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METAMODELING FOR VARIABLE ANNUITY VALUATION: 10 YEARS BEYOND KRIGING

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ABSTRACT

Variable annuities are retirement insurance products created by insurance companies that contain financial guarantees. To mitigate the financial risks associated with these guarantees, insurance companies have adopted dynamic hedging, which is a risk management technique. However, dynamic hedging is associated with computationally intensive valuations of variable annuity policies. Recently, metamodeling approaches have been developed to address the computational problems. A typical metamodeling approach consists of two components: an experimental design method and a metamodel. In this paper, we give a survey of metamodeling approaches developed in the past ten years. For each metamodeling approach, we will describe the experimental design method and the metamodel.

1 INTRODUCTION

Variable annuities are retirement insurance products that are created by insurance companies to address concerns many people have about outliving their assets (Ledlie et al. 2008; Feng et al. 2022). Variable annuities were first introduced in 1950s in the U.S. and have grown in popularity during the past two decades. The popularity of variable annuities in the U.S. can be seen from Figure 1, which shows the annual variable annuity sales in the U.S. from 2000 to 2021. From the figure, we see that the sales peaked in 2007 with annual sales of 184 billion and then decreased to around 100 billion. In 2021, the annual sales increased again.

When a person purchases a variable annuity policy from an insurer, the person actually enters a contract with the insurer. The person is often called a policyholder. Under a variable annuity policy, the policyholder agrees to make one lump-sum or a series of purchase payments to the insurer and in turn, the insurer agrees to make benefit payments to the policyholder beginning immediately or at some future date. A typical variable annuity policy has two phases (Hardy 2003): the accumulation phase and the payout phase. During the accumulation phase, the policyholder builds assets for retirement by investing the purchase payments in mutual funds provided by the insurer. During the payout phase, the policyholder receives benefit payments from the insurer. The benefit payments can be a lump-sum, periodic withdrawals or an ongoing income stream.

A main feature of variable annuities is that they contain guarantees, which can be bought as add-ons by policyholders to protect their purchase payments from the downside of investment risks. There are two main types of guarantees (Hardy 2003): guaranteed minimum death benefit (GMDB) and guaranteed minimum living benefit (GMLB). A GMDB is the most basic guarantee type and guarantees that the beneficiaries receive a minimum amount upon the death of the policyholder during the term of the policy. There are different types of GMLB such as guaranteed minimum accumulation benefit (GMAB), guaranteed minimum income benefit (GMIB), and guaranteed minimum withdrawal benefit (GMWB). Unlike a GMDB, a GMLB

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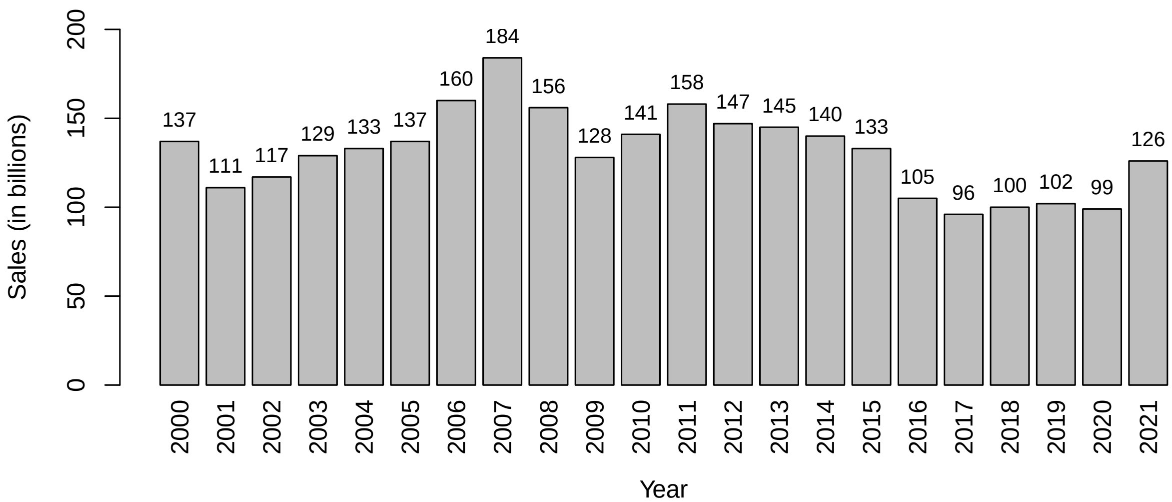

Figure 1: VA sales in the U.S. from 2000 to 2021. The numbers are obtained from LIMRA Secure Retirement Institute.

is not triggered by the death of the policyholder. A GMLB is typically triggered on policy anniversaries. Different GMLBs have different specifications on how and when the guarantee amount is determined.

