

**COMBINATION OF SIMULATED ANNEALING ALGORITHM AND MINIMUM HORIZONTAL LINE ALGORITHM TO SOLVE TWO-DIMENSIONAL PALLET LOADING PROBLEM**

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**ABSTRACT**

Major logistics companies have been paying most of attentions on the maximization of pallet usage efficiency. The two-dimensional pallet loading problem can be transformed into the orthogonal arrangement of two-dimensional irregular rectangular pieces. This paper mainly studies the hybrid heuristic algorithm combining simulated annealing (SA) algorithm and the minimum horizontal line (MHL) algorithm to solve the two-dimensional pallet loading problem. Since the local optimization ability of the simulated annealing algorithm is relatively strong, and the global search ability of the lowest level algorithm is relatively strong, the two algorithms are combined to solve the two-dimensional pallet loading problem. Experiments show that the hybrid heuristic algorithm proposed in this paper is nearly 17 times faster than the integer programming model in terms of convergence time, and it is also improved compared with the genetic algorithm in terms of convergence time and space utilization.

**1 INTRODUCTION**

With the rapid development of the logistics industry, modern logistics storage methods have gradually developed from traditional transportation based on bulk and manual handling to large-scale, standardized and intelligent development. Pallets have gradually become one of the important symbols to measure the level of logistics efficiency. The two-dimensional pallet loading problem is a typical two-dimensional rectangular strip packing problem (2DR-SPP). The two-dimensional pallet loading problem is that

rectangular goods of different sizes are loaded into trays with fixed width and unlimited height without overlap, but the height is required to be minimized. Tray area, according to the laws of the Germans Janer design utilization increased by 5%, its packaging costs by about 10%, therefore study how to orthogonal and do not overlap in a tray stacking the highest number of different sizes of rectangular box, namely the pallet loading problem, to reduce logistics cost, raise the use efficiency of the tray is of great significance. For this kind of highest complexity optimization problem, the commonly used methods include precise algorithms and heuristic algorithms. The precise algorithm mainly solves small-scale problems. Optimization problem, when solving medium and large-scale loading problems, the exact algorithm cannot give the optimal solution or approximate optimal solution in a limited time, and the constraints and the amount of computation are very huge, so in recent years, the heuristic-based approximation Algorithms to solve the two-dimensional strip bin packing problem are increasingly studied.

Two-dimensional pallet loading and unloading is a very important part of cargo port loading and unloading. The level of loading and unloading utilization will directly affect the cost of the port. Therefore, more and more enterprises are beginning to seek to maximize the utilization of pallet loading. In the loading problem, the precise algorithm of integer programming is usually used to solve it. However, due to the gradual increase of the freight volume, the precise algorithm cannot effectively obtain the optimal solution due to the increase of its constraints. In recent years, heuristic algorithms have played an important role in solving medium and large-scale loading problems. Hi-fi M (1998) proposed an accurate algorithm based on the branch and bound method, which has good performance in solving small-scale problems. However, for medium and large-scale problems, due to the increasing constraints and increasing complexity, it is difficult to obtain approximate solutions in a relatively short time. Lesh N. et al. (2004) presented an exact algorithm based on an integer programming model, but the exhaustive method can only solve small-scale optimization problems. For medium-to-large-scale problems, the exhaustive method has many constraints and takes a long time to calculate. It is impossible to obtain the global optimal solution in a short time. It can be seen that the exact algorithm is only suitable for the small-scale two-dimensional strip bin packing problem with low complexity. Burke E.K. et al. (2004) introduced a new algorithm. The algorithm will randomly identify the state of the current partial solution and select a higher fitness. to avoid falling into a local optimal solution. Baker (1980) and others proposed the BL algorithm, which takes the coordinates of the lower left corner as the base point, and stipulates that the rectangle should be placed as far as possible to the left of the two-dimensional strip during the lowering process. Chazelle introduced the BLF algorithm (2006) to improve the defect that the BL algorithm is easy to be high on the left and low on the right. When placing the next rectangle to move to the left, the priority is to arrange downward and then to the left until it cannot be arranged to the left.

