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A SIMHEURISTIC ALGORITHM FOR RELIABLE ASSET AND LIABILITY MANAGEMENT UNDER UNCERTAINTY SCENARIOS

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ABSTRACT

The management of assets and liabilities is of critical importance for insurance companies and banks. Complex decisions need to be made regarding how to assign assets to liabilities in such a way that the overall benefit is maximised over a time horizon. In addition, the risk of not being able to cover the liabilities at any given time must be kept under a certain threshold level. This optimisation challenge is known in the literature as the asset and liability management (ALM) problem. In this work, we propose a biased-randomized (BR) algorithm to solve a deterministic version of the ALM problem. Firstly, we outline a greedy heuristic. Secondly, we transform it into a BR algorithm by employing skewed probability distributions. The BR algorithm is then extended into a simheuristic by incorporating Monte-Carlo simulation to deal with the stochastic version of the problem.

1 INTRODUCTION

Financial institutions have to face some critical risk-management processes. Among such processes, asset and liability management (ALM) is of paramount importance due to its potential consequences. ALM consists of a range of techniques necessary to invest adequately, so that the firm's long-term liabilities are met (Ziemba et al. 1998). For an insurance company, a liability constitutes the legal responsibility to repay the insurance contributions that the customer has been making over an agreed length of time, which are increased by the interest rate. This is a typical transaction of pension or life insurance intended to secure retirement income, which gives rise to a three-tier financial problem. First, the insurance company receives the customer's premium. Second, the company invests this premium in the long term, so that the financial benefit envisaged in the insurance policy is secured. Third, in the event of the customer's retirement or death, the insurance company needs to have sufficient funds to meet its liability to the customer. While the aforementioned financial problem unfolds, the insurance company is confronted with a range of risks, which arise either from its role as a financial intermediary or due to complex regulations as well as economic and social policies. If the insurer's obligation to the customer is not honoured, its default becomes a likely scenario. A default can be very costly for the firm, since it can inflict a loss of credibility and reputation. On the one hand, it can face legal action from its creditors. As a result the insurer may be forced to pay hefty fines by the regulatory body. On the other hand, the firm's market share may diminish as its customers may switch to other insurers.

It is thus not surprising that the ALM problem has been widely studied in the literature. As interest rates vary over time, the present value of both assets and liabilities responds to such variation. Consequently, optimal and smart asset management solutions become critical to the insurer, who seeks to ensure that the liabilities can be met at the time when they are required, while at the same time, the value of the firm is maximised. In practical applications, one of most popular solutions to this asset management problem is cash-flow matching (Iyengar and Ma 2009), whose main objective is to ensure the timely payment of the liabilities. This approach minimises the number of contractual breaches. In some European countries, the legislation does not envisage any specific mechanism to ensure that the firm's obligations are met. Instead, capital is regulated by targeting the value of the reserves that the company needs to build on its balance sheet. In general, regulations impose a specific interest rate to calculate the provisions of the firm's liabilities over the short and medium term. Sufficient provisions are required to achieve the solvency of the firm. Furthermore, if the firm's manager can prove that its assets are adequate to cover its liabilities in the long term, the firm is granted permission to use a higher interest rate in its provisions. This allows its capital value on the balance sheet to be lower.

Heuristic and metaheuristic algorithms have become a new standard when dealing with complex and large-scale portfolio optimisation and risk management problems (Soler-Dominguez et al. 2017; Doering et al. 2019). Hence, in this paper we first propose a constructive heuristic to solve the deterministic version of the ALM problem. The greedy behaviour of the heuristic is then relaxed by using a skewed probability distribution, thus transforming it into a biased-randomized (BR) algorithm. This BR algorithm is able to generate many promising solutions to the deterministic version of the ALM problem. Finally, this probabilistic algorithm is extended into a full simheuristic one (Juan et al. 2018) in order to deal with the stochastic version of the ALM problem, in which liability values are modelled as random variables. Thus, our simheuristic algorithm finds out which assets of a firm's portfolio can be efficiently used to reduce the risk of liability default while minimising the monetary cost for the company. The rest of the paper is structured as follows: Section 2 provides a brief literature review on ALM. Section 3 discusses the typical cash-flow behaviour in both assets and liabilities. Section 4 provides a formal model for the optimisation problem being analysed. Section 5 presents recent work on BR algorithms and simheuristics. Section 6 proposes a greedy heuristic as an initial solving method and its extension to a BR algorithm and a full simheuristic. A series of computational experiments are carried out in Section 7, while Section 8 provides an analysis of the obtained results. Finally, Section 9 highlights the most relevant findings of our work and points out future research lines.