The guarantees embedded in variable annuities are financial guarantees and are quite different from other insurance risks such as mortality risks. As a result, the guarantee risks cannot be adequately addressed by traditional pooling methods (Boyle and Hardy 1997). If the stock market goes down, the insurers lose money on all the variable annuity policies that invest heavily in the equity market. In fact, top variable annuity issuers lost money in the 2007/2008 financial crisis. We can see this from Figure 2, which shows the stock prices of five top issuers of variable annuities during the period from 2007 to 2021. From the figure we see that the stock prices of all these insurance companies declined significantly during the 2007/2008 financial crisis.

Insurers adopted dynamic hedging to mitigate the financial risks associated with the guarantees. However, dynamic hedging requires calculating the fair market values of the guarantees in a timely manner. This can be a challenging task, especially when an insurer has a large portfolio of variable annuity business. Metamodeling techniques have been developed recently to address the computational challenges. In this paper, we review the metamodeling techniques published in academic venues.

The remaining part of the paper is organized as follows. In Section 2, we introduce in detail the computational problems associated with dynamic hedging of variable annuities. In Section 3, we review various metamodeling approaches developed recently. In Section 4, we conclude the paper with some remarks.

2 CHALLENGES OF VARIABLE ANNUITY VALUATION

Using dynamic hedging to mitigate the financial risks associated with variable annuity guarantees requires quantifying the risks. This usually involves calculating the fair market values of the guarantees for a large portfolio of variable annuity policies. In addition, the fair market values need to be calculated in a timely manner so that insurer can take timely actions.

Unlike standard options traded in exchanges, however, the guarantees embedded in variable annuities are complex. Their fair market values cannot be calculated in closed form by using formulas. In practice, insurance companies rely on the Monte Carlo method to calculate the fair market values of the guarantees. The Monte Carlo method is flexible and can be used to value different types of guarantees. In general, the Monte Carlo method works as follows. First, a large number of economic paths (e.g., 1000 paths) are

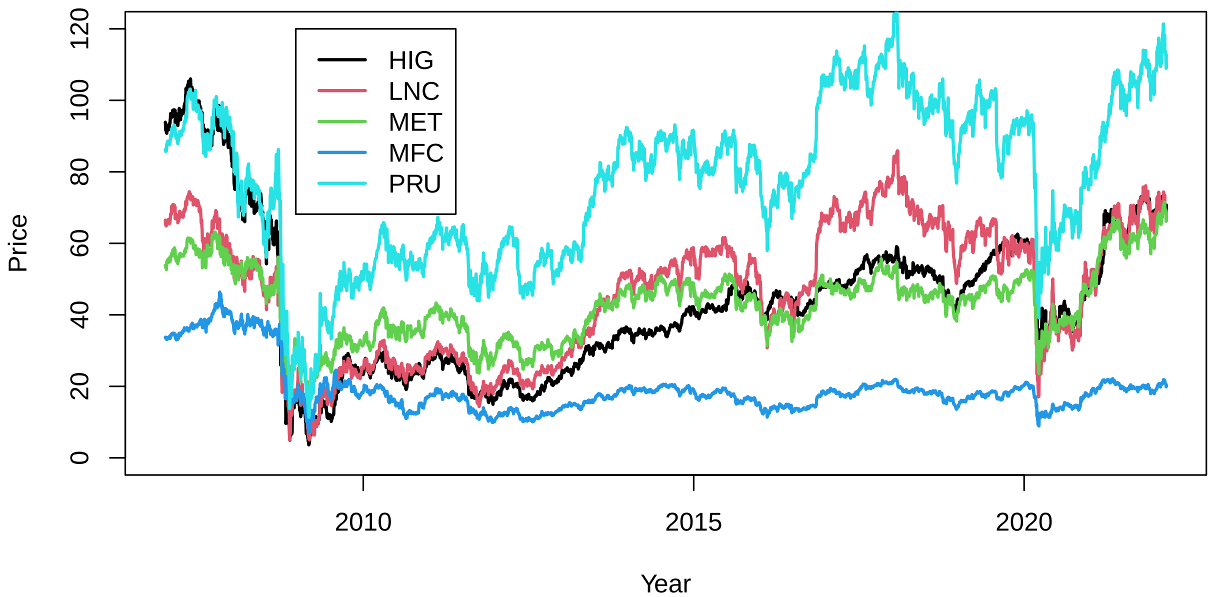

Figure 2: The stock prices of five insurance companies from 2007 to 2021. These insurance companies were top issuers of variable annuities.

projected over a long period of time (e.g., 30 years) in to the future. Then cash flows of the guarantees embedded in a variable annuity policy are calculated at all time steps of all paths. Finally, the average present value is calculated as the fair market value of the guarantees embedded in the variable annuity policy. The process is repeated for each variable annuity policy in the portfolio.

