ON THE SCARCITY OF OBSERVATIONS WHEN MODELLING RANDOM INPUTS AND THE QUALITY OF SOLUTIONS TO STOCHASTIC OPTIMISATION PROBLEMS

Canan G. Corlu

Metropolitan College Boston University 1010 Commonwealth Avenue Boston, MA, 02215, USA Javier Panadero Angel A. Juan

Universitat Oberta de Catalunya – IN3 Euncet Business School Av. Carl Friedrich Gauss 5 Castelldefels, 08860, SPAIN

Bhakti Stephan Onggo

CORMSIS – Southampton Business School University of Southampton University Road Southampton, SO17 1BJ, UK

ABSTRACT

Most of the literature on supply chain management assumes that the demand distributions and their parameters are known with certainty. However, this may not be the case in practice since decision makers may have access to limited amounts of historical demand data only. In this case, treating the demand distributions and their parameters as the true distributions is risky, and it may lead to sub-optimal decisions. To demonstrate this, this paper considers an inventory-routing problem with stochastic demands, in which the retailers have access to limited amounts of historical demand data. We use simheuristic method to solve the optimisation problem and investigate the impact of the limited amount of demand data on the quality of the simheuristic solutions to the underlying optimisation problem. Our experiment illustrates the potential impact of input uncertainty on the quality of the solution provided by a simheuristic algorithm.

1 INTRODUCTION

Most of the literature on supply chain management assumes that the demand distributions and their parameters are known with certainty. This may not be the case in practice. There may be cases where data collection is infeasible, or feasible but too expensive. Although more data have become available in the era of big data, there are still some practical situations in which decision makers have access to a limited amount of data. For example, when launching new products with shorter life cycles, the sales data does not exist and companies have to rely on historical sales data from similar products. Hence, there is no guarantee that the data will be representative. Even in cases where decision makers have access to abundant data, firms may prefer to use the most recent data to be able to capture the ever changing market conditions. The phenomenon "big data dreams, small data reality" that was introduced by Brian Lewis –chief data scientist and co-founder of Fractal Sciences– in the January/February 2014 issue of the *Analytics* magazine refers to this practical situation (Lewis 2014).

A typical practice when modelling random inputs in simulation is to find the best-fit probability distribution –including its parameter values– using a maximum likelihood estimation (MLE) method. The maximum likelihood estimates of the parameters of a probability distribution can be shown to approach their true values when the number of historical data points approach infinity (Rohatgi and Saleh 2015). However, we are rarely fortunate to be in that asymptotic situation and we often have access to only limited amounts of historical data. In that case, the use of the maximum likelihood estimates (finite-sample estimates) as if they are the true parameter values leads to parameter uncertainty. Obviously, as the length of the historical data approaches infinity, parameter uncertainty disappears. However, it is there in the presence of limited amounts of data.

In combinatorial optimisation problems with stochastic inputs that need to be modelled using a probability distribution, this lack of observations might cause an incorrect selection of the underlying probability distribution or its parameters. Figure 1 illustrates this scenario: let's assume that the customer's real demand is a random variable following a log-normal probability distribution with parameters $\mu = 2.34$ and $\sigma = 0.83$; however, if only a random sample of 10 observations of this demand were available to the decision maker, the best-fit model would correspond to a log-normal probability distribution with parameters $\mu = 2.198$ and $\sigma = 1.153$. In other words, due to the lack of data, the best-fit model that can be obtained might differ from the real probability distribution that governs the problem stochastic input. As we collect more data (e.g. 100 and 10,000 observations), the estimated parameters become closer to the true parameters. Notice that the log-normal probability distribution is frequently used in the literature to model random variables that can only take positive values (Faulin et al. 2008).

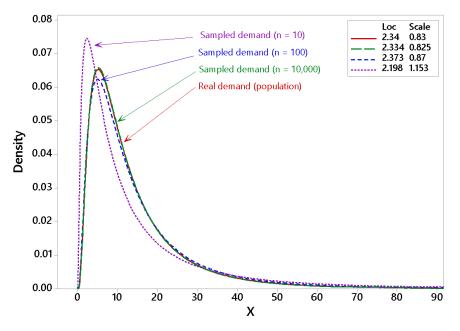


Figure 1: Illustrating a deviation from the real model due to a limited sample size.

