ABSTRACT
The goal is to study the urban traffic signal control problem as a discrete-event simulation. We explore a network-based design and verify the sample-path behavior of the average cycle time as the objective function. We compare the performance of different Simulation Optimization solvers available on SimOpt library and discuss takeaways for traffic control structures.

1 INTRODUCTION
This work explores modeling the urban traffic with a discrete-event simulation within the SimOpt library framework (Eckman et al. 2019) and finding the best offset between traffic signals. Classically, there are several different approaches to find optimal control of the traffic signals (Osorio and Chong 2012; Ito et al. 2019). Osorio and Chong (2012) focus on finding the green time durations among three main control variables, namely, cycle length, green splits, and offsets. Ito et al. (2019) used two types of decision variables (a coordination pattern and offsets) with hierarchical relationships. A coordination pattern determines which pairs of intersections are coordinated, and offsets are the time difference between the beginning of the green light. Unlike the existing models, we use multiple offsets as decision variables to relax the assumption of all traffic signals being coordinated the same and hence renders a larger domain.

2 MODELING AND VERIFICATION
The model is an $n \times m$ grid network that forms an $n \times m - 1$ dimensional problem. The structure is inspired by Manhattan’s grid with a few long two-way vertical arterial roads and a number of one-way crossing streets with alternating directions. The nodes in the network describe intersections, origin points, and destination points. Cars wishing to travel from the origin point $i$ to the destination point $j$ externally arrive at point $i$ following a Poisson process with rate $\lambda_i$, with $\lambda_i$ being larger if $i$ is an entrance to the main arterial streets. The route for each car upon entry is determined via the shortest path from $i$ to $j$. Edges describe the queues between intersections: $e_{ij}$ is the edge between node $i$ and $j$ with its value being equal to the length of that street. Each of the directed edges has an infinite capacity to avoid what is known as manufacturing blocking. We assume that the service time, which is the time it takes to travel the edges and cross intersections, is modeled with three quantities based on whether the car has to reduce its velocity. The objective is to choose the individual offsets that minimize the average cycle time over a finite horizon of length $H$. For a warm-up period, the first $k$ cars are not included in the average cycle time. Hence, the average cycle time is calculated by $W = (l - k)^{-1} \sum_{t=k+1}^{l} T_t$, where $T_t$ is the simulated travel time from the $t^{th}$ car’s origin to its destination. The $l^{th}$ car is the last one departing the system before $H$. Thus the average cycle time does not include the cycle time of cars that are still in the system at the end of the simulation.
To lower the variance in the optimization, we employ common random numbers for the car arrivals. The sample-path objective function shows two characteristics (Figure 1). The first is the periodicity: based on its definition, if the offset equals the duration of green and inter-green lights, it is equivalent to zero offsets, implying that objective function repeats its pattern. The second is that the objective function shows zig-zagging decline but increases linearly after reaching its minimum value. We verify these observations with the physical behavior of the system.

Figure 1: The sample-path average wait time for a one-dimensional (a) and two-dimensional problem (b).

3 OPTIMIZATION AND COMPARISON OF SOLVERS’ PERFORMANCE

We use the solvers implemented in the SimOpt library to solve the problem and compare the results. The SimOpt solver library contains direct-search methods (e.g., Nelder-Mead), gradient-based methods (e.g., GASSO), and model-based methods (e.g., ASTRO-DF). While the direct-search methods can converge to non-stationary points, the gradient-based and model-based methods converge to stationary points.

There are three procedures in SimOpt. First, we run \( x \) number of macro-replications for each solver and problem via the `runwrapper`. Within a macro-replication, a solver solves the problem until the pre-defined budget is exhausted. At each solution, the function is evaluated by \( z \) number of replications to estimate the objective function with sample average approximation, which varies per solver. Second, the `postwrapper` runs \( y \) post-replications at the intermediate solutions of each macro-replication for getting more accurate objective function value without bias. Third, the `plotwrapper` records and plots the sample mean, sample variance, and quantiles at each of the solutions reported at the budget points across the macro-replications. For a fair comparison and variance reduction, the SimOpt uses common random numbers for macro-replication and post-replications. We analyze the solvers’ performance using the confidence intervals on each solver’s produced solutions to understand their convergence behavior.

ACKNOWLEDGEMENT

The authors are thankful for comments and suggestions from Shane Henderson and David Eckman.

REFERENCES