2 RECENT WORK ON ASSET AND LIABILITY MANAGEMENT

The scientific literature on ALM is quite extensive and covers several decades. Due to space limitations, we focus on research published over the last two decades. Stochastic programming models have been widely used to improve financial operations and risk management. Hence, building on multi-stage stochastic programming to model a pension fund, Kouwenberg (2001) develops scenario-generation methods for the ALM. Gondzio and Kouwenberg (2001) combine decomposition methods and high-performance computing to cope with large-scale instances of the problem. These authors simulated over 4 million scenarios, 12 million constraints, and 24 million variables to study a pension fund. Dempster et al. (2003) combine dynamic stochastic optimisation with Monte Carlo simulation to analyse an ALM problem involving global asset classes and contribution pension plans. Arguably, their approach can also be used to manage financial planning problems related to insurance firms, risk capital allocation, and corporate investment, among others. Additional applications and case studies on ALM can be found in Zenios and Ziemba (2007). Also, Kouwenberg and Zenios (2008) review stochastic programming models for ALM. Among other issues, they analyse the performance of these models when applied to pension funds, discussing both their advantages and limitations.

More recently, Ferstl and Weissensteiner (2011) consider a multi-stage ALM under time-varying investment opportunities. To minimise the conditional value-at-risk of shareholder value, the authors utilise stochastic linear programming and a decomposition of the benefits in dynamic re-allocation. Examples of specialised books dedicated to ALM are Bauer et al. (2006), Adam (2008), Mitra and Schwaiger (2011), and Choudhry (2011). Gülpinar and Pachamanova (2013) present an ALM model based on robust optimisation techniques. Their model incorporates a time-varying aspect of investment opportunities. These authors perform a series of computational studies with real market data in order to compare the performance of their approach to that of classical stochastic programming. More recent approaches to ALM focus on the mean-variance ALM with constant elasticity of variance (Zhang and Chen 2016), random coefficients (Wei and Wang 2017), or stochastic volatility (Li et al. 2018). Fernández et al. (2018) introduce a stochastic ALM model for a life insurance company. They use GPUs to run Monte Carlo simulations in parallel. Dutta et al. (2019) employ big data analytics and stochastic linear programming in ALM under uncertain scenarios. The authors study the relevance of employing a large number of scenarios in solving the stochastic ALM problem. Finally, Li et al. (2019) use a multi-period mean-variance model to analyse an ALM problem with probability constraints. In their model, investors seek to control for the probability of bankruptcy, while the process is influenced by uncertainty in the cash flows.

3 CASH FLOWS OF LIABILITIES AND ASSETS

Under an insurance policy, the insurer is liable to pay whenever the event described in the contract takes place. This is a 'must' obligation that the insurer has to honour. Otherwise, the company would face a hefty monetary fine, its reputation would be severely damaged, and its administrators could be taken to court. The insurer's liabilities comprise all policies subscribed by its customers. This aggregation results in an irregular and difficult-to-predict cash flow structure. Indeed, each policy has a different maturity and size, and is bound to a set of conditions. Figure 1 illustrates a series of liabilities (L) and assets (A) with different monetary values (vertical axis) and time occurrence (horizontal axis).