The main goal of this paper is to investigate how the inaccuracy in the estimates of the probability distribution caused by the limited data can affect the quality of solutions to stochastic optimisation problems. In order to numerically quantify this 'inputs inaccuracy effect over the solution', we make use of a numerical example based on the inventory routing problem (IRP), which combines a vehicle routing problem (VRP) with an inventory management problem –a more detailed explanation on the IRP is given in Section 2. Being an extension of the VRP, the IRP is also *NP-hard* (Lenstra and Kan 1981). Hence, several heuristic-based approaches have been proposed for both the deterministic version –where the demands are assumed to be exactly known– and the stochastic version –where the demands are assumed to follow a probability

distribution. Among other methods, simheuristic algorithms have been used to deal with different variants of the stochastic IRP (Onggo et al. 2019). Simheuristics integrate simulation with optimisation heuristics to provide high-quality solutions to stochastic optimisation problems (Juan et al. 2018).

The remainder of the paper is organised as follows. Section 2 describes in more detail the stochastic IRP employed to test the effect of small-size samples in the quality of the solutions. Section 3 reviews related work on both the stochastic IRP and the issue of parameter uncertainty in stochastic simulations. Section 4 illustrates our numerical study, and Section 5 concludes by highlighting the main contributions of this work and potential future research directions.

2 TESTING EXAMPLE: THE STOCHASTIC INVENTORY ROUTING PROBLEM

As described in Gruler et al. (2018), in IRP with stochastic demands, a product is distributed from a central warehouse to several distribution centres (DCs), where stock-outs might occur. A stock-out happens at a DC whenever the aggregated customers' demand associated with that DC exceeds the available inventory at the DC. In that case, a penalty cost is charged. This cost is equivalent to a refill round-trip from the warehouse to that DC. We assume that the initial inventory level at each DC is known, and the goal of the optimisation problem is to minimise the total expected cost, which includes routing and inventory costs. This IRP is illustrated in Figure 2. Because the IRP includes both inventory management decisions and

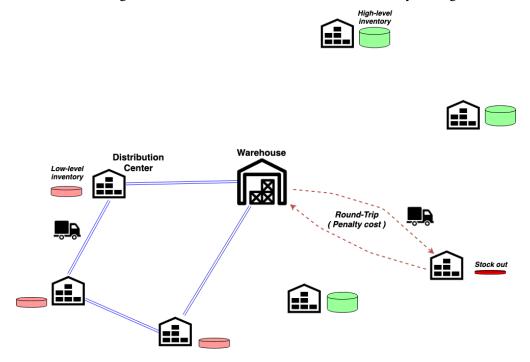


Figure 2: A simple example of the inventory routing problem.

routing decisions, a typical IRP aims to minimise the summation of the routing costs and the inventory costs. The following lays out the operational plan and calculation of the related costs. The operational plan starts with finding the optimum replenishment levels at each DC, which has a direct impact on the quantities to be delivered from the warehouse to each DC. More specifically, if the current stock level at DC *i*, s_i , is smaller than the replenishment level at DC *i*, r_i , the difference between these two quantities $(q_i = r_i - s_i)$ is delivered to the DC. Otherwise, no delivery takes place. Letting $N \in \{0, 1, 2, ..., n\}$ denote the set composed of all DCs including the warehouse (represented as node 0), we determine q_i as defined in Equation (1):

$$q_i = \begin{cases} r_i - s_i & \text{if } s_i < r_i \\ 0 & \text{if } s_i \ge r_i \end{cases} \quad \forall i \in N \setminus \{0\}$$

$$(1)$$

The next step is finding the optimum routes. Notice that we only visit a DC if the quantity to be delivered to that centre is greater than zero; i.e, $q_i > 0$. We assume the availability of *K* homogeneous vehicles (i.e., vehicles with the same loading capacity) located at the warehouse. Let *S* be the set of edges that connect the nodes in $N \in \{0, 1, 2, ..., n\}$, and y_{ij}^k to be a binary variable that takes the value of 1 if the edge (i, j) is visited by vehicle $k \in K$, and $c_{ij} = c_{ji} > 0$ is the cost of travelling from *i* to *j*. Then the total routing cost is represented as defined in Equation (2):