In this paper, the two-dimensional pallet loading problem is transformed into a two-dimensional rectangular strip packing problem. All rectangular boxes must be placed in a fixed width. In the strip with limited height, the rectangular box can be rotated by  $90^\circ$ , without overlap. Because the simulated annealing (SA) algorithm has a strong local search ability, and the lowest level algorithm has a strong global search ability. This paper combines the SA and the hybrid heuristic algorithm of the minimum horizontal line (MHL) algorithm is used to solve the two-dimensional pallet loading problem, and compared with the integer programming (IP) model and the genetic algorithm (GA). The experiments show that the hybrid heuristic algorithm proposed in this paper is faster than the IP model in terms of convergence speed. Because the GA is easy to fall into the problem of the local optimal solution, the hybrid heuristic algorithm has a convergence speed of 10s-16s faster than the GA, and is 5% higher than the GA in terms of utilization. The research in this paper bring a new solution to the 2D pallet loading problem with faster convergence and higher pallet utilization in solving medium to large-scale port loading problems.

In the following parts, this paper introduces the IP model in Section 2, combines the SA algorithm and the MHL algorithms to design the solution idea of the hybrid heuristic algorithm in Section 3, and finally simulate and compare the examples in the experiments of Section 4.

## 2 RESEARCH BACKGROUND

### 2.1 Integer Programming Model

Xiaoying Qi proposed an improved lower-left IP model. Compared with the lower-left model, this model has fewer constraints and greatly speeds up the time to obtain the optimal solution. This paper introduces this model, and builds a two-dimensional strip discrete model so that 2DR-SPP can be discretized. For each rectangular box, it can be rotated but not overlapped. The rotation refers to parallel to the strip. The strip box is either perpendicular to the strip box, nor can it overflow outside the strip box. The goal is to use the strip with the lowest height to fit all the rectangular boxes. Figure 1 below shows the two cases where the rectangular boxes overflow and the rectangular boxes overlap.

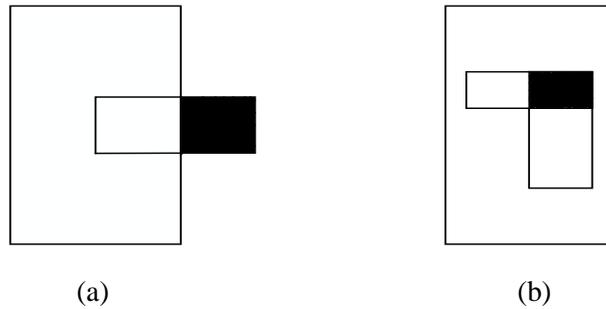


Figure 1: (a) The rectangle overflows to the outside of the strip. (b) Some parts of the two rectangles overlap.

#### Parametric variables:

$(n_i, m_i)$  lower left cell coordinates of rectangle  $i$ ,  $i \in I, n \in N, m \in M$

$(n_i^*, m_i^*)$  upper-right cell coordinates of rectangle  $i$ ,  $i \in I$

$Z$  the number of rows the strip box is occupied by the rectangle

$y_i$  0-1 variable  $y_i = 1$  indicates the non-rotated placement of the rectangle  $i$ , otherwise 0,  $i \in I$

$S_i^T$  the placeable area of the lower left cell when rectangle  $i$  is rotated and placed,  $i \in I$

$h_i$  the height of each rectangle after  $90^\circ$  rotation,  $i \in I$

$x_{inm}$  represents the cell  $(n,m)$  of the strip box occupied by the lower-left cell of rectangle  $I$ , when occupied  
 $x_{inm} = 1$ , otherwise  $x_{inm} = 0$

$\Omega_i$  the placement area of the lower-left cell of rectangle  $i$ ,  $i \in I$

#### Objective function and constraints :

$$\min Z \tag{1}$$

$$Z \geq (\sum_{(n,m) \in \Omega_i} n \cdot x_{inm} + h_i - 1), (i \in I') \tag{2}$$

$$\sum_{(n,m) \in S_i} x_{inm} + \sum_{(n',m') \in S_j} x_{i'n'm'} = 1, (i \in I, i' = i + |I|) \tag{3}$$

$$\sum_{i \in I'} \sum_{(n', m') \in S; n-h_t+1 \leq n' \leq n; m-l_t+1 \leq m' \leq m} x_{i'n'm'} \leq 1, ((n, m) \in S) \quad (4)$$

$$Z \geq \left\lceil \frac{\sum_{i \in I} h_i \cdot l_i}{L} \right\rceil \quad (5)$$