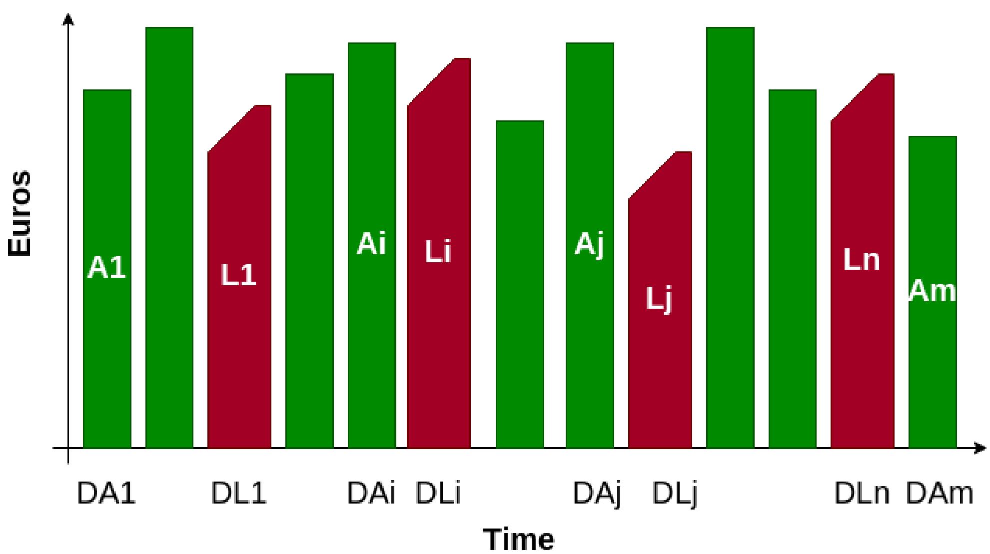

Figure 1: An illustrative representation of different assets and liabilities over time.

Once the structure of liabilities and assets is known over a time horizon, the manager is tasked to select a set of assets to cover the liabilities in each period. Because of the opportunity cost of these assets, the total value of those selected should be the minimum required, since these assets remain 'frozen' and cannot be used for any other purpose. In other words, once the assets that will cover the firm's liabilities have been selected, they cannot be used in any other transaction. Therefore, this results in an optimisation problem, in which a set of minimum-value assets has to be determined to cover the firm's liabilities. Corporate and government bonds are the predominant asset classes in the insurance market, since returns on a bond market investment can be accurately predicted in advance. The static assumption makes it simpler to predict the value of assets, as opposed to the value of liabilities. It is also worth noting that assets feature a significantly shorter span time than liabilities. For instance, while insurance contracts cover the customer's retirement or full life – which can span over 45 years – typical maturities of bond market instruments do not extend beyond 30 years. This generates a maturity mismatch between assets and liabilities. In addition, while liability cash flows might arise at any moment in time, the cash-flow structure of assets is more concentrated around some particular time periods.

4 MODELLING A STOCHASTIC ASSET-LIABILITY MANAGEMENT PROBLEM

In this paper, we first propose a deterministic and relatively simple version of the ALM problem, which is then extended into a stochastic version by modelling liability values as random variables instead of assuming that they are constant values. The deterministic version allow us to use exact methods to generate optimal solutions –at least in some small-sized instances. It is possible then to compare the results of our BR algorithm against these optimal values, which contributes to validate, at least in the deterministic scenario, the quality of the proposed methodology.

Given a set of liabilities L and a set of assets A, the binary variable x_{al} takes the value 1 if asset $a \in A$ is employed to cover liability $l \in L$, being 0 otherwise. Let us denote by t_a the time at which asset $a \in A$ becomes available, and by v_a its value at that time. Similarly, let t_l represent the maturity date of liability $l \in L$, and v_l the associated value to be covered. Our initial goal is to find the asset-to-liability mapping, $(a_{l1}, a_{l2}, \ldots, a_{l|L|})$, that minimises the aggregated net present value (NPV) of the assets employed to cover all of our liabilities, i.e.:

$$min \sum_{a \in A} \sum_{l \in L} \frac{v_a}{(1+d)^{t_a}} x_{al},$$

where d is the discount factor used for calculating the NPV of an asset. Also, we need to make sure that each liability $l \in L$ is covered by exactly one asset $a \in A$, i.e.:

$$\sum_{a \in A} x_{al} = 1, \ \forall l \in L.$$

Likewise, we need to ensure that no asset is assigned to more than one liability:

$$\sum_{l \in L} x_{al} \leq 1, \ \forall a \in A.$$

Also, for each liability $l \in L$, the asset assigned to l needs to be available on or before t_l, i.e.:

$$\sum_{a \in A} x_{al} t_a \leq t_l, \ \forall l \in L.$$

Likewise, it is required that if an asset $a \in A$ is selected to cover a liability $l \in L$, then the monetary value of a (at the time it becomes available) has to be equal or higher than the monetary value of l (at its maturity date):