Although the Monte Carlo method is flexible, it is extremely time-consuming to value a large portfolio of variable annuity policies because every policy needs to be projected over many economic paths for a long time horizon Dardis (2016). For example, (Gan and Valdez 2017b) developed a Monte Carlo method to calculate the fair market values for a portfolio of 190,000 synthetic variable annuity policies. The Monte Carlo method used 1,000 risk-neutral paths projected for a 30-year projection horizon with monthly time steps. The total number of cash flow projections for this portfolio is:

$$1,000 \times 12 \times 30 \times 190,000 = 6.84 \times 10^{10}.$$

It took a single CPU (Central Processing Unit) about 4 hours to calculate the fair market values for the portfolio. The aforementioned runtime is the runtime for just one market scenario. In practice, an insurer needs to see how the fair market value of the portfolio changes under different market scenarios. This will need additional runs of the Monte Carlo method and requires additional runtime.

Using Monte Carlo to quantify risks for dynamic hedging of variable annuities leads to two main computational problems (Gan and Valdez 2019a). The first computational problem arises from daily hedging. For the daily hedging purpose, Greeks on a set of pre-defined market scenarios are calculated overnight and are used to interpolate the Greeks in real-time (Gan and Lin 2017). Greeks are referred to as sensitivities of the fair market values of the guarantees on market factors. There are different ways to calculate the Greeks (Cathcart et al. 2015). In practice, the bump method is commonly used to calculate the

Greeks. For example, the partial dollar delta of the guarantees on a market factor is calculated as follows:

$$Delta^{(l)} = \frac{V_0\left(PA_0^{(1)},\ldots,PA_0^{(l-1)},(1+s)PA_0^{(l)},PA_0^{(l+1)},\ldots,PA_0^{(k)}\right)}{2s} - \frac{V_0\left(PA_0^{(1)},\ldots,PA_0^{(l-1)},(1-s)PA_0^{(l)},PA_0^{(l+1)},\ldots,PA_0^{(k)}\right)}{2s},$$

where s is the shock amount (e.g., 1%) applied to the partial account value, $PA_0^{(l)}$ denotes the partial account value in the lth investment fund, and $V_0(\cdot,\cdots,\cdot)$ denotes the fair market value expressed as a function of partial account values. Here the partial account value refers to the account value of a particular investment fund invested by a variable annuity policy. Calculating the Greeks for a large portfolio of variable annuity policies is extremely time-consuming. If we calculate Greeks for the aforementioned portfolio of 190,000 variable annuity polices at 50 market scenarios, for example, we would need $2\times 50\times 4 = 400$ hours of runtime on a single CPU.

The second computational problem related to dynamic hedging arises from an insurer's quarterly financial reporting. Without hedging of the risks, an insurer needs to put a large amount of capital on reserves, which are used to back future payouts on the guarantees. Reserves for variable annuity guarantees are not static and are in fact sensitive to market conditions (Drexler et al. 2017). Dynamic hedging allows an insurer to release some capital from reserves for other purpose and stabilizes the insurer's funding needs. To reflect the effect of dynamic hedging in quarterly financial reporting, insurers usually employ a stochastic-on-stochastic (also known as nested simulation) framework.

Stochastic-on-stochastic valuation involves two levels of simulation (Reynolds and Man 2008; Dang et al. 2019): at the first level, outer loop real-world paths are projected; at the second level, inner loop risk-neutral paths are projected. Figure 3 shows a sketch of stochastic-on-stochastic valuation. The outer loop involves projecting the variable annuity guarantees along real-world paths, which reflect realistic assumptions about the market. At each node of an outer loop path, the guarantees are projected along a large number of risk-neutral paths, which reflect unrealistic assumptions that investors are risk-neutral.

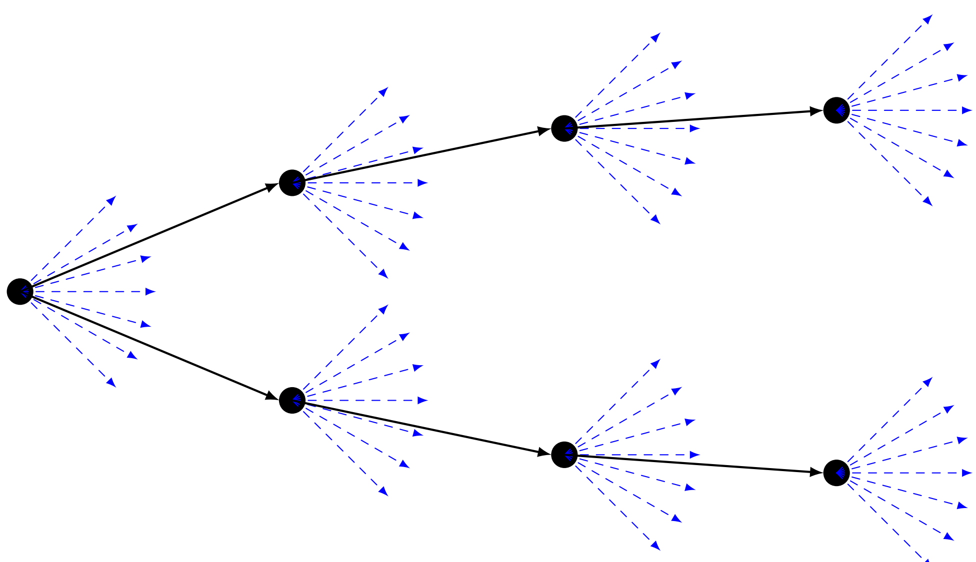

Figure 3: A sketch of stochastic-on-stochastic valuation. Outer loop paths are denoted by solid arrows and inner loop paths are denoted by dashed arrows in blue.

