastic states, implying that computational complexity increases only linearly allowing expansion of dimensionality to be feasible conditional on computational resources. Finally, our approach can be further generalized to consider true uncertainty with robust methods, and to consider multiple stochastic processes with intertemporal correlation.

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