$$\sum_{a \in A} x_{al} v_a \geq v_l, \ \forall l \in L.$$

Finally, we can add the binary variables to complete the model:

$$x_{al} \in \{0,1\}, \ \forall a \in A, \forall l \in L.$$

In this paper, we will also consider a stochastic version in which the value of any liability $l \in L$ is modelled as a positive random variable, V_l. Therefore, Equation (4) will be transformed in minimising the expected aggregated NPV, while Equation (4) will be substituted by the following probabilistic constraint:

$$Pr\left[\sum_{a \in A} x_{al} v_a \geq V_l\right] \geq p, \ \forall l \in L,$$

where p is a user-defined probability related to the reliability level required of a solution in order to avoid costly defaults. Actually, given a solution of the problem, $s = (a_{l1}, a_{l2}, \ldots, a_{l|L|})$, its associated reliability level, $R(s)$ can be computed as: $R(s) = \prod_{l \in L} Pr(v_{a_l} \geq V_l)$.

5 RECENT WORK ON BIASED-RANDOMIZED ALGORITHMS AND SIMHEURISTICS

By combining skewed probability distributions with Monte Carlo simulation, biased-randomized techniques can be used to transform a greedy heuristic into a probabilistic algorithm. One of the main advantages of BR algorithms is their ability to generate multiple promising solutions that still follow the logic behind the original heuristic (Juan et al. 2009). BR techniques have been successfully used during the last years to solve different rich and realistic variants of vehicle routing problems (Fikar et al. 2016), permutation flow-shop problems (Ferrer et al. 2016), location routing problems (Quintero-Araujo et al. 2017), facility location problems (Pages-Bernaus et al. 2019), waste collection problems (Gruler et al. 2017), horizontal cooperation problems (Quintero-Araujo et al. 2019), and constrained portfolio optimisation problems (Kizys et al. 2019).

A different concept, also combining simulation with heuristic optimisation, is that of simheuristics (Rabe et al. 2020). Simheuristics can be seen as an extension of metaheuristics, since a simulation module is integrated inside the metaheuristic framework to efficiently deal with *NP-hard* and large-scale stochastic optimisation problems (Ferone et al. 2019). Notice that simheuristics might employ any type of simulation, e.g., discrete-event, agent-based, or Monte Carlo. These algorithms have also been used recently in multiple sectors, including: flow-shop scheduling (Hatami et al. 2018), waste collection management (Gruler et al. 2017), vehicle routing (Gonzalez-Martin et al. 2018; Guimarans et al. 2018), Internet computing (Cabrera et al. 2014), finance (Panadero et al. 2020), e-commerce (Pages-Bernaus et al. 2019), system reliability (Faulin et al. 2008), and inventory routing (Gruler et al. 2018; Gruler et al. 2020). All in all, both BR algorithms and simheuristics demonstrate the great potential that simulation has when combined with heuristics and metaheuristics, either for solving *NP-hard* deterministic optimisation problems as well as to cope with their stochastic counterparts.

6 FROM A GREEDY HEURISTIC TO A SIMHEURISTIC

Figure 2 offers an overview of our solving approach. First, a fast constructive heuristic is designed to solve the deterministic version of the problem. The heuristic completes the following steps: *(i)* it sorts the list of liabilities from the most challenging ones (i.e., those with higher values to cover) to the less challenging ones; *(ii)* it computes the NPV for each asset; and *(iii)* for each liability in the sorted list, it chooses the asset with the minimum NPV among a list including the ones that occur on or before the maturity date of the liability, with a value exceeding that of the liability. Covering the largest liabilities first as efficiently as possible helps to reduce the value of the frozen assets. Algorithm 1 provides the pseudo-code of this greedy heuristic. In the second step, the previous heuristic is transformed into a biased-randomized algorithm by using a Geometric probability distribution to introduce a small random deviation in the order in which the assets are selected from the sorted-by-NPV list (but still respecting the time-precedence constraint). The single parameter of the Geometric distribution $\alpha \in (0,1]$ defines the probability that the first element of the sorted-by-NPV list is selected, subsequent elements have a diminishing probability of being selected. The BR algorithm is capable of generating multiple solutions per second, all of them following the heuristic criterion (but with greed biased randomization), with some of them outperforming the solution provided by the greedy heuristic itself. Now, in the final step the most promising solutions generated by the BR algorithm are sent to a Monte Carlo simulation process, where a number of executions are run using randomly generated values for the stochastic variables V_l ($\forall l \in L$).

