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FROM CONCEPTUALIZATION OF HYBRID MODELLING & SIMULATION TO EMPIRICAL STUDIES IN HYBRID MODELLING

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ABSTRACT

The combined application of simulation techniques is referred to as Hybrid Simulation (HS). The primary focus of HS is on the integrative application of techniques developed within the M&S field for better representation of the system under scrutiny. In contrast, Hybrid Modelling (HM) focuses on cross-cutting hybrid models that combine theories, frameworks, techniques and established research approaches that have been tried and tested and have existed as extant knowledge within academic disciplines such as Computing Science, Operations Research and Social Sciences. In previous work, the authors present a conceptual framework and classification of HS and HM. However, the translation of this framework toward developing HM models can be challenging. To address this gap, we discuss five empirical HM studies conducted by the authors and map them to the existing classification. The paper will serve as a reference point for developing HM studies that extend the theory and practice of M&S.

1 INTRODUCTION

Modelling & Simulation (M&S) is widely applied in areas such as manufacturing (Jahangirian et al. 2010), supply chain (Terzi and Cavalieri, 2004) and healthcare (Brailsford et al. 2009; Mustafee et al. 2010) with the end goal of supporting better and more informed decision making; as a tool for operations management; and as a decision support system. However, the primary objective of the use of simulation methods differs between disciplines. In Social Sciences, for example, scientists from disciplines such as Sociology and Psychology predominantly rely on lab-based and field experiments to test cause-effect relationships (Bhattacherjee, 2012). However, social scientists also increasingly use computational experiments through social simulations (Suleiman et al. 2012) to explore and test hypotheses concerning aspects of collective action, group dynamics, and governance under various assumptions of governance and public issues (Cioffi-Revilla, 2014). Indeed, social simulation is a distinct sub-field of M&S, with its scholarly societies (e.g., the *European Social Simulation Association*), conferences (e.g., *Social Simulation Conference*) and journals (e.g., the *Journal of Artificial Societies and Social Simulation*).

Social science researchers use techniques like agent-based simulation (ABS) to study complex social systems, e.g., the emergence of social behaviour (Gilbert, 1995), crowd behaviour (Wijermans et al. 2013), religious behaviour (Shults, 2019), as well as dynamics in demographics (Silverman et al. 2013), technology diffusion (Kiesling et al. 2012) and policy implications (Downing et al. 2000). In doing so, they have arguably expanded the very applied nature of M&S, especially its use in the context of Operations Management (OM). They have established theoretical relevance to impact practice in social science research. However, although the objective of undertaking computational experiments differs within the OM and Social Sciences disciplines, the distinct stages of an M&S study largely remain the same. For example, Figure 1 overlays the typical phases of an M&S study (e.g., conceptual modelling, model implementation, scenario development) with terminologies used in the context of social simulations, e.g., referent system, process formalization, explanans (the model), testable predictions, observation and feedback.

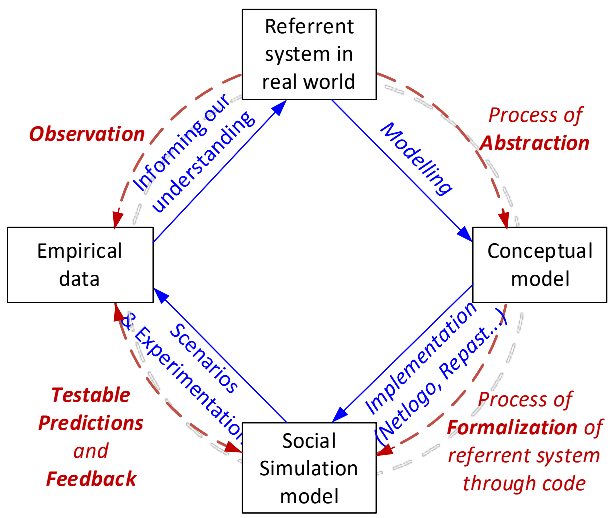

Figure 1: Overlay of the typical stages of an M&S study [blue] with terminologies used in Social Simulation [brown] – adapted from Cioffi-Revilla (2017) and Mustafee and Powell (2018).