This model is an improved lower-left corner positioning model. (1) The objective function is to take the minimum number of rows occupied by the rectangular box. (2) indicates that only one of the rectangles and can be selected to enter the tape box. (3) ensures that the rectangular boxes do not overflow and do not overlap each other. (4) uses the minimum number of cell rows required for the strip box, that is, the theoretical optimal value to delimit the goal of the problem.

After simplification, although the complexity of the integer programming model is greatly reduced, (1)-(4) contain a large number of constraints and variables when faced with large and medium-sized problems. As the scale of the problem increases, the amount of computation will change explosively, and the optimal solution cannot be obtained in a short time. Therefore, this paper introduces a heuristic algorithm, and in the heuristic algorithm, because the simulated annealing algorithm has a good local search ability in solving the operational optimization problem, and can jump out of the local optimal solution with a certain probability, and the minimum horizontal line algorithm has a good Global search ability, so I combine them to continue to solve the problem.

### 2.2 Minimum Horizontal Line Algorithm

The main idea of solving the two-dimensional strip boxing problem based on the minimum horizontal line (MHL) algorithm is to first set the bottom edge of the bar as the initial lowest horizontal line, and put it into each rectangular box in turn according to the lowest principle. When the current lowest horizontal line is present, it will be raised to the horizontal line of the previous height until the arrangement ends. The technical flow chart of the lowest horizontal line algorithm is shown in Figure 2.

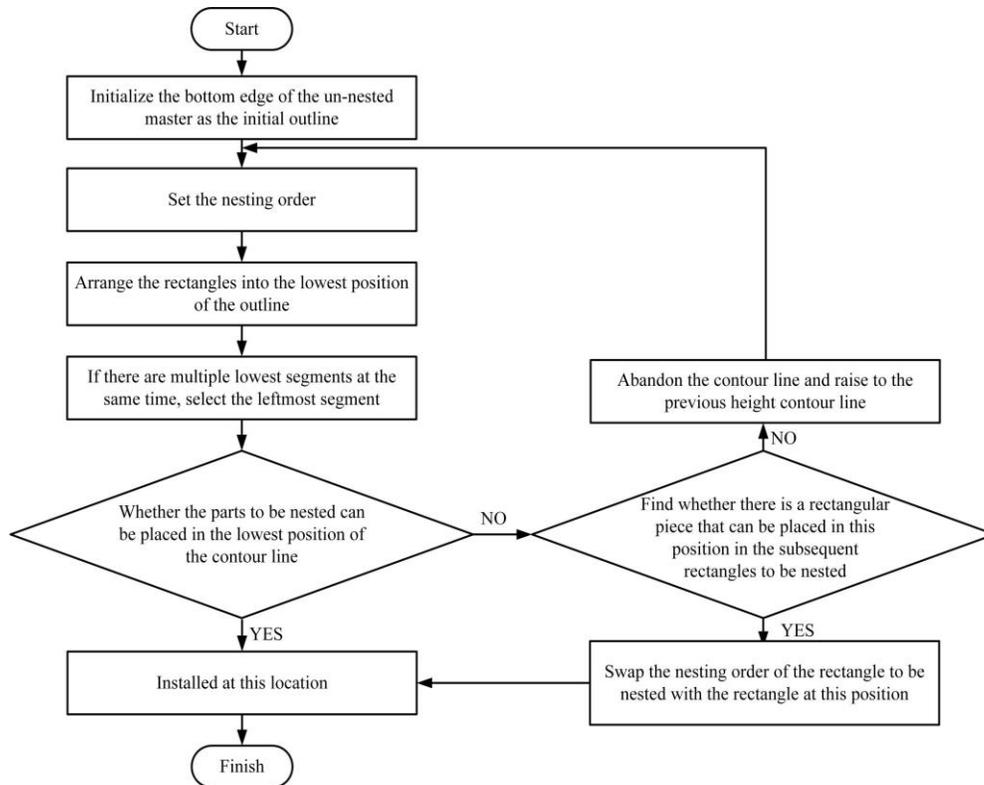


Figure 2: The technical flow chart of the MHL algorithm.