Since stochastic-on-stochastic valuation involves two nested simulations, it is extremely time-consuming. For example, if we apply stochastic-on-stochastic valuation on the aforementioned portfolio of 190,000 policies with 1000 outer loop paths, 1000 inner loop paths, and 360 monthly time steps, then we would need $1000 \times 360 \times 4 = 1440000$ hours or 164.30 years to complete the calculation on a single CPU.

Approaches to addressing the aforementioned computational challenges can be broadly divided into two types: hardware approaches and software approaches. In hardware approaches, multiple CPUs and GPUs (Graphics Processing Units) have been used to value a portfolio of variable annuity policies in parallel. In software approaches, mathematical models and algorithms are designed to speed up the computation. In practice, both types of approaches are used together to help reduce the runtime of valuing a large portfolio of variable annuity policies. In this paper, we provide a review of a particular type of software approaches called metamodeling approaches. For other types of software approaches, readers are referred to (Gan and Valdez 2019a).

3 METAMODELING APPROACHES

The main idea of metemodeling approaches is to reduce the number of variable annuity policies to be valued by the valuation model. A typical metamodeling approach involves the following four major steps:

1. select a small number of representative policies from a portfolio of variable annuity policies;
2. use the Monte Carlo method to calculate the fair market values of the representative policies;
3. build a regression model based on the representative policies and their fair market values;
4. use the regression model to estimate the fair market value for every variable annuity policy in the portfolio.

In the first step, the method used to select representative policies is referred to as the experimental design method. In the third step, the regression model is called a metamodel because it is a model of the valuation model. In practice, the valuation model for variable annuities is usually a Monte Carlos simulation model and is time-consuming to value a large portfolio of variable annuity policies. The metamodel is built to approximate the valuation model and is much faster than the valuation model. The metamodel can be any predictive models such as regression models and neural network models. Figure 4 shows the sketch of a typical metamodeling approach.

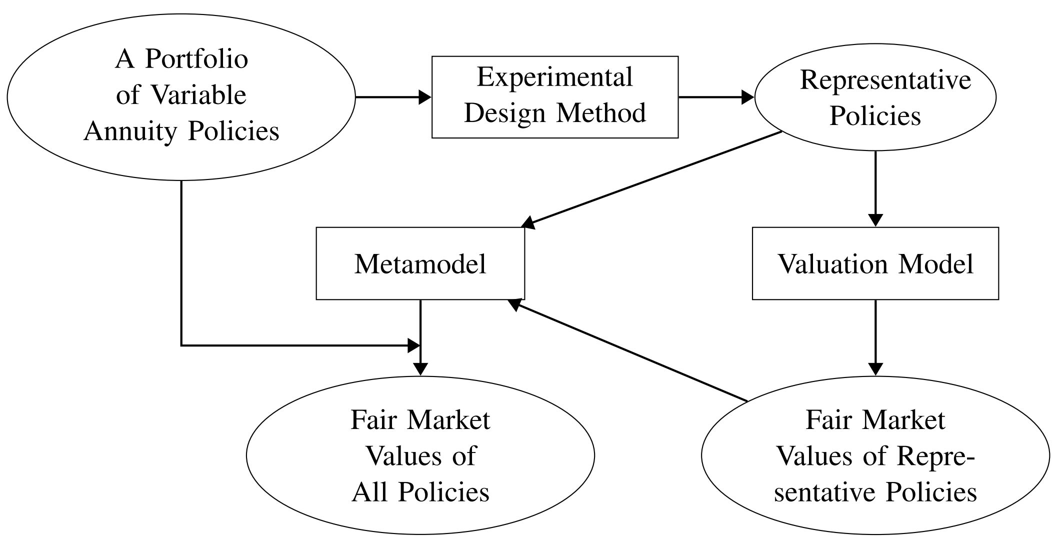

Figure 4: A typical metamodeling approach for estimating the fair market values of guarantees embedded in variable annuities.

Compared to other approaches used to solve the computational problems, metamodeling approaches have the following advantages:

- Metamodeling approaches are scalable. If a portfolio of variable annuity policies doubles its size, we do not necessarily need to double the number of representative policies. As a result, the computational time of metamodeling approaches will not increase much.
- Metamodeling approaches can be used to validate the valuation model. It is error-prone to create a valuation model for thousands of different policies. The results from metamodeling approaches can be used to identify any problems with the valuation model.