1.1 Hybrid Simulation and Hybrid Modelling

M&S techniques such as Discrete-event Simulation (DES), Agent-based Simulation (ABS), Monte Carlo Simulation (MCS) and System Dynamics (SD) are widely used in OM and strategic planning. For healthcare, the techniques have been applied to modelling A&E departments, outpatient clinics, healthcare supply chains, etc. (Brailsford et al. 2009; Katsaliaki and Mustafee, 2011). However, the techniques have mainly been applied in isolation. This is also true for social simulations where a social system is modelled based on a single simulation methodology and using a single technique, e.g., ABS, Cellular Automata social simulations, and micro-analytical models (Gilbert and Troitzsch, 2005). There is a consensus in the simulation community that the combined application of methods can overcome the unavoidable limitations of any single approach and thereby enable synergies across techniques to engender improved insights. The search for the best possible representation and analysis of the system under scrutiny has, thus, led to an increasing number of studies that integrate multiple simulation techniques for the development of new hybrid simulation models, e.g., refer to the literature review paper on hybrid M&S by Brailsford et al. (2019).

Hybrid Modelling (HM) extends Hybrid Simulation (HS). The primary focus of HS is on the combined application of M&S methods and techniques that have been developed predominantly within the M&S field. In contrast, HM focuses on cross-cutting hybrid models that combine theories, frameworks, techniques and established research approaches that have been tried and tested and have existed as extant knowledge within distinct academic disciplines (Mustafee et al. 2018, 2020; Tolk et al. 2021). By absorbing research approaches developed within diverse fields of study (*not limited to only M&S techniques*), researchers and practitioners can assess how best to deploy these hybrid methods to complement (rather than supplement) the techniques traditionally used within the M&S knowledge domain.

1.2 Existing Work and Contribution to the Literature on Hybrid Modelling

The focus of this paper is on HM. In previous work, the authors have presented a high-level classification of hybrid modelling and simulation and identified four different model types – *Type A-D* (Mustafee et al. 2018). In Mustafee et al. (2020), based on a general review of the literature on HS and HM, the original classification (Mustafee et al. 2018) was extended to include a new *Type E* hybrid model. The classification can help develop the conceptual understanding of M&S and cross-disciplinary methods and help promote the development of HMs. However, researchers may experience a gap in translating the conceptual understanding into an empirical HM study. An approach to address this is to reflect on existing HM studies from the authors' perspective that have previously used cross-disciplinary methods in an M&S study, and map the empirical studies to the existing classification. Towards this, the paper presents five HM studies conducted by the authors of this paper. Three of the five studies pre-date the classification work by Mustafee et al. (2018, 2020); one study is work-in-progress. Thus, to an extent, the paper is retrospective in its discussion of earlier work through the lens of HM and its classification. Indeed, a couple of studies are on *Parallel and Distributed Computing* and *Grid Computing* (primarily associated with Computer Science and Applied Computing disciplines). These studies, and several others reported in Mustafee et al. (2020), were conducted when the terms HS and HM were not popular in our field. Nevertheless, although these papers may not have included the "hybrid" term, at an intrinsic level, these were arguably hybrid studies that spanned across disciplines. The five studies discussed in this paper are from M&S, Hard and Soft OR and Applied Computing disciplines.

1.3 Paper Structure

The remainder of the paper is organized as follows. Section 2 presents an overview of the conceptualization and classification of hybrid M&S, including both HS and HM. As the focus of this paper is on HM and cross-disciplinary engagement, in Section 3, we restrict our discussion to five empirical studies that have applied M&S techniques with approaches from Operational Research (OR) - we distinguish between Soft OR and Hard OR - and Computer Science/Applied Computing (CS/AS). For every study, an overview is presented, followed by the justification of the choice of HM methods and mapping with the hybrid classification framework. Section 4 is the paper's concluding section; it provides a summary of the paper and pointers for future research.