Algorithm 1 Greedy Heuristic.

```
Sort liabilities list L from higher to lower v_l (∀l ∈ L)
sol ← ∅
for each liability l in L do
   Consider A* = {a ∈ A : t_a ≤ t_l ∩ v_a ≥ v_l}
   a_l ← argmin{NPV(v_a) : a ∈ A*}
   A ← delete a_l
   sol ← add a_l
end for
return sol
```

The simulation does not only estimate the expected cost associated with any of the promising solutions generated by the BR algorithm, but it also provides estimates to other key performance indicators, e.g.: the variability of the values that each solution generates (which can be used to compare different solutions in a multiple boxplot), or the reliability level of each solution –measured as the probability that the corresponding assets-to-liability mapping can be implemented in a stochastic environment without suffering any default (i.e., the probability that all selected assets have successfully covered the assigned liabilities).

7 COMPUTATIONAL EXPERIMENTS

The proposed approach has been implemented as a Java application running on a CPU with 3.60 GHz and 16 GB of RAM. Several instances have been generated. Table 1 provides some details on the number of assets and liabilities for each instance, as well as the associated discount rate and value modifier –when applicable. Assets and liabilities have been randomly distributed over time using a uniform probability distribution from 0 to 100 and from 50 to 150, respectively. Likewise, values for assets and liabilities have been randomly generated using a uniform probability distribution from 0 to 1 and from 0 to 0.5, respectively. This approach results in feasible instances in which it is possible to cover all of the liabilities. Additionally, asset values in instance 4 have been modified to consider a scenario where their value increases over time, i.e.: given an asset $a \in A$ with a value v_a at time t_a, a new value v'_a is computed $v'_a = v_a(t_a/T)$, with $T = \max\{t_a : a \in A\}$. Likewise, a scenario with decreasing asset values is considered in instance 5 by using $v'_a = v_a(1-t/T)$. As specified in Table 1, similar modifications have been performed on liability values for instances 6 and 7.

Some initial experiments have allowed us to set the parameter α associated with the geometric probability distribution that drives the BR algorithm. In this case, a reasonably good performance seems to be reached when $\alpha \in (0.70, 0.95)$. In the stochastic scenario, and in order to search for more reliable solutions, a 'safety-stock' value has been considered. Thus, given a liability $l \in L$, only assets with a value exceeding $E[V_l] + \lambda\sigma_{Vl}$ can be selected to cover l, where σ_{Vl} refers to the standard deviation of V_l and $\lambda \in \{0,1,2,3\}$. Notice that the higher the value of λ, the more reliable the solution will be – i.e., a solution built with a relatively high value of λ will be able to 'absorb' a higher degree of variability in V_l without suffering a default. However, it is also true that increasing λ comes at the cost of using assets with a higher value, which will tend to increase the objective function. A total of 10 seconds per instance has been allowed. Each 'promising' solution s generated by the BR algorithm is sent to the Monte Carlo simulation module. In our experiments, a solution was considered to be promising if its deterministic value was equal or better than the one provided by the greedy heuristic. Once in the simulation module, a total of 250 runs are executed per solution. These runs employ random observations from the probability distribution modelling