The accuracy of a metamodeling approach depends on its two components: the experimental design method and the metamodel. Research on metamodeling has focused on these two components. Table 1 shows a list of papers related to metamodeling of variable annuities. The table also shows the experimental design method and the metamodel investigated in each paper.

Table 1: Some metamodeling approaches published in the past ten years.

Publication	**Experimental Design**	**Metamodel**
Gan (2013)	Clustering	Kriging
Gan and Lin (2015)	Clustering	Kriging
Gan (2015)	LHS	Kriging
Hejazi and Jackson (2016)	Uniform sampling	Neural network
Gan and Valdez (2016)	Clustering, LHS	GB2 regression
Gan and Valdez (2017a)	Clustering	Gamma regression
Gan and Lin (2017)	LHS, conditional LHS	Kriging
Hejazi et al. (2017)	Uniform sampling	Kriging, IDW, RBF
Gan and Huang (2017)	Clustering	Kriging
Xu et al. (2018)	Random sampling	Neural network, regression trees
Gan and Valdez (2018b)	Conditional LHS	GB2 regression
Gan et al. (2018)	Clustering	Regression trees
Gan (2018)	Random sampling	Lasso
Cheng et al. (2019a)	Clustering	Kriging
Cheng et al. (2019b)	Clustering	Neural network
Gan and Valdez (2019b)	Clustering	Kriging
Feng et al. (2020)	Clustering	Cluster size multiple
Gweon et al. (2020)	Conditional LHS	Regression trees
Lin and Yang (2020)	Cube sampling	Spline regression
Gweon and Li (2021)	Adaptive selection	Regression trees
Quan et al. (2021)	Clustering	Regression trees
Liu and Tan (2021)	Quasi-Monte Carlo	Taylor approximation

3.1 Models

In this subsection, we shall review the metamodeling approaches that have been developed in the past ten years. Papers (e.g., Dang et al. (2019), Goudenège et al. (2020)) that are related to the aforementioned computational problems but do not use metamodeling approaches are not included in the review.

Gan (2013) was one of the first papers that proposed the metamodeling approach to address the computational problems associated with the valuation of large variable annuity portfolios. In this paper, a clustering algorithm called the k-prototypes algorithm was used as the experimental design method to select representative variable annuity policies. The k-prototypes algorithm is an extension of the well-known k-means algorithm that can handle both numerical and categorical features. The ordinary kriging method was used as the metamodel to estimate the fair market values of variable annuity policies.

The ordinary kriging method used in (Gan 2013) is commonly used in geostatistics. Let $\mathbf{z}_1$, $\mathbf{z}_2$, ..., $\mathbf{z}_k$ denote the representative policies selected by the clustering algorithm. For $j = 1, 2, \ldots, k$, let y_j denote the fair value corresponding to the jth representative policy that is obtained from the valuation model such as the Monte Carlo method. For an arbitrary policy $\mathbf{x}_i$ in the portfolio, the ordinary kriging method estimates the fair value for $\mathbf{x}_i$ as follows:

$$\hat{y}_i = \sum_{j=1}^{k} w_{ij} y_j,$$

where w_{ij}'s are kriging weights determined by the following linear equation system:

$$\begin{pmatrix} V_{11} & V_{12} & \cdots & V_{1k} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ V_{k1} & V_{k2} & \cdots & V_{kk} & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{pmatrix} \begin{pmatrix} w_{i1} \\ \vdots \\ w_{ik} \\ \theta_i \end{pmatrix} = \begin{pmatrix} D_{i1} \\ \vdots \\ D_{ik} \\ 1 \end{pmatrix}.$$

Intuitively, the kriging method assigns more weight to policies nearby to the policy of interest than those farther away. Here θ_i is a control variable used to make sure the sum of the kriging weights is equal to one. In addition, V_{rs}'s and D_{is}'s are determined according to the distances between representative policies and the distances between representative policies and $\mathbf{x}_i$, respectively. For example, Gan (2013) used the following equations:

$$V_{rs} = \alpha + \exp\left(-\frac{3}{\beta} d(\mathbf{z}_r, \mathbf{z}_s)\right), \quad D_{is} = \alpha + \exp\left(-\frac{3}{\beta} d(\mathbf{x}_i, \mathbf{z}_s)\right), \quad r, s = 1, 2, \ldots, k,$$

where $\alpha \geq 0$, $\beta > 0$, and $d(\cdot, \cdot)$ is the distance function used in the k-prototypes algorithm. The ordinary kriging method is simple to implement and is quite robust.