2 CLASSIFICATION OF HYBRID MODELLING AND SIMULATION

A simulation study consists of several stages, for example, *problem formulation* which outlines the objectives of a simulation study, *conceptual modelling* which helps determine the scope of the system to be modelled and the level of detail, *model formalism* which could be mathematical or logical, *data collection* – this could be both primary or secondary data, *input data analysis* to determine the probability distributions and other quantitative elements that are necessary for the execution of a stochastic model, *model building* using ABS, DES, SD, MCS or a combination of M&S approaches (the latter referred to as hybrid simulation), *verification* to ascertain the technical correctness of the model, *validation* to determine if the results generated by the base model reflects the behavior of the real or the intended system of interest, development of *scenarios for experimentation, model execution* (including replication), *output analysis* – this often takes the form of comparing the results of the base model and the experimental in relation to the KPIs identified in the problem formulation stage of the study, *documentation and reporting* (to formally present the results of the simulation study to the stakeholders), and *implementation of the results* of the simulation study. Except for the implementation stage, which is mainly driven by the problem owners and may depend on the various facets of trust (Harper et al. 2021), the M&S study is usually led by the modelers (researchers, simulation consultants) working along with the stakeholders.

The distinct stages of an M&S study (as outlined above) provide opportunities for the combined application of M&S methods, including hybrid simulation (HS), with frameworks, methods, techniques and research approaches that have been developed in disciplines outside M&S; in other words, an opportunity for the development of hybrid models (HM). For example, in the problem formulation and conceptual modelling stage, Soft OR approaches such as cognitive mapping, group model building, soft systems methodology (SSM), qualitative system dynamics (QSD); input data analysis can benefit from Machine Learning and other forms of inductive algorithms developed in the field of Data Science, and mathematical models and optimization methods in the area of Hard OR; model formalism can extend the traditional DEVS approach and incorporate UML, and other Software Engineering approaches; model execution can extend its traditional reliance on historical distributions to also include integration of real-time data feeds from sensors, IoT and other Industry 4.0 technologies; model experimentation can include technologies and methods developed in Computer Science/Applied Computing, for example, distributed simulation, grid computing, cloud computing (Powell and Mustafee, 2017).

The authors make a distinction between HS and HM; however, there is a lack of consensus in the community as to the usage of the term HM, viz-a-viz, HS. Indeed, the term "hybrid modelling and simulation" is often used to refer to HS or the combined application of multiple simulation techniques (with no reference to the use of M&S with cross-disciplinary methods – what we define as HM) (Figure 2). With the objective of differentiating the terms, Mustafee and Powell (2018) presented a unified conceptual representation of HS and HM. Mingers and Brocklesby's (1997) definitions of paradigm, methodology, technique and tool were adapted to make them relevant to the discussion on HS and HM.