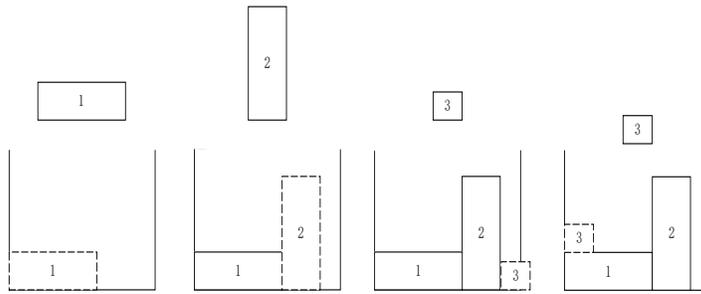


Figure 3: Example of the lowest level algorithm.

As shown in Figure 3, I want to put the rectangular box 1.2.3 into the tape box. After the rectangular boxes 1 and 2 are installed, the rectangular box 3 should be placed at the small right corner of the tape box according to the lowest horizontal line algorithm. In contrast, the position of the lower right corner is not enough to meet the placement of the rectangular box 3, so select the lowest horizontal line of a higher level, and place it on the upper side of the rectangular box 1 and to the left.

### 3 HYBRID HEURISTIC ALGORITHM BASED ON SIMULATED ANNEALING ALGORITHM AND MINIMUM HORIZONTAL LINE

#### 3.1 Overview

The use of heuristic algorithms to solve the two-dimensional strip bin packing problem has been ongoing. Chen (2007) proposed an AL intelligent two-stage search algorithm. The first stage selects the initial arrangement position according to the principle of occupying the lower left corner, and the second stage optimizes its existing position according to its fitness and selects the final position. Zhang et al. (2006) introduced a heuristic recursive algorithm HR (heuristic recursive algorithm), which is a method based on the combination of heuristic strategy and recursive model. Optimal solution. Chen Duanbing et al. (2007) proposed a greedy algorithm for the two-dimensional rectangular box packing problem based on the principle of occupying corners. The algorithm tries to ensure that each step is the optimal solution, so the local optimal solution can be obtained in a relatively short time. Chen et al. (2007) proposed a two-stage search algorithm to solve the two-dimensional strip rectangle packing problem. Compared with the single algorithm, this algorithm is more efficient and easier to obtain the global optimum. Cuil. (2008) gave A recursive mode heuristic branch-and-bound algorithm HRBB (heuristic recursive branch-and-bound algorithm), which applies the recursive structure to the branch-and-bound method and combines them to solve the two-dimensional strip bin packing problem. Jia Zhixin et al. (2002) proposed a LHL (lowest-horizontal-line) layout algorithm. They strictly follow a certain arrangement sequence in the process of arranging two-dimensional rectangles, which is easy to cause waste of two-dimensional containers. Tian et al. (2014) used the heuristic genetic algorithm to solve the two-dimensional strip packing problem. This algorithm can quickly find the optimal solution when solving large and medium-scale problems, but because of its fast convergence speed, it is easy to fall into Local optimal solution. Yu Bi, Peng Qiaozi, Wang Yingpin et al. (2015) systematically summarized and evaluated the parameters and process of simulated annealing algorithm.

#### 3.2 2D Pallet Loading Hybrid Heuristic Design Flow

The simulated annealing algorithm (SA) is more stable than other intelligent algorithms because of its good local search ability, and because of its initial solution and neighborhood search ability, and the simulated annealing algorithm has a certain probability to accept the worst solution set, so that it is easier to jump out

of the local optimal solution and achieve the global optimal. However, because there is a certain probability of accepting the worst solution set, it is possible to miss the global optimal solution, and the global search ability of the lowest level algorithm is better, so I combined the two algorithms to solve the two-dimensional pallet loading problem. The following is the process design scheme and flow chart 4 of the hybrid heuristic algorithm in this paper.