Gan and Lin (2015) extended the idea of Gan (2013) to address the computational problem associated with nested simulation of large variable annuity portfolios. Gan and Lin (2015) tried to use metamodeling to speed up the calculation of Greeks along a single out-loop path. To do that, a small number of representative policies were selected by the k-prototypes algorithm and the Greeks of each representative policy were obtained at each time step of the out-loop path under consideration. Then a metamodel called the UKFD (Universal kriging for functional data) was used to estimate the Greeks of all policies at each time step of the out-loop path. In this metamodel, the Greeks along an out-loop path were treated as functional data. This metamodeling approach can be repeated for each out-loop path. However, if there are 1000 out-loop paths, the same procedure has to be repeated 1000 times. As a result, the metamodeling approach proposed by Gan and Lin (2015) is limited in terms of reducing the runtime of nested valuation.

Gan (2015) followed up the study of Gan (2013) by trying a different experimental design method to select representative policy. In particular, Gan (2015) proposed to use the Latin hypercube sampling method to select representative policies. The k-prototypes algorithm tends to find representative policies in dense areas of the portfolio. The Latin hypercube sampling method ignores the density of policy distribution in the portfolio and finds representative policies in the whole space that maximize the minimum distance of representative policies.

Hejazi and Jackson (2016) proposed to use a variant of the Nadaraya-Watson estimator to approximate the Greeks of an arbitrary variable annuity policy. The variant of Nadaraya-Watson estimator is given by

$$\hat{y}_i = \sum_{r=1}^{k} \frac{G_{h_r}(d(\mathbf{x}_i, \mathbf{z}_r) y_r}{\sum_{s=1}^{k} G_{h_s}(d(\mathbf{x}_i, \mathbf{z}_s)},$$

where $K_h(\cdot)$ is a a nonlinear differentiable function with a parameter h. Hejazi and Jackson (2016) proposed a feed-forward neural network to find good G_{h_r} functions and h_r values. Uniform sampling was used to select representative policies.

Gan and Valdez (2018b) proposed to use the GB2 (generalized beta of the second kind) distribution to model the fair market values, which exhibit fat-tails. The density function of the GB2 distribution is given by:

$$f(z) = \frac{|a|}{bB(p,q)} \left(\frac{z}{b}\right)^{ap-1} \left[1 + \left(\frac{z}{b}\right)^{a}\right]^{-p-q}, \quad z > 0,$$

where $a \neq 0$, $p > 0$, $q > 0$, $b > 0$, and $B(p,q)$ is the Beta function. The GB2 distribution has three shape parameters (i.e., a, p, and q) and one scale parameter (i.e., b). The GB2 distribution can capture the skewness of the fair market values. The conditional Latin hypercube sampling was used to select representative policies. The difference between Latin hypercube sampling and the conditional Latin hypercube sampling is that representative policies selected by the former may not be policies from the portfolio. The conditional Latin hypercube sampling method selects policies in the portfolio as representative policies.

Gan and Valdez (2016) compared several experimental design methods used in metamodeling approaches. In particular, Gan and Valdez (2016) compared random sampling, low-discrepancy sequences, the TFCM (truncated fuzzy c-means) clustering algorithm, Latin hypercube sampling, and conditional Latin hypercube sampling. The metamodel was the GB2 regression model proposed in Gan and Valdez (2018b). They found that the data clustering method and the conditional Latin hypercube sampling method are better than other experimental design methods in terms of accuracy. However, those two experimental design methods are slower than other methods.

Gan and Valdez (2017a) studied the use of copula to model the dependence between partial Greeks, which are used to hedge different risk factors (e.g., large cap equity risk, small cap equity risk, interest rate risk). Instead of building a metamodel for each partial Greek, the copula allows building a single metamodel for all partial Greeks. The authors considered the independence copula, the Gaussian copula, the t copula, the Gumbel copula, and the Clayton copula. The Gamma distribution was used as the marginal distribution. However, Gan and Valdez (2017a) found that modeling the dependence structures in the metamodels does not improve the prediction accuracy at the portfolio level.

Gan and Lin (2017) proposed a two-level metamodeling approach that can be used to estimate Greeks in real time. The first-level metamodel is used to estimate the Greeks at some well-chosen market conditions. In the first level, the conditional Latin hypercube sampling was used to select representative policies and the universal kriging method was used as the metamodel. The second-level metamodel is used to estimate the Greeks at the current market level based on the pre-calculated Greeks. In the second level, Latin hypercube sampling was used to select market conditions from a space and the ordinary kriging was used as the metamodel.

Hejazi et al. (2017) proposed a spatial interpolation framework to estimate the Greeks of variable annuity policies. In particular, the authors compared kriging, inverse distance weighting (IDW), and radial basis functions and found that the kriging method with the spherical variogram model is quite robust in terms of providing accurate estimations.

Gan and Huang (2017) proposed a metamodeling approach as a data mining framework to estimate the Greeks. In the data mining framework, a data clustering algorithm called the TFCM++ (Truncated Fuzzy *c*-means) algorithm was used to select representative policies and the ordinary kriging method was used to predict the fair market values of the guarantees embedded in variable annuity policies. The TFCM++ algorithm is an extension of the TFCM algorithm with an improved method to select initial cluster centers.