$$RC = \sum_{(i,j)\in S} \sum_{k\in K} c_{ij} \cdot y_{ij}^k$$
⁽²⁾

The final step is the realisation of the customer demands and the computation of the inventory costs. Notice that there are two (mutually exclusive) components of the inventory costs: holding cost and stock-out cost. More specifically, if the demand is less than the summation of the quantity delivered to the DC (i.e., q_i) and the current stock level of that DC (i.e., s_i), then a holding cost is incurred. Otherwise, a stock-out occurs, in which case, we assume that the only penalty cost is a round-trip cost from the warehouse to that DC. We formulate the inventory costs at a single center as given in Equation (3):

$$IC_{i} = \begin{cases} h_{i}(q_{i}+s_{i}-d_{i}) & \text{if } q_{i}+s_{i} \ge d_{i} \\ 2 \cdot c_{i0} & \text{if } q_{i}+s_{i} < d_{i} \end{cases} \quad \forall i \in N \setminus \{0\}$$

$$(3)$$

In this representation, h_i stands for the holding cost for each unit held at DC *i* and c_{i0} represents the cost of a trip from the warehouse to DC *i*. Because demand is random, we are interested in expected inventory costs over all DCs, which we represent as described in Equation (4):

$$E[I] = \sum_{i \in N \setminus \{0\}} E[IC_i]$$
(4)

The objective function that we want to minimise is the the total expected cost, i.e. the sum of Equations (2) and (4). Notice that we are considering in this paper a single-period inventory routing problem. Also, it is assumed that there is infinite inventory at the warehouse, and that there is no cost of holding inventory at the warehouse.

3 RELATED WORK

In this section we discuss relevant literature in the two streams of research: (i) the stochastic IRP and simheuristic algorithms for solving it; and (ii) work on parameter uncertainty in stochastic simulations.

Although the deterministic IRP has been studied extensively, the focus on stochastic IRP has remained limited. Among the notable solution approaches proposed for stochastic IRPs are Markov decision process (Adelman 2004), dynamic programming (Kleywegt et al. 2004), scenario trees (Hvattum et al. 2009), dynamic programming (Bertazzi et al. 2013) and metaheuristic methods (Huang and Lin 2010; Bertazzi et al. 2015). Juan et al. (2014) propose a simheuristic approach for the stochastic IRP. It integrates a multi-start metaheuristic with Monte Carlo simulation to solve a single-period IRP with stochastic demands. Later, Gruler et al. (2018) provide an enhanced simheuristic algorithm, while Gruler et al. (2020) extend it to the multi-period stochastic IRP. Recently, Onggo et al. (2019) propose a simheuristic algorithm for solving the multi-period IRP with stochastic demands and perishable products. However, these simheuristic approaches assume that the demand distributions and their parameters are known with certainty. This paper relaxes this assumption, and studies how the existence of limited observations to model the input random variables may affect the quality of the IRP solutions.

The estimation of the demand parameters from limited amounts of historical demand data, and the use of these demand parameters as if they are the "true" demand parameters estimated from an infinite amount of data leads to an uncertainty that is referred to as parameter uncertainty in the stochastic simulation literature. There has been a significant body of research that studies proper modeling of the parameter uncertainty. See review papers Henderson (2003), Chick (2006), Barton (2012), Song et al. (2014), Lam (2016), and Song and Nelson (2017) for an excellent review of the methods that are available to account for parameter uncertainty in stochastic simulations. The danger of ignoring parameter uncertainty has also been investigated in different contexts. The ones more related to our work are Muñoz et al. (2013), Muñoz and Muñoz (2015), Akcay and Corlu (2017), and Corlu et al. (2019). These authors study the impact of demand parameter uncertainty in inventory management simulations. To the best of our knowledge, our paper is the first one that investigates the implications of demand parameter uncertainty on the quality of solution in the context of an inventory routing problem.

4 COMPUTATIONAL EXPERIMENTS

In order to illustrate the impact of the limited amount of demand data on the quality of the solutions to the IRP, we perform a set of experiments using the VRP instances proposed by Augerat et al. (1998) and adapted for the IRP by Juan et al. (2014). The data set consists of 27 small- and medium-sized test instances, with a number of nodes ranging from 32 to 80, and a fleet of 5–10 homogeneous vehicles. In the adapted instances, each node represents a DC.