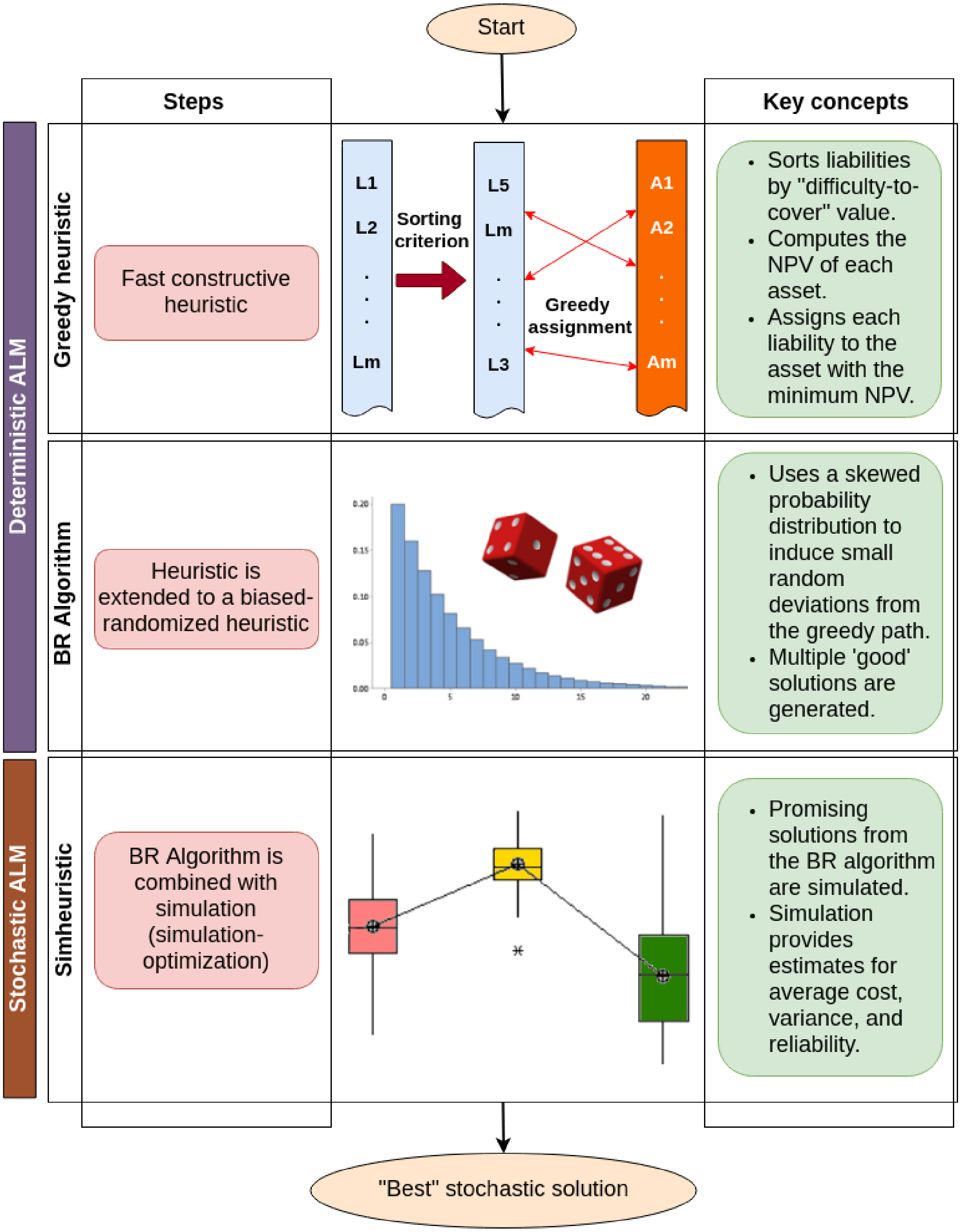

Figure 2: Schematic representation of our solving approach.

Table 1: Characteristics of the set of instances.

#	Instance	# assets	# liabilities	Discount rate	Asset value modifier	Liability value modifier
1	Control_Instance	1000	200	0.05	-	-
2	Large_x3	3000	600	0.05	-	-
3	Large_x5	5000	1000	0.05	-	-
4	Asset_Value_Increases	1000	200	0.05	t/T	-
5	Assets_Value_Decreases	1000	200	0.05	$1-(t/T)$	-
6	Liability_Value_Increases	1000	200	0.05	-	t/T
7	Liability_Value_Decreases	1000	200	0.05	-	$1-(t/T)$
8	Reduced_Discount_Rate	1000	200	0.005	-	-
9	Liabilities_x2	1000	400	0.05	-	-

each V_l, $\forall l \in L$. Given a solution $s = (a_{l1}, a_{l2}, \ldots, a_{l|L|})$, for each asset a_l assigned to a liability l, a penalty cost p_l is incurred whenever $V_l > v_{a_l}$ (this cost represents the 'failure-to-cover-a-liability' situation). In that case, $p_l = 2(V_l - v_{a_l})$. Therefore, under the stochastic scenario, the total cost of a solution is computed as the aggregated NPV of its assets plus all the penalty costs incurred. In summary, several statistics can be obtained from the simulation component, among others: *(i)* expected total cost of a solution s; and *(ii)* reliability of s, computed as the percentage of runs in which the mapping has been implemented without any default.