Xu et al. (2018) proposed a moment matching Monte Carlo method to speed up stochastic-on-stochastic valuation of variable annuities. Instead of generating real-world scenario in a normal way, the authors used a moment matching method to generate real-world scenarios, which were then used to estimate Greeks, value at risk (VaR), and conditional value at risk (cVaR) for a single variable annuity policy. The theory behind the idea is the Johnson curve, which can convert any continuous random variable into a standard normal random variable based on the first four moments. The moment matching method was shown to be much faster than nested Monte Carlo simulation. The authors also proposed a metamodeling approach to calculate Greeks, VaR, and CVaR for a portfolio of variable annuity policies.

Gan et al. (2018) explored the use of tree-based models as the metamodel for the valuation of large variable annuity portfolios. In this paper, a hierarchical k-means algorithm was used to select representative policies. This clustering algorithm first divides the whole portfolio into two clusters. Then it keeps dividing the largest cluster into two clusters until a desired number of clusters is reached. The tree-based model `rpart` from the R package `rpart` was used as the metamodel.

Gan (2018) investigated the use of linear regression models with interaction effects as the metamodel for the valuation of large variable annuity portfolios. A linear model with first-order interactions has the following form:

$$E[Y|X_1, X_2, \ldots, X_p] = \beta_0 + \sum_{j=1}^{p} \beta_j X_j + \sum_{s<t} \beta_{s:t} X_s X_t,$$

where the term $X_s X_t$ models the interaction effect between X_s and X_t. Including interactions will increase the number of predictors and may lead to overfitting. Regularization was employed to avoid overfitting. The R package `glinternet` was used to fit a linear interaction model with regularization. In this paper, random sampling was used to select representative policies.

Cheng et al. (2019a) proposed a deep neighbor embedding method to select representative policies. A deep neural network was used to embed variable annuity policies in a low-dimensional space. The low-dimensional representation can preserve similarities among policies in both policy features and the historical performance. The k-means algorithm was applied to the low-dimensional representations to select representative policies. In this paper, the kriging method was used as the metamodel.

Cheng et al. (2019b) explored the use of transfer learning for the valuation of large variable annuity portfolios. The transfer learning method is able to handle more representative policies than the kriging method, which involves matrix inversion.

Feng et al. (2020) proposed a metamodeling approach by using a simple random sampling and clustering Method to select representative policies and a simple cluster size multiple metamodel. The method for selecting representative policies consists of two stages. In the first stage, a random sample is drawn from the portfolio of variable annuity policies. In the second stage, a clustering algorithm is applied to the sample to select representative policies. In the cluster size multiple metamodel, the fair market value of an arbitrary policy is predicted to be the fair market value of its nearest representative policy.

Gweon et al. (2020) proposed a bias-corrected bagging method as the metamodel for the valuation of large variable annuity portfolio. The bias-corrected bagging method can reduce the prediction bias arising from the bagging approach, which involves many regression models trained on the bootstrap samples to improve the prediction accuracy and reduce prediction variance. The bias-corrected bagging method trains two sequential bagging models. In the first bagging model, the fair market value is used as the response variable. In the second bagging model, the prediction bias is used as the response variable. Mathematically, the bias-corrected bagging method predicts the fair market value of an arbitrary policy $\mathbf{x}$ as follows:

$$\hat{y}^* = \hat{f}(\mathbf{x}, D) - \hat{B}(\mathbf{x}, D),$$

where $D = \{(\mathbf{z}_1, y_1), \ldots, (\mathbf{z}_k, y_k)\}$ denotes the training data. The term $\hat{f}(\mathbf{x}, D)$ is obtained from the first bagging model and the term $\hat{B}(\mathbf{x}, D)$ is obtained from the second bagging model.

Lin and Yang (2020) proposed a method based on surrogate models (i.e., metamodels) to speed up nested simulation for portfolios of variable annuities. The method can reduce the number of outer-loops and the number of policies for the nested simulation. The reduction of outer-loops was achieved through a spline regression model. To reduce the number of policies, the method used a model-assisted finite population estimation framework. The surrogate model was a linear model given by:

$$L_i(s) = \mathbf{x}_i' \mathbf{b}(s) + e_i(s),$$

where $L_i(s)$ is the liability estimated from the spline model of the ith policy at the sth outer-loop simulation, $\mathbf{x}_i$ denotes the features of the ith policies, and $e_i(s)$ represents the prediction error.