To measure how the solution results are affected by the inaccuracy in the estimation of the demand distribution –due to the limited amount of demand data–, we have designed four different scenarios of data (*P*, *L*, *M*, and *S*) representing random samples of demands. Scenario *P* represents the population; scenario *L* represents the situation in which a large amount of data (10,000 observations) is available; scenario *M* represents the situation in which a medium amount of data (100 observations) is available; finally, scenario *S* represents the situation in which only a small amount of data (10 observations) is available. For the purpose of our experiments, since the population follows a log-normal probability distribution, all scenarios use log-normal probability distribution. For each scenario, we use the maximum likelihood estimates; hence, each distribution uses a slightly different set of values for the μ and σ parameters. As shown in Figure 1, the more observations we collect, the more accurate the estimation is.

Table 1 shows the results. The first group of columns indicate the tests and scenarios (i.e. the test id, the instance name, the scenario, and the probability distribution). Columns five and six show the fitted parameters. In the second group of columns, we show the quality of the simheuristic solution based on the best-fitted distribution parameters. Columns seven to nine indicate the estimated costs provided by the simheuristic algorithm. The result shows that the quality of the solution of the large sample is very close to that of the population. The solutions from medium and small sample are non-optimal. In column ten, we compare the solutions to the best-known solution (BKS) reported in Juan et al. (2014). The gap shows that the fewer data points we have, the less optimal the simheuristic result is. When we have a large amount of data (scenarios L, simheuristic produces an accurate estimation and hence, the optimality of the solution is not significantly affected). The final column shows the computational time needed to generate the solution –by default, a maximum time of 120 seconds was given to each test. The result shows that the computation time is not affected by the inaccuracy of the estimated parameters.

Figure 3 shows the box plot associated with the gap values across all instances shown in column ten in Table 1. It clearly depicts the effect of sample size over the quality of the solutions generated by our simheuristic approach. When the sample size is 10, the total cost of the optimum solution found by the simheuristic algorithm, on average, is 1.78% worse than the solution obtained using the true parameters' values. In some applications, a 1.78% difference may not matter. However, the objective of this experiment is to show that input uncertainty may have an impact on performance. Furthermore, in the experiments, we only consider one source of uncertainty (i.e. demand). As we include more sources of uncertainty (e.g. travel cost), the impact would be more significant. Finally, the experiments show that collecting more

					-			-		
#Test	Instance	Scenario	Distribution	mu	sigma	Routing	Stock	Total	Gap(%)	Time
						Cost [1]	Cost [2]	Cost [3]	BKS	(sec.)
1	A-n65-k9	Population	LogNormal	2.340	0.830	795.4	1,815.7	2,611.1	0.0%	17.79
		L (sample 1e4)	LogNormal	2.334	0.825	849.9	1,751.1	2,601.1	0.1%	25.09
		M (sample 100)	LogNormal	2.373	0.870	874.8	1,835.3	2,710.1	0.3%	66.64
		S (sample 10)	LogNormal	2.189	1.153	806.2	1,843.1	2,847.3	1.2%	67.24
2	A-n80-k10	Population	LogNormal	2.340	0.830	1,603.4	4,123.1	5,726.5	0.0%	31.86
		L (sample 1e4)	LogNormal	2.334	0.825	1,538.2	4,167.5	5,705.7	0.0%	33.74
		M (sample 100)	LogNormal	2.373	0.870	1,684.4	4,216.5	5,900.9	0.3%	21.58
		S (sample 10)	LogNormal	2.189	1.153	1,329.8	4,374.6	5,973.5	2.9%	8.91
3	A-n63-k9	Population	LogNormal	2.340	0.830	1,180.0	2,670.2	3,850.2	0.0%	18.38
		L (sample 1e4)	LogNormal	2.334	0.825	1,155.8	2,687.6	3,843.3	0.6%	26.93
		M (sample 100)	LogNormal	2.373	0.870	1,191.4	2,819.9	4,011.3	0.9%	83.49
		S (sample 10)	LogNormal	2.189	1.153	1,160.2	2,746.9	3,907.1	2.4%	78.09
4	B-n67-k10	Population	LogNormal	2.340	0.830	776.8	1,541.6	2,318.4	0.0%	23.32
		L (sample 1e4)	LogNormal	2.334	0.825	768.4	1,534.0	2,302.4	0.1%	77.25
		M (sample 100)	LogNormal	2.373	0.870	770.3	1,632.9	2,403.2	0.3%	35.68
		S (sample 10)	LogNormal	2.189	1.153	781.9	1,591.0	2,372.9	2.5%	95.65
5	B-n68-k9	Population	LogNormal	2.340	0.830	1,072.4	2,310.7	3,383.1	0.0%	73.80
		L (sample 1e4)	LogNormal	2.334	0.825	1,077.2	2,283.1	3,360.3	0.3%	5.70
		M (sample 100)	LogNormal	2.373	0.870	1,072.8	2,449.8	3,522.5	0.4%	65.26
		S (sample 10)	LogNormal	2.189	1.153	1,010.8	2,419.2	3,430.0	0.6%	96.57
6	B-n78-k10	Population	LogNormal	2.340	0.830	999.4	2,600.1	3,599.5	0.0%	57.35
		L (sample 1e4)	LogNormal	2.334	0.825	1,068.8	2,523.5	3,592.2	0.2%	14.50
		M (sample 100)	LogNormal	2.373	0.870	1,034.3	2,696.6	3,730.9	1.0%	3.83
		S (sample 10)	LogNormal	2.189	1.153	943.3	2,730.0	3,673.2	1.1%	21.58
-										