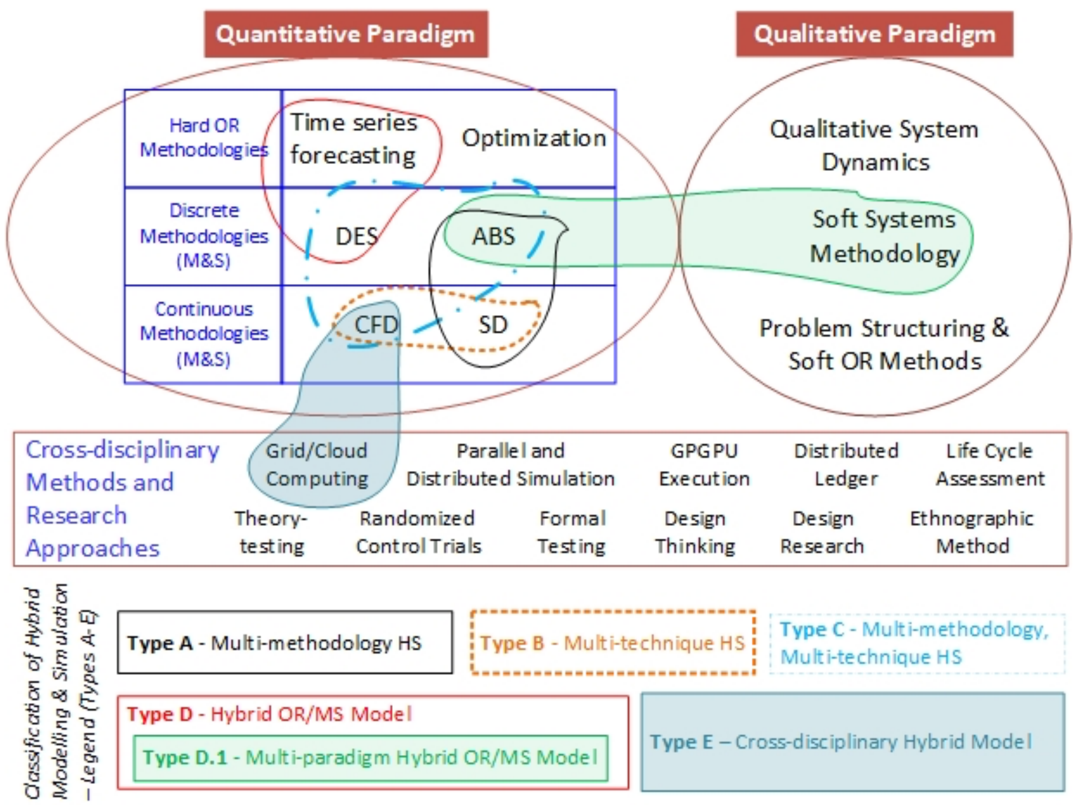

Figure 2: The conceptual representation of Hybrid M&S (left) - adapted from Mustafee et al. (2020). Note that the techniques identified in the figure are not exhaustive; for example, there are a plethora of Hard OR techniques that can be included under Hard OR methodologies.

The classification (Figure 2) refers to the qualitative and the quantitative paradigm; within the quantitative paradigm, it distinguishes discrete and continuous M&S methodologies and Hard OR. As techniques exist within methodologies, examples of discrete techniques include ABS and DES; computational fluid dynamics (CFD) and SD are examples of continuous techniques. Figure 2 also includes two examples of Hard OR techniques - forecasting and optimization. The reader is referred to Mustafee and Katsaliaki (2020) for a study on Operations Research/Management Science (OR/MS) techniques that are widely used in the literature. Readers should note that the techniques identified in Figure 2 are not exhaustive. Mustafee and Powell (2018) identified four model types and one sub-type (*Model Types A-D and Model Sub-Type D.1*). Mustafee et al. (2020) extended the original classification with the addition of *Type E Cross-disciplinary HM,* as it was felt that the M&S and OR/MS focus of the original paper ignored hybridity that also included research approaches, methods and techniques from wider disciplines. We list the model types that make up the classification of hybrid modelling and simulation below. For an outline of studies mapped to the model types, the reader is referred to Mustafee et al. (2020).

- ***Hybrid Simulation*** (HS) is defined by *Model Types A-C*.
- ***Hybrid Model*** (HM) is defined by *Model Types D, D.1* and *E*.
- ***Type A*** is *Multi-Methodology HS* developed using techniques from both continuous and discrete methodologies, for example, an HS comprising of SD+ABS or SD+DES.
- ***Type B*** *Multi-Technique HS* is based on either continuous or discrete methodologies, for example, an HS consisting of both discrete (ABS + DES) or continuous techniques (CFD + SD).
- ***Type C*** *Multi-Methodology, Multi-Technique HS* uses a combination of techniques from both discrete and continuous methodologies, with at least two techniques from either of the underlying M&S methodologies. Studies that include the combined application of SD+DES+ABS are examples of *Type C* HS.

Types D, D.1, and *Type E* are HMs.