In this paper, the coding method of the simulated annealing algorithm to solve the two-dimensional pallet loading problem adopts the random generation method to obtain the initial solution, and the decoding method is to use the bottom horizontal line algorithm to perform nesting decoding, and exchange two adjacent rectangular pieces at random for neighborhood. Search to ensure the randomness of the new solution and avoid jumping into the local optimal solution. The exponential cooling method is used to increase the efficiency of the simulated annealing algorithm. The initial temperature is set to  $900^{\circ}$ , the termination temperature is  $1^{\circ}$ , and the maximum number of cycles is 180 times.

Step 1: Set the filling order of a random rectangular box as the initial solution, and give the initial temperature, the termination temperature, and the number of iterations.

Step 2: Randomly select two adjacent rectangular pieces to exchange the arrangement order to obtain a new solution, and use the lowest horizontal line algorithm to arrange the new solution to obtain the volume utilization rate of the strip box.

Step 3: Set the evaluation function as the ratio of the unused area of the bar box to  $f(x)=S(x)$ , and calculate the ratio of the unused area of the new solution to the unused area of the old solution. If  $\Delta f < 0$ , it means that the new solution has a higher volume utilization rate, go to Step 4, otherwise, according to the probability selection principle, according to the probability selection principle  $\exp(-\Delta f / t_k) > \text{random}[0,1]$  whether to accept the new solution.

Step 4: Let  $t_{k+1} = \alpha t_k$ ,  $k \rightarrow k+1$ ,  $\alpha \in [0,1]$ , this formula is a process of simulated annealing cooling, we set  $\alpha$  to 0.8 to judge whether the set termination temperature is reached, if The output optimal solution is reached, otherwise go to Step2.