Table 1: Results obtained from four scenarios based on sample size (population, large, medium, and small).

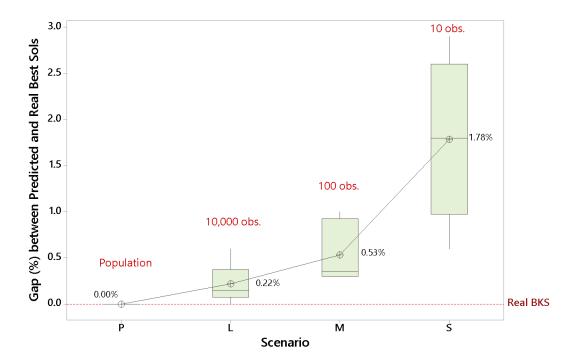


Figure 3: Gaps (in %) between the predicted BKS and the real BKS for each scenario.

data beyond certain point will not significantly improve the quality of the solution. This is shown when the sample size is 10,000, the quality of the simheuristic solution is very close to the solution obtained using the true parameters' values. Hence, collecting more data beyond this point will not significantly improve the quality of the solution. This can guide decision makers when deciding budget for further data collection.

5 CONCLUSIONS

In this paper, we have analysed how a limited number of observations might cause inaccuracies when modelling random inputs using probability distributions and, in turn, lead to sub-optimal solutions to stochastic optimisation problems. To illustrate these concepts, we have employed a testing example based on the inventory routing problem (IRP) with stochastic demands and stock-outs. Different sizes are considered for the random sample of observations. For each of these sizes, a different set of parameters is estimated. Thus, we can observe that for parameter values without the necessary level of accuracy, the solution generated by the simheuristic algorithm might be biased and reflect an incorrect total expected cost. Hence, treating the demand distributions and the parameters that are obtained from limited amount of data as the true distributions is risky, and may lead to sub-optimal decisions. It should be noticed that this issue will affect any optimisation method that makes use of best-fit probability distributions. Still, this error is much lower than when researchers employ methods that can only work under a reduced set of probability distributions and, hence, do not derive them from real-life data.

As future work, we plan to extend our analysis to more complex stochastic optimisation problems and additional sources of uncertainty. From that extensive study, we expect to be able to generate general conclusions on the trade-off between the sample size and the error that the solution might contain.

ACKNOWLEDGMENTS

This work has been partially funded by the IoF2020 project of the European Union (grant agreement no. 731884), the Spanish Ministry of Science, Innovation, and Universities (PID2019-111100RB-C21, RED2018-102642-T), and the Erasmus+ Program (2019-I-ES01-KA103-062602). We would like to thank the three reviewers for their constructive feedback.

REFERENCES

Adelman, D. 2004. "A Price-directed Approach to Stochastic Inventory/Routing". Operations Research 52(4):499-514.