- ***Type D*** *Hybrid OR/MS Model* is the combined application of simulation (SD, DES, ABS, or an HS) with Hard OR/MS (quantitative).
- ***Type D.1*** *Multi-paradigm Hybrid OR/MS Model,* which is a sub-type of Model *Type D*, is the combined application of a qualitative technique from the area of Soft OR, for example, Soft Systems Methodology, together with computer simulation (SD, DES, ABS, or an HS).
- ***Type E*** *Multi-paradigm Hybrid OR/MS Model* is the combined application of simulation (SD, DES, ABS, or an HS) with cross-disciplinary techniques from fields such as Social Sciences, Economics and Computer Science. *Type E* models go beyond only the use of simulation with OR/MS (as is the case with *Model Type D, D.1*). in multiple stages of an M&S study.

3 HYBRID MODELLING STUDIES

In the context of HM, we define cross-disciplinary engagement as applying M&S techniques with methods from broader disciplines. The five HM studies are subsequently referred to as #Study 1, #Study 2, .. (Table 1, Column 1). The studies have used different combinations of HM methods. For example #Study 1 is from the transportation sector; it uses an optimization algorithm (CLO) with ABS (Table 1, Column 2); The use of CLO was specific to input data analysis since the output of the algorithm was used to initialize agents (Table 1, Column 3). #Study 2 is an example from the healthcare domain; it uses forecasting, a Hard (quantitative) OR technique, along with a DES model. #Study 3 is the combined application of a Soft (qualitative) OR approach called Qualitative System Dynamics (QSD) to conceptualize a DES model that focuses on sustainable development and the triple bottom line. While the first three studies combined simulation with either a Hard or a Soft OR approach, #Study 4 and #Study 5 used Monte Carlo Simulation (MCS) and a DES with Grid Computing and Distributed Simulation, respectively. Both Grid computing and Parallel and Distributed Simulation (PADS) were developed in the field of Computer Science/Applied Computing. Table 1 provides a summary of the methods, application area and the M&S stage in which hybrid methods were applied; it also maps the studies with the classification of HS and HM presented in Section 2 and based on Mustafee et al. (2020).

Table 1: Mapping of the HM Studies with Classification of Hybrid M&S, Discipline, Application Area, M&S Study Phase and References/WIP.

Study & **Model Type**	Description of Methods that constitute the HM – **Discipline** – *Application*	**M&S Study Phase(s)** where the Hybrid method was applied	Reference
#1 Model Type **D**	Cutting and Packing Optimisation (CPA) and ABS – ***Hard OR*** – *Transportation*	**Input Data Analysis stage**– output of CPA used as input for a AnyLogic™ ABS.	Mustafee and Bischoff (2013)
#2 Model Type **D**	Forecasting and DES – **Hard OR** – *Healthcare*	**Input Data Analysis and Execution stage**– output of the forecasting model determined the distribution, and used as input for a Simul8™ DES, with Visual Logic to change distributions at runtime.	Harper and Mustafee (2022)
#3 Model Type **D.1**	QSD and DES – **Soft OR** – *Sustainable Development/Healthcare*	**Model Conceptualization stage** – a QSD model developed in Vensim™ used in the conceptualization of a Simul8™ DES model.	Work in Progress
#4 Model Type **E**	Desktop Grid and MCS – **CS/Applied Computing** – *Financial Services*	**Experimentation stage** - Monte Carlo Simulations (MCS) developed in Excel™ executed over a desktop grid.	Mustafee and Taylor (2009)
#5 Model Type **E**	Distributed Sim and DES – **CS/Applied Computing** – *Healthcare*	**Implementation and Experimentation stages** – Distributed Simul8™ DES models used with the IEEE 1516 HLA standard.	Mustafee et al. (2009)

3.1 Study 1 – HM Using Cutting and Packing Optimisation and ABS

Study 1 is an example of Type D as it is the combined application of a Hard OR technique with ABS. More specifically, the Hard OR technique is Container Loading Algorithm (CLA), which is a particular category of Cutting and Packing Optimization. CLA has been used with ABS to analyze the trade-offs between loading efficiency of containers along with key transport considerations.