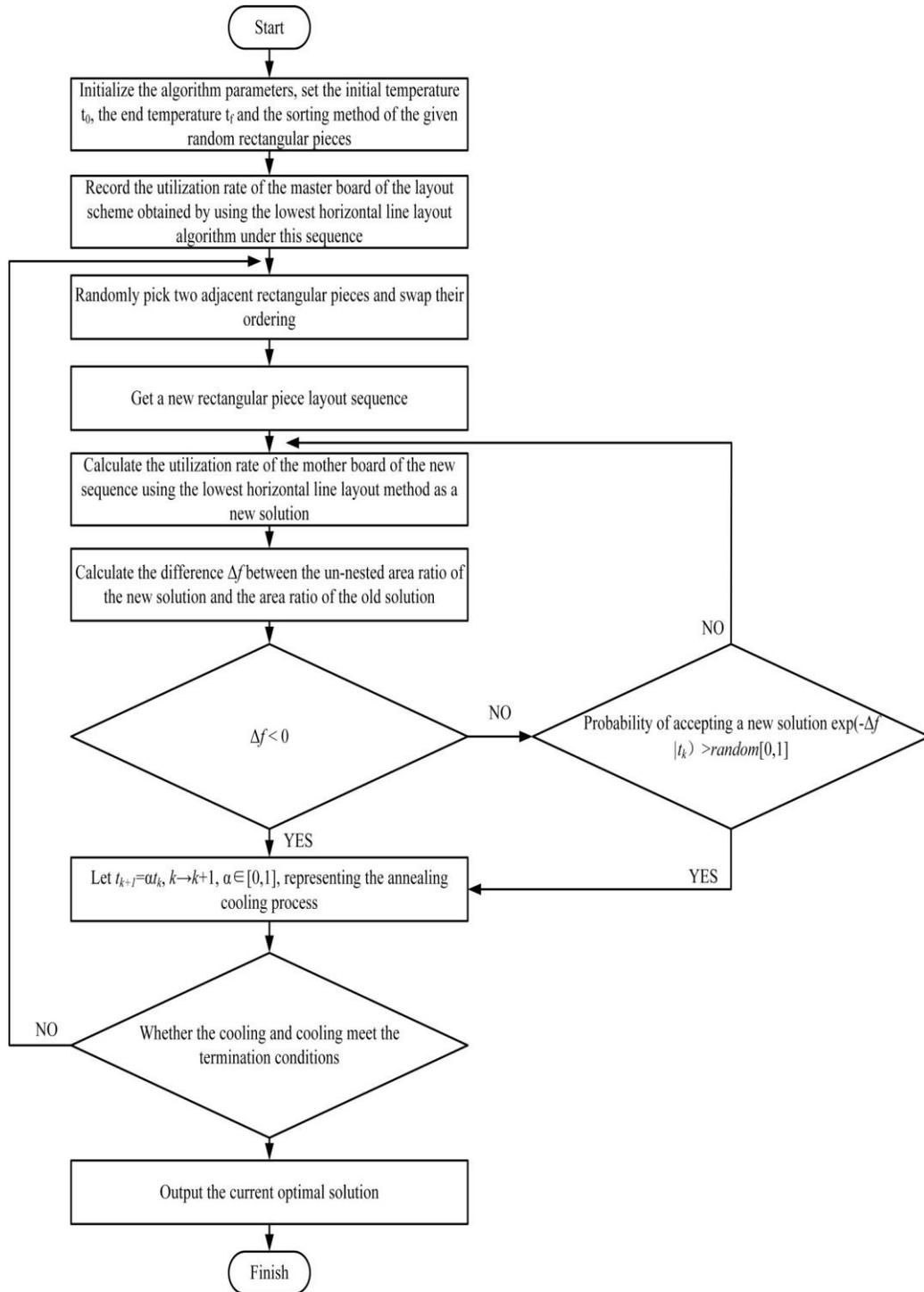


Figure 4: Technology roadmap for hybrid heuristic algorithms based on simulated annealing algorithm and minimum horizon algorithm.

## 4 NUMERICAL EXAMPLE AND SIMULATION

### 4.1 Problem Statement

In order to verify the effectiveness and superiority of the algorithm, we apply the hybrid heuristic algorithm mentioned in this paper to the example in the literature. The example data is shown in Table 1.

Table 1 : The width and height of each rectangular box in the example.

Serial number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
High	1	2	3	2	5	6	5	4	3	9	4	1	2	3	2	2	1	2	2	1	1
Width	5	2	2	7	1	6	10	3	2	5	2	1	3	1	6	2	2	1	1	1	1

### 4.2 Parameter Setting

The simulated annealing algorithm used in this paper is an annealing algorithm based on the exponential cooling method,  $t_k = \alpha t_{k+1}$ . Among them,  $\alpha$  we take 0.9, the initial temperature 0-900°, the termination temperature  $t_n = 1^\circ$ , and the number of iterations is 180.

### 4.3 Optimization and Comparison

After calculation, the minimum number of rows and time combined with the simulated annealing algorithm and the lowest horizontal line algorithm are obtained, and compared with the literature [3], the results are shown in Table 2, and the visual layout is shown in Figure 5 and Figure 6.

Table2 :Comparison of the two algorithms in the example.

Algorithm name	Time (Sec.)	High (No.)
The algorithm of literature [3]	112.23	23
Hybrid heuristics	6.62	22

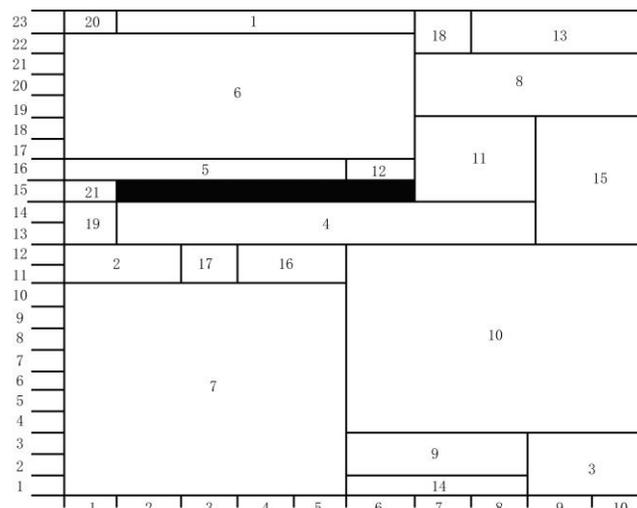


Figure 5: The visual layout of the model algorithm in the lower left corner of the literature.