- Akcay, A., and C. G. Corlu. 2017. "Simulation of Inventory Systems with Unknown Input Models: A Data-driven Approach". International Journal of Production Research 55(19):5826–5840.
- Augerat, P., J.-M. Belenguer, E. Benavent, A. Corbéran, and D. Naddef. 1998. "Separating Capacity Constraints in the CVRP Using Tabu Search". *European Journal of Operational Research* 106(2-3):546–557.
- Barton, R. R. 2012. "Input Uncertainty in Output Analysis". In *Proceedings of the 2012 Winter Simulation Conference*, edited by C. Laroque, J. Himmelspach, R.Pasupathy, O. Rose, and A. M. Uhrmacher, 67–78. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.
- Bertazzi, L., A. Bosco, F. Guerriero, and D. Lagana. 2013. "A Stochastic Inventory Routing Problem with Stock-out". Transportation Research Part C: Emerging Technologies 27:89–107.
- Bertazzi, L., A. Bosco, and D. Laganà. 2015. "Managing Stochastic Demand in an Inventory Routing Problem with Transportation Procurement". Omega 56:112–121.
- Chick, S. E. 2006. "Bayesian Ideas and Discrete Event Simulation: Why, What and How". In Proceedings of the 2006 Winter Simulation Conference, edited by L. F. Perrone, F. P. Wieland, J. Liu, B. G. Lawson, D. M. Nicol, and R. M. Fujimoto, 96–106. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.
- Corlu, C. G., B. Biller, and S. Tayur. 2019. "Driving Inventory System Simulations with Limited Demand Data: Insights from the Newsvendor Problem". *Journal of Simulation* 13(2):152–162.
- Faulin, J., A. A. Juan, C. Serrat, and V. Bargueno. 2008. "Predicting availability functions in time-dependent complex systems with SAEDES simulation algorithms". *Reliability Engineering & System Safety* 93(11):1761–1771.

- Gruler, A., J. Panadero, J. de Armas, J. A. Moreno, and A. A. Juan. 2018. "Combining Variable Neighborhood Search with Simulation for the Inventory Routing Problem with Stochastic Demands and Stock-outs". *Computers & Industrial Engineering* 123:278–288.
- Gruler, A., J. Panadero, J. de Armas, J. A. Moreno, and A. A. Juan. 2020. "A Variable Neighborhood Search Simheuristic for the Multiperiod Inventory Routing Problem with Stochastic Demands". *International Transactions in Operational Research* 27(1):314–335.
- Henderson, S. G. 2003. "Input Model Uncertainty: Why Do We Care and What Should We Do About It?". In *Proceedings of the 2003 Winter Simulation Conference*, edited by S. Chick, P. J. Sanchez, D. Ferrin, and D. J. Morrice, 90–100. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.
- Huang, S.-H., and P.-C. Lin. 2010. "A Modified Ant Colony Optimization Algorithm for Multi-Item Inventory Routing Problems with Demand Uncertainty". *Transportation Research Part E: Logistics and Transportation Review* 46(5):598–611.
- Hvattum, L. M., A. Løkketangen, and G. Laporte. 2009. "Scenario Tree-Based Heuristics for Stochastic Inventory-Routing Problems". INFORMS Journal on Computing 21(2):268–285.
- Juan, A. A., S. E. Grasman, J. Caceres-Cruz, and T. Bektaş. 2014. "A Simheuristic Algorithm for the Single-Period Stochastic Inventory-Routing Problem with Stock-outs". Simulation Modelling Practice and Theory 46:40–52.
- Juan, A. A., W. D. Kelton, C. S. Currie, and J. Faulin. 2018. "Simheuristics Applications: Dealing with Uncertainty in Logistics, Transportation, and Other Supply Chain Areas". In *Proceedings of the 2018 Winter Simulation Conference*, edited by M. Rabe, A. A. Juan, N. Mustafee, A. Skoogh, S. Jain, and B. Johansson, 3048–3059. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.
- Kleywegt, A. J., V. S. Nori, and M. W. Savelsbergh. 2004. "Dynamic Programming Approximations for a Stochastic Inventory Routing Problem". *Transportation Science* 38(1):42–70.
- Lam, H. 2016. "Advanced Tutorial: Input Uncertainty and Robust Analysis in Stochastic Simulation". In *Proceedings of the 2016 Winter Simulation Conference*, edited by T. M. K. Roeder, P. I. Frazier, R. Szechtman, E. Zhou, T. Huschka, and S. E. Chick, 178–192. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.
- Lenstra, J. K., and A. R. Kan. 1981. "Complexity of Vehicle Routing and Scheduling Problems". Networks 11(2):221-227.
- Lewis, B. 2014. "Big Data Dreams, Small Data Reality". Analytics Available at: https://doi.org/10.1287/LYTX.2014.01.11.
- Muñoz, D. F., and D. G. Muñoz. 2015. "Solving the Newsvendor Problem Under Parametric Uncertainty Using Simulation". In Proceedings of the 2015 Winter Simulation Conference, edited by L. Yilmaz, W. K. V. Chan, I. Moon, T. M. K. Roeder, C. Macal, and M. D. Rossetti, 2078–2087. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.
- Muñoz, D. F., D. G. Muñoz, and A. Ramírez-López. 2013. "On the Incorporation of Parameter Uncertainty for Inventory Management Using Simulation". International Transactions in Operational Research 20(4):493–513.
- Onggo, B. S., J. Panadero, C. G. Corlu, and A. A. Juan. 2019. "Agri-Food Supply Chains with Stochastic Demands: A Multi-Period Inventory Routing Problem with Perishable Products". Simulation Modelling Practice and Theory 97:101970.
- Rohatgi, V. K., and A. M. E. Saleh. 2015. An Introduction to Probability and Statistics. Hoboken, New Jersey: John Wiley & Sons.
- Song, E., and B. L. Nelson. 2017. "Input Model Risk". In Advances in Modeling and Simulation, edited by A. Tolk, J. Fowler, G. Shao, and E. Yücesan, 63–80. Cham, Switzerland: Springer.
- Song, E., B. L. Nelson, and C. D. Pegden. 2014. "Advanced Tutorial: Input Uncertainty Quantification". In Proceedings of the 2014 Winter Simulation Conference, edited by A. Tolk, S. Y. Diallo, I. O. Ryzhov, L. Yilmaz, S. Buckley, and J. A. Miller, 162–176. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.