Table 2 provides the experimental results. The first column contains the instance number (same as in Table 1). The second column contains the value of λ used to compute the 'safety-stock' as described before. For $\lambda = 0$, the adjacent column contains the optimal value for each instance as provided by the popular IBM Cplex solver (this value refers just to the deterministic scenario, and it allows us to validate the results provided by the greedy heuristic). The next three columns refer to the greedy heuristic: *BDS-D* represents the best-deterministic solution evaluated in a deterministic scenario, while *BDS-S* represents the cost of the same asset-to-liability mapping plan being evaluated in a stochastic scenario. The reliability of the mapping is also provided. Likewise, for the BR algorithm, the corresponding columns show the best-stochastic solution under the uncertainty (*BSS-S*), and its associated reliability. Finally, some gaps between pairs of columns are also provided.

8 ANALYSIS OF RESULTS

As it can be seen in Table 2, the greedy heuristic is providing reasonably good solutions when compared with the optimal ones given by Cplex for the deterministic scenario with $\lambda = 0$. Notice, however, that Cplex is not able to solve all instances since it gets an "out of memory" (OoM) error for instance 3 (which justifies the need of using heuristics even for the deterministic case). Also, notice that the cost of the greedy mapping is quite different in the deterministic scenario (*BDS-D*) and in the stochastic one (*BDS-S*), as can be easily appreciated in Figure 3 (instance 8 has been removed from this multi-boxplot figure since its values were outliers that make a clear view difficult in this case). In other words, it is not possible to use the deterministic cost as a good estimate of the stochastic one – therefore, a simulation component is required while solving the stochastic scenario. Regarding reliability values, one can observe that these values rise as λ increases. However, increasing the 'safety stock' will also lead to selecting more expensive assets and, accordingly, to solutions with a typically higher NPV cost. Moreover, in the case of instance 4 no feasible solution is obtained when λ is set to its maximum level (i.e., for $\lambda = 3$ the algorithm cannot find assets with the requested high value). Finally, a relevant result is that the best-stochastic solution provided by the BR algorithm for the stochastic scenario (*BSS-S*) is frequently able to outperform the equivalent *BDS-S*. This justifies the need for using the BR algorithm, which provides different alternative solutions for the simulation component to evaluate.

Table 2: Results obtained for each instance and λ value.