CLAs are used for various container loading problems, such as loading cargo consisting of identical items (homogenous cargo), heterogeneous cargo and loading a consignment of goods into either a single container or multiple containers (Bischoff and Ratcliff, 1995). The HM presented in Mustafee and Bischoff (2013) relates to cargo consisting of weakly heterogeneous items of rectangular shape (defined by their length, breadth and height attributes), which have to be placed in a container of pre-defined size. The objective of the optimization element of the work was to develop load plan construction heuristics to determine the spatial coordinates of cargo placement within the container. However, computer simulation could add a second dimension of analysis when considering additional transport-related considerations which change *dynamically over time* (transportation time of the container). Thus, the objective of the ABS model was to identify the trade-off between loading efficiency (expressed as a %) and practical considerations like cargo stability or the risk of cross-contamination of perishable cargo.

We now describe the hybrid approach in terms of the realization of the model. The load plans that the CLA generated were used to initialize an agent-based model developed in AnyLogic™. For each item listed in the load plan, an agent was created with dimensions that reflected the length, breadth and height contained in the loading plan. Furthermore, the agents were related to one another based on the x, y and z coordinates that were generated by the load plan heuristics; this was important to model the effect of cross-contamination over time. Finally, agents were coded with a property called "freshness index" which decreased over time; after a threshold lower bound of freshness was reached, the inter-agent communication started the process of cross-contamination. The logic incorporated in the ABS model determined whether two or more agents (which represented spatially located boxes of specific dimensions) were "in contact" through a comparison on their respective x, y and z-axis (this data was contained in the CLA load plan; it micmiks physical contact between boxes that were being transported in a container).

The study concluded that an HM that uses Hard OR optimization in the input data analysis stage of an M&S survey could be plausibly used to initialize ABS models, and their combined applications will provide further insights to the stakeholders. The realization of the stated objective of the study (Mustafee and Bischoff, 2013) required the deployment of an integrative approach that permitted the investigation of the trade-off between loading efficiency with considerations related to the dynamic evolution of time, the latter being modelled in an ABS.

3.2 Study 2 – HM Using Forecasting and DES

Study 2 is an example of a sequential *Type D* model. This study is reported in Harper and Mustafee (2022). The first stage developed long-term forecasts of uncertain demand for three endoscopic procedures used for the diagnosis, treatment, and surveillance of conditions of the stomach, oesophagus and bowel. These forecasts were inputted into a DES model to investigate strategic capacity planning interventions. The model, scenarios, and key outputs were developed in close collaboration with a large healthcare organization, which was experiencing unexpected rises in demand for services caused by national policy changes. Endoscopy demand is relatively unaffected by changes in disease prevalence or socioeconomic factors, but can be heavily impacted by policy changes, technical advances, fluctuations in government priority, and public expectations (Brown et al. 2015). The objective was to support the organization to future-proof their service, given increasing pressure nationally to accommodate improved cancer outcomes. Stakeholders were interested in large perturbations in demand, as scenarios represented considerable time and financial investments.

The HM was implemented using demand projections accounting for both internal (historical demand, bowel cancer screening) and external (future policy change, demographics) environments to create linear demand profiles. These were inputted into a DES to investigate capacity planning scenarios over a 10-year period, such as six-day working, reconfiguration of wait-time targets, and increased theatre efficiency. Within the healthcare organization, there existed a good understanding of potential interventions for strategic change, but limited awareness of future demand, and the impact of interventions on future demand. The efficiency of delivery of healthcare services relies on the coordination of demand and capacity, but forecasting studies often predict demand without regard for future capacity constraints. Likewise, capacity planning requires strategic decision-making, therefore planning tools should allow decision-makers to examine the consequences of changing demand. For this reason, the collaborating team determined that an HM would better address the problem than one method alone, and reported changes to the real-world system provided evidence that this approach was successful.