AUTHOR BIOGRAPHIES

CANAN G. CORLU is an Associate Professor of Administrative Sciences Department of Metropolitan College at Boston University. She holds a Ph.D. in Operations Management from the Tepper School of Business at Carnegie Mellon University. Her research interest is in the area of the design and analysis of stochastic simulations with applications in operations management. Her email address is canan@bu.edu.

JAVIER PANADERO is an Assistant Professor of Simulation and High Performance Computing in the Computer Science, Multimedia and Telecommunication Department at the Universitat Oberta de Catalunya (Barcelona, Spain). He is also a Lecturer at the Euncet Bussines School, and a member of the ICSO@IN3 research group. He holds a Ph.D. and a M.S. in Computer Science. His major research areas are: high performance computing, modeling and analysis of parallel applications, and simheuristics. He has co-authored more than 18 articles published in journals and conference proceedings. His website address is http://www.javierpanadero.com and his email address is jpanaderom@uoc.edu.

BHAKTI STEPHAN ONGGO is an Associate Professor of Business Analytics at the Centre for Operational Research, Management Sciences and Information Systems (CORMSIS), Southampton Business School, University of Southampton, UK.

His research interest is in simulation modelling (symbiotic simulation, hybrid simulation, agent-based simulation, discrete-event simulation) with applications in operations and supply chain management. He is the associate editor for the Journal of Simulation and the chair of The OR Society's Simulation SIG. His email address is b.s.s.onggo@soton.ac.uk and his website is http://bsonggo.wordpress.com.

ANGEL A. JUAN is a Full Professor of Operations Research & Industrial Engineering in the Computer Science Dept. at the Universitat Oberta de Catalunya (Barcelona, Spain). He is also the Director of the ICSO research group at the Internet Interdisciplinary Institute and Lecturer at the Euncet Business School. Dr. Juan holds a Ph.D. in Industrial Engineering and an M.Sc. in Mathematics. He completed a predoctoral internship at Harvard University and postdoctoral internships at Massachusetts Institute of Technology and Georgia Institute of Technology. His main research interests include applications of simheuristics and learnheuristics in computational logistics and finance, as well as mathematical e-learning. He has published about 100 articles in JCR-indexed journals and over 225 papers indexed in Scopus. His website address is http://ajuanp.wordpress.com and his email address is ajuanp@uoc.edu.