			Greedy			BR Algorithm		Gaps			
#	λ	Cplex (1)	BDS-D (2)	BDS-S (3)	Rel. (4)	BSS-S (5)	Rel. (6)	(2) - (1)	(3) - (2)	(5) - (3)	(6) - (4)
1	0	1.25	1.26	2.83	0.00	2.68	0.00	0%	125%	-5.2%	0.00%
1	1	–	1.34	1.59	0.00	1.56	0.00		18%	-1.7%	0.00%
1	2	–	1.43	1.45	0.27	1.45	0.24		1%	-0.4%	-13.24%
1	3	–	1.56	1.56	0.92	1.56	0.95		0%	0.0%	3.93%
2	0	3.73	3.73	10.22	0.00	9.94	0.00	0%	174%	-2.8%	0.00%
2	1	–	4.03	5.19	0.00	5.12	0.00		29%	-1.4%	0.00%
2	2	–	4.33	4.43	0.00	4.42	0.00		2%	-0.2%	0.00%
2	3	–	4.61	4.62	0.69	4.61	0.70		0%	-0.1%	0.58%
3	0	OoM	6.19	17.45	0.00	17.15	0.00	OoM	182%	-1.7%	0.00%
3	1	–	6.70	8.81	0.00	8.73	0.00		32%	-0.9%	0.00%
3	2	–	7.23	7.42	0.00	7.41	0.00		3%	-0.1%	0.00%
3	3	–	7.69	7.70	0.47	7.70	0.47		0%	0.0%	0.00%
4	0	1.22	1.23	2.75	0.00	2.61	0.00	1%	124%	-5.3%	0.00%
4	1	–	1.31	1.56	0.00	1.55	0.00		19%	-0.8%	0.00%
4	2	–	1.42	1.44	0.24	1.44	0.25		1%	-0.2%	3.33%
4	3		Inf.	Inf.		Inf.					
5	0	3.66	3.66	5.87	0.00	5.74	0.00	0%	60%	-2.1%	0.00%
5	1	–	4.27	4.69	0.00	4.66	0.00		10%	-0.5%	0.00%
5	2	–	5.02	5.05	0.12	5.05	0.10		1%	-0.1%	-10.34%
5	3	–	5.85	5.85	0.84	5.85	0.88		0%	0.0%	4.74%
6	0	5.99	7.97	8.49	0.00	8.21	0.00	33%	6%	-3.3%	0.00%
6	1	–	8.28	8.35	0.00	8.09	0.00		1%	-3.1%	0.00%
6	2	–	8.17	8.18	0.42	7.98	0.39		0%	-2.4%	-8.49%
6	3	–	8.76	8.76	0.94	8.53	0.95		0%	-2.6%	0.42%
7	0	10.06	10.18	10.74	0.00	10.65	0.00	1%	6%	-0.8%	0.00%
7	1	–	10.88	10.97	0.00	10.83	0.00		1%	-1.3%	0.00%
7	2	–	11.23	11.24	0.47	11.23	0.45		0%	-0.1%	-4.27%
7	3	–	11.71	11.71	0.95	11.65	0.97		0%	-0.5%	1.68%
8	0	33.99	34.01	36.36	0.00	36.29	0.00	0%	7%	-0.2%	0.00%
8	1	–	37.10	37.50	0.00	37.49	0.00		1%	0.0%	0.00%
8	2	–	39.98	40.01	0.11	40.00	0.12		0%	0.0%	7.14%
8	3	–	42.82	42.82	0.85	42.81	0.90		0%	0.0%	5.16%
9	0	3.58	3.63	6.98	0.00	6.78	0.00	1%	92%	-2.7%	0.00%
9	1	–	3.98	4.56	0.00	4.52	0.00		15%	-0.9%	0.00%
9	2	–	4.24	4.29	0.04	4.28	0.04		1%	-0.2%	0.00%
9	3	–	4.58	4.59	0.83	4.58	0.84		0%	-0.1%	0.48%

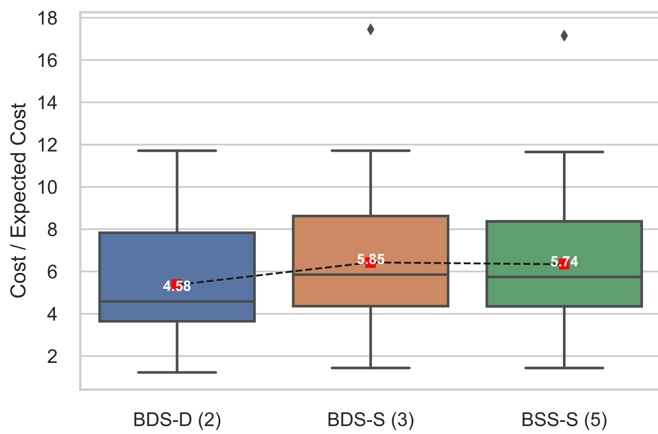

Figure 3: A boxplot comparison of different solutions.

9 CONCLUSIONS

This paper proposes a simheuristic approach for a stochastic version of the asset and liability management problem. The paper first addresses the deterministic version of the problem by introducing a greedy heuristic as well as a biased-randomized algorithm. The latter is then extended into a full simheuristic by integrating simulation into the optimisation framework. Our method is flexible and it can be easily extended for new constraints, such as those due to financial regulations or the firm's strategic plans.

The results show that the best deterministic mapping of assets to liabilities is far from being an optimal solution when uncertainty is present. Hence, simulation-optimisation methods become necessary to generate high-quality solutions whenever some components of the asset and liability management problem need to be modelled as random variables instead of deterministic values. Also, according to our computational experiments, the savings generated by the simheuristic are noticeable. Considering that the insurance market is strongly regulated, having an efficient, flexible, and easy-to-implement method to select the proper assets inside a firm's portfolio is extremely important.

As future work, we plan to: *(i)* extend our probabilistic algorithm into a full metaheuristic; *(ii)* include additional characteristics in the model so it fully represents the real-life problem that insurance companies and other financial institutions have to face; and *(iii)* introduce and test the algorithm in real-life bench-mark instances.

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