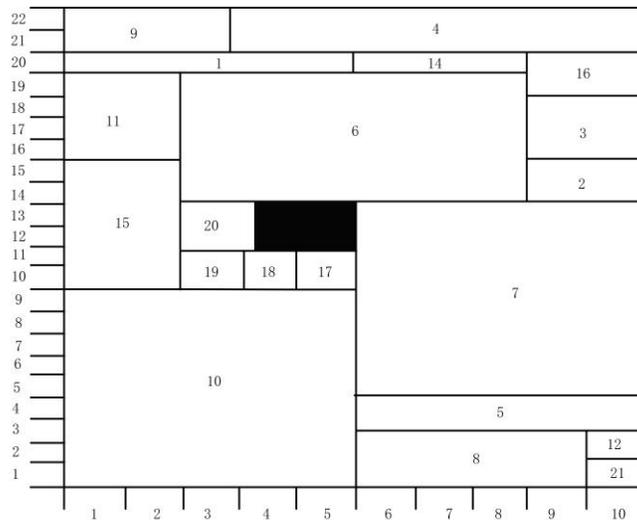


Figure 6: Hybrid GA visualization layout.

The results show that although the height of the hybrid genetic algorithm is not much different from that of the lower left model, the solution time is much smaller than that of the lower left model. The hybrid algorithm model is more efficient and more suitable for practice.

In order to compare the difference between the hybrid heuristic algorithm combining the simulated annealing algorithm and the minimum horizon algorithm described and other heuristic algorithms in terms of strip utilization and solution time, this paper and the calculation example solved by the genetic algorithm in comparisons were made. The calculation example data is shown in Table 3, and the comparison results are shown in Table 4.

Table 3: Reference example data.

Size	12*6	12*4	8*6	6*4	6*2	3*4	4*2	3*2	6*1	2*2	3*1	2*1
Case 1	2	1	2	4	3	4	2	2	1	1	2	1
Case 2	3	3	2	4	5	4	4	0	2	0	2	1
Case 3	3	3	5	4	2	2	4	2	4	4	2	1
Case 4	5	4	3	3	4	4	3	4	4	2	1	3
Case 5	4	3	6	8	4	2	3	8	1	2	3	1

Table 4: Comparison of GA and hybrid heuristics algorithm.

Algorithm name serial number	Hybrid heuristics		GA	
	Time	Utilization	Time	Utilization
Case 1	6.68	96.27%	6.12	96.61%
Case 2	11.32	97.42%	11.69	93.52%
Case 3	12.55	95.14%	14.01	92.99%
Case 4	16.68	98.55%	18.23	91.54%
Case 5	17.52	95.96%	19.28	91.55%

Compared with the GA, the hybrid heuristic algorithm based on the combination of SA algorithm and the MHL algorithm proposed in this paper is slightly slower for small-scale problems in terms of convergence time, but with the increase of scale, it gradually surpasses GA. From the perspective of two-dimensional space utilization, the hybrid heuristic algorithm proposed in this paper can jump out of the local optimal solution more than the genetic algorithm as the scale increases, so as to achieve the global optimal.

## 5 CONCLUSION

Starting from the actual loading problem in ports, this paper introduces a hybrid heuristic algorithm based on simulated annealing algorithm and minimum horizon algorithm to solve the two-dimensional pallet loading problem. Due to the local optimization of the simulated annealing algorithm and the characteristics of jumping out of the local optimum with a certain probability, as well as the strong global search ability of the lowest level algorithm, the hybrid heuristic algorithm combined with the latter is compared with the integer programming model in small-scale problems has a huge advantage in time. In comparison with the genetic algorithm, although the convergence speed of the genetic algorithm is faster than the hybrid heuristic algorithm mentioned in this paper when the scale is small, as the scale increases, the convergence speed is gradually lower than that of the hybrid heuristic algorithm in this paper. The genetic algorithm is easy to fall into the local optimal solution, and the utilization rate of two-dimensional space is not as good as that of the hybrid heuristic algorithm in this paper. Next, we introduce the method to the 3D pallet loading problem and continue to discuss the advantages and disadvantages of the algorithm.

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