Gweon and Li (2021) proposed an active learning framework to speed up the valuation of large variable annuity portfolios. In the active learning framework, representative policies were select iteratively and adaptively. At the beginning, a small number of representative policies are selected randomly from the portfolio and used to build a regression model. Then the regression model is used to evaluate the informativeness of other policies in the portfolio and select a subset of the most informative policies, which need to be valued by the Monte Carlo method. The process is repeated until the computational budget for valuing the policies runs out. The bagging method is used as the metamodel to estimate the fair market values of the guarantees. The informativeness is measured through the prediction ambiguity, which is roughly proportional to the prediction variance.

Quan et al. (2021) explored the use of tree-based models as metamodels for the valuation of variable annuity guarantees. The authors considered traditional regression trees, tree ensembles, and trees based on unbiased recursive partitioning. In addition, they compared tree-based models with existing metamodels such as the ordinary kriging model and the GB2 (generalized beta of the second kind) regression model in terms of prediction performance. They found that tree-based models can produce accurate predictions and the gradient boosting method is superior to other tree models in terms of prediction accuracy.

Liu and Tan (2021) proposed the use of mesh methods to select representative policies for the valuation of large variable annuity portfolio. In particular, the authors studies standard mesh methods and conditional mesh method produced by Latin hypercube sampling and quasi-Monte Carlo. Once the mesh is constructed, the Taylor approximation is used to approximate the fair market value of a variable annuity policy from its neighbors.

3.2 Validation Measures

To validate a metamodeling approach, we usually need the following three validation measures: runtime, accuracy at the portfolio level, and accuracy at the policy level.

Runtime is an important validation measure for metamodeling approaches because the purpose of metamodeling approaches is to reduce the runtime of valuing large portfolios of variable annuity policies. Runtime should be further decomposed to reflect the runtime used by the experimental design method and that used by the metamodel.

In additional to runtime, prediction accuracy is another important validation measure. A metamodeling approach should be fast and accurate in predictions. Accuracy of a metamodeling approach needs to be measured at the portfolio level and at the individual policy level. Since risk management of variable annuities is done for the whole portfolio rather than individual policies, it is important to know the accuracy at the portfolio level. Measuring accuracy at the policy level helps us to see how a metamodeling approach performs in terms of predicting individual policies.

A widely used validation measure to measure the accuracy at the portfolio level is the percentage error given by $PE = \sum_{i=1}^{n}(\widehat{y}_i - y_i)/\sum_{i=1}^{n} y_i$, where y_i and $\widehat{y}_i$ denote the fair market values of the ith policy obtained from the valuation model (e.g., Monte Carlo simulation model) and the metamodel model, respectively, n is the total number of policies in the portfolio. A lower absolute value of PE indicates a better result.

A common validation measure to measure the accuracy at the individual policy level is the R^2, which is defined as $R^2 = 1 - \sum_{i=1}^{n}(\widehat{y}_i - y_i)^2/\sum_{i=1}^{n}(y_i - \mu)^2$, where μ is the average fair market value, i.e., $\mu = \frac{1}{n}\sum_{i=1}^{n} y_i$. A higher value of R^2 means a better result.

3.3 Public Data

Ideally, metamodeling approaches should be tested by using real datasets from insurance companies. However, it is extremely difficult for researchers to obtain real datasets from insurance companies. To facilitate the development and dissemination of metamodeling approaches, synthetic data have been produced.

For example, Gan and Valdez (2017b) created a large synthetic portfolio of variable annuity policies and implements a simple Monte Carlo simulation model for valuing the synthetic portfolio. This portfolio contains 190,000 synthetic variable annuity policies, whose fair market values and Greeks are also obtained from the simple Monte Carlo simulation model. Due to space constraints, I will not describe the data in this paper. Readers are referred to (Gan and Valdez 2017b) for detailed description of the data.

For nested simulation, Gan and Valdez (2018a) created a synthetic dataset with Greeks along 1,000 real-world paths, which are generated from a regime-switching model. The portfolio of synthetic variable annuity policies is extracted from the portfolio created by Gan and Valdez (2017b) and contains 38,000 policies. This dataset contains seriatim results, which are important for developing and testing metamodeling approaches.

4 CONCLUSIONS

Variable annuities are popular retirement products created by insurance companies. A major feature of variable annuities is that they contain financial guarantees, which caused huge losses to large insurers in the 2007/2008 financial crisis. Dynamic hedging has been adopted by many insurance companies to mitigate the financial risks of the guarantees. However, dynamic hedging requires valuation of variable annuities on a daily basis and stochastic-on-stochastic valuation on a quarterly basis. Valuation of a large variable annuity policies is computationally intensive. In this paper, we gave a survey of metamodeling approaches that have been developed recently to address these computational problems.

From the survey, we found that most metamodeling approaches were developed to address the computational problem arising from the daily hedging of variable annuities. Only a few metamodeling approaches were developed to address the computational problem associated with stochastic-on-stochastic valuation. In addition, these approaches are complicated and rely on certain assumptions of the guarantees. As a result, applying these metamodeling approaches in practice can be difficult. In the future, there still lots of work can be done to solve the computational problem on nested simulation.

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