3.3 Study 3 – HM Using Qualitative SD and DES

Study 3 follows *Model Type D.1* and is an example of bridging paradigms. The qualitative and quantitative elements of the model are QSD and DES, respectively. It is a work-in-progress and based on the conceptualization of TBL modelling first presented in Fakhimi (2016). This research investigates the use of the Soft OR QSD method to inform the development of a conceptual model for DES; the model focuses on sustainability analysis and the Triple Bottom-line (TBL).

The authors define a TBL-based model as an abstraction of an underlying system of interest to analyze productivity criteria (e.g., resource utilization, service time), along with environmental and social criteria.

As TBL systems are complex, inter-dependent and have multiple system outputs, they may benefit from the HM approach since it leverages the precision of the quantitative DES model and the ability of QSD to capture multiple perspectives and the effect of unquantifiable variables.

The author assesses the suitability of Type D.1 HM using a case study on haematology outpatients' clinics. The original case study is based on Mustafee et al. (2011) and was developed with healthcare stakeholders from the *Abertawe Bro Morgannwg University Health Board* (ABMUHB) in Wales. The original study investigated the outpatient capacity for specialist haematology services and tested strategies for service consolidation. Its primary focus was on the efficiency aspect of the operation (e.g., improving throughput, reducing *Referral To Treatment* waiting time). However, it did not consider the direct cost implications, the environmental implications (e.g., patients travelling further) and societal considerations (e.g., preference of patients to be treated locally). Yet, according to the *NHS Commissioning for Sustainable Development Plan*, any decisions NHS healthcare unites should contribute to the NHS sustainable development strategy (2012). This plan proposes striking the right balance between the three key areas of economic, environmental and social responsibilities when making commissioning decisions. Study 3 thus uses Mustafee et al. (2011) case study to assess the application of the DES-QSD HM, developed in Vensim™ and Simul8™, to analyze the suitability strategy of restructuring the haematology OPD service in ABMUHB from the perspective of the TBL framework (environmental, social and economic responsibilities). More generally, the case study is a means to investigate whether the combined approach can potentially reduce the oversimplification (including possible misrepresentation) of the underlying TBL-based system when only a conventional DES model is used.

The authors conclude that QSD provides enhanced visibility of interrelationships between tangible factors (i.e., measurable and productivity-based factors) and intangible factors (i.e., those related to social responsibility and ethical values). Conceptual models developed using this Soft OR method will benefit the practice of modelling TBL-based systems.

3.4 Study 4 – HM Using Monte Carlo Simulation (MCS) with Desktop Grid Computing

Grid and desktop grid computing are research areas in distributed computing, a specialist area of the Computer Science/Applied Computing discipline. While much of grid computing research focussed on meeting the needs of large virtual organizations, desktop grid computing or desktop grids addressed the potential of harvesting the idle computing resources of desktop PCs (Choi et al. 2004). Study 4 is an example of a cross-disciplinary HM where desktop grid computing enabled faster execution of credit risk simulations of counterparty transactions from the banking industry. The credit risk simulations were Monte Carlo simulations. In this *Type E* HM, the hybridity was expressed in the experimentation stage of an M&S study.

The WinGrid middleware was developed by Mustafee (2007) as a software contribution from PhD; the middleware aggregated the computing power of idle workstations that were connected through the LAN. The MCS was developed by the stakeholder (a European bank) using a package called SunGard™. The objective was to use the *WinGrid* infrastructure to automatically farm multiple instances of the MCS (referred to as jobs) to idle PCs in a network (worker nodes); to execute the simulations in worker nodes; interrupt the jobs when the internal state of the worker changed from *idle* to *busy*, and vice-versa; and aggregate simulation results from the worker to the master node. Several experiments investigated the speed-up achieved using varying configurations of the WinGrid nodes and using different workloads. This demonstrated the efficacy of using an HM based on a distributed computing middleware to farm idle computing cycles to execute an embarrassingly parallel task (such as the case with the MCS model) in the experimentation stage of an M&S study.

Grid and desktop grid computing has now largely been superseded by Cloud computing. However, the need for faster computation persists; this could take the faster of faster processors (like GPGPU), supercomputing architectures, or indeed distributed architecture with commodity hardware, as was the case with the desktop grid middleware called *WinGrid* that was implemented in Mustafee and Taylor (2009).

3.5 Study 5 – HM Using DES with Parallel and Distributed Simulation

Similar to #Study 4, the final study is also an example of *Model Type E*. It is an application of DES with standards and techniques developed in a very specialized area of Computer Science/Applied Computing called *Parallel and Distributed Simulation* (PADS). Since the late 1970s, this field has studied approaches to distributing a simulation across many computers and linking together and reusing existing simulations running on one or more processors (Fujimoto, 2000). Coordinated execution of such distributed models over different computers requires specialist distributed computing software (also referred to as distributed simulation middleware).

In this work, published in Mustafee et al. (2009), a distributed simulation standard called the *IEEE 1516 High-Level Architecture* (IEEE, 2010) and the *DMSO 1.5NG Run-time Infrastructure* were used to execute distributed simulation models of a blood supply chain in the UK. The objective of the work was faster execution of large and complex models developed using DES; thus, the hybrid element of the model was used in the experimentation stage of the M&S study. However, unlike Study 4, the IEEE 1516 standard had to be considered while implementing the distributed DES models specific to the different echelons of the blood supply chain. Thus, the hybrid element of this work was also necessary for the model implementation stage (refer to Table 1) of the M&S study!

The original model consisted of one Simul8™ DES model that represented the processes specific to blood donation, processing, testing and issuance (PTI) of blood units, ordering strategies of hospitals of different sizes, local stock management, and regular and emergency transportation of blood. The original model was developed as part of a PhD study and is reported in Katsaliaki and Brailsford (2007). However, as the model grew in size and complexity, it resulted in an exponential increase in runtime. For example, it took 36 hours to simulate a conventional DES model that represented the processes of one blood supply centre and four hospitals (Mustafee et al. 2009). Thus, exploring strategies that could help reduce the execution time became necessary. This led to the development of an HM consisting of five separate Simul8™ models, each modelling the processes of the blood supply centre and the four hospitals, respectively. Experiments were performed using two HLA time management protocols – *Time Advance Request* (TAR) and *Next Event Request* (NER) – which enabled the synchronized execution of the distributed simulation federation over a total of five computers without causality errors.

Compared to the exponential rise in the runtime for a conventional DES model, the distributed experiment results confirmed a substantially smaller and smoother rise in the runtime for both the TAR and NER time management algorithms. This demonstrated that a *Type E* HM using distributed simulation could be a feasible option for the execution of simulations that experience non-trivial runtimes.

4 CONCLUSION

The paper presents five hybrid modelling (HM) studies conducted by the authors over the years. Of the five studies, four have been published, and one study is a work in progress. Several authors have also conducted similar HM studies that have extended beyond the application of mere hybrid simulation (HS) and have sought to extend the disciplinary boundaries of M&S by embracing new research approaches, conceptualizations and frameworks, methods, tools and techniques that have been developed in other disciplines. Indeed, Mustafee et al. (2020) have classified several of these studies according to the Model Types A-E. However, for this paper, we restrict the discussion to our studies since the objective was to help modellers view, from the authors' perspective, the translation from the conceptualization of an HM (Figure 2) to the realization of an empirical HM study.

As mentioned earlier in the paper, some of the studies pre-date the framework and the classification of HS and HM, first reported in Mustafee and Powell (2018) and since extended by the inclusion of cross-disciplinary HM in Mustafee et al. (2021). So to an extent, the framework development can be viewed as an inductive approach; learning from data and our experience with HM studies and introducing conceptualization that may offer clarity in terminology, for example, how does conventional simulation differ from HS? what is the difference between HS and HM? What does the bridging of paradigm mean with respect to a simulation study? Further, the conceptualization offers recognition of the opportunity of including hybrid methods in several of the stages of an M&S study. Future work could include the extension of the classification framework and proof-of-concept studies which present the validation of different combinations of multi-, inter-and transdisciplinary research approaches (Tolk et al. 2021) used along with M&S approaches and techniques.

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