Proceedings of the 2022 Winter Simulation Conference
B. Feng, G. Pedrielli, Y. Peng, S. Shashaani, E. Song, C.G. Corlu, L.H. Lee, E.P. Chew, T. Roeder, and P. Lendermann, eds.

FEEDER SHIP ROUTING PROBLEM WITH TIDAL TIME WINDOWS

Yuan Gao

School of Maritime Economics and Management
Dalian Maritime University
1 Linghai Road
Dalian, 116031, P.R.CHINA

Zhuo Sun

School of Transportation Engineering
Dalian Maritime University
1 Linghai Road
Dalian, 116031, P.R.CHINA

ABSTRACT

In a shipping network, feeder ships need to fulfill the demand for cargo transportation between a hub port and its feeder ports. The objective of the Feeder Ship Routing Problem (FSRP) is to minimize the transportation cost, while the feasible time of entering and leaving a port is affected by tide and load due to the limitation of the waterway depth. Different from classic VRPTW(Vehicle Routing Problem with Time Windows), the tidal time windows in this study change by the route of the ship, bringing a challenge to solve the problem. This paper studies an FSRP with nonlinear time windows and solved by Column Generation after a model simplification by Dantzig-Wolfe Decomposition. Numerical experiments and sensitivity analyses proved that the algorithm is effective and that considering tidal influence can effectively reduce the operation cost of the fleet.

1 INTRODUCTION

The shipping market, especially the dry bulk commodities, is recovering from the COVID-19 pandemic (UNCTAD, 2021). Transportation among hub ports maritime network is carried by large liner ships due to economies of scale(Zheng et al., 2015), while cargos between hub ports and feeder ports often need to be carried by feeder ships which are smaller and more flexible to meet the dynamic change of demand, hydrography and other conditions. Feeder Ship Routing Problem (FSRP) needs to be regularly solved in order to reduce transportation costs according to market changes.

Hemmati et al.(2014) categorized ship transportation into two main parts: liner transportation and tramp and industrial ship transportation. In that paper, tramp and industrial ship transportation is described as a pickup and delivery problem with time windows (PDPTW). The Feeder Ship Routing Problem is a special case of tramp ship routing: cargo needs to be picked up from or delivered to the hub port and can be regarded as a Vehicle Routing Problem(VRP). Ricardo et al.(2018) describe the Feeder Ship Routing Problem as a VRP and consider the joint optimization of route and speed. The Feeder Ship Routing Problem studied in this paper considers tidal time windows and simultaneous pickup and delivery.

During the routing of a single ship, the ship's access to a port is greatly influenced by the ship's draft and the water depth of the navigation channel, which is called the draft limit. Although the berths usually have sufficient water depth, the relatively shallow water depth of the navigation channels often becomes a bottleneck for the feasibility of a ship's berthing and unberthing (Yu et al.,2017). Unsal et al. (2019) studied an integrated port planning problem in which the departure time of the ship should meet the tidal time window. Rakke et al. (2012) studied the routing problem of a single ship considering the static draft limitation, and Arnesen et al. (2017) considered the same problem considering pickup and delivery. Gelareh et al.(2019) considered a selective traveling salesman problem with draft limits, in which most profitable routes are selected since a route that satisfies all needs may not exist. However, none of the presenting papers considered that the depth of the navigation channel might change by the rising and receding tides. As shown in Figure 1, the ship draft is related to the ship's weight (weight of the ship + weight of the cargo), and its correspondence can be roughly considered linear by referring to the hydrostatic equation of the ship; while the depth of navigation channel depends on its design depth and the influence of the tide, which can be predicted by using the harmonic function of multiple trigonometric functions(Meena et al., 2015). Lightly loaded ships with low draft can be berthed and unberthed during any time of the day, while heavily loaded ships are more easily stranded during low tide and usually need to wait until the depth of the navigation channel is adequate, which is referred to as the " Tidal Time Windows " in this study.

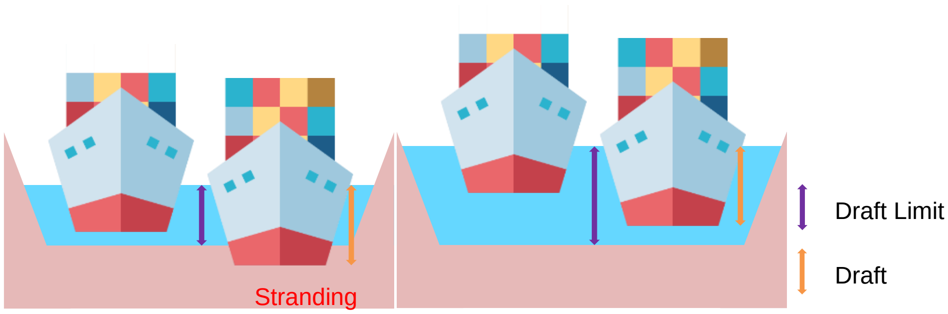

Figure 1: Ship draft and navigation channel draft limit.

The Feeder Ship Routing Problem with tidal time windows (FSRPTTW) can be regarded as vehicle routing problems with time windows (VRPTW). The time windows can be classified into Hard Time Window and Soft Time Window: Pan et al. (2021) studied the problem of a coffee sales company in Singapore that replenished its retail stores because the replenishment time allowed for each retail store is fixed, so the authors restricted it as a Hard Time Window. Soft time windows, also known as Flexible Time Window, differ from the hard time window in that it allows vehicles to arrive outside the time window but incurs a penalty cost (Tas et al., 2014). In addition, a customer may have multiple discrete time windows and allow to be served in any one of them, which is called multiple time windows. Belhaiza et al.(2014) studied the problem of multiple time windows that allow vehicles to arrive early but need to wait. Li et al. (2020) studied "synchronized multiple time windows", in which a customer can be visited by multiple vehicles and within multiple time windows. However, all the vehicles must arrive at the same time window. The "tidal time window" in this study is a hard, periodic, multi-time window, and the size of the time window will change with the increase or decrease of the ship's load, which presents a nonlinear characteristic and brings challenges to solve.

The vehicle routing problem with time windows is a mixed integer programming problem, and a large-scale problem cannot be exactly solved in polynomial complexity time, so the existing research usually uses a column generation algorithm and heuristic algorithm. For example, Wang et al.(2019) used the column generation algorithm to solve the vehicle routing problem for the cooperative operation of UAVs and trucks, and compared it with the Gurobi solver, which solves the mathematical model exactly; Li et al.(2020) mentioned above also used the column generation algorithm to solve for the demand of the vehicle routing problem with synchronized time windows and split demand, the results achieved the same or even lower compared with the exact solution using Cplex. Heuristic algorithms are often used to solve medium to large-scale vehicle routing problems because of their fast speed and low resource consumption. Pureza et al. (2012) tried adaptive tabu search (ATS) with ant colony algorithm (ACO) for solving the practical problem of tobacco and alcohol transportation in Brazil, whose innovation is that they considered the number of crows as a decision variable in constructing a nonlinear service time; Martins et al.(2019) used the Adaptive Large Scale Neighborhood Search (ALNS) algorithm to solve the problem of transporting fresh goods in separate compartments, where the time window is related to the preservation conditions of fresh goods rather than from customer preferences; Azi et al.(2014) used the ALNS algorithm to solve the problem of transporting perishable goods, where all the time windows have the same rear edge considering the characteristics of perishable goods. Berghida et al.(2015) solved a complex vehicle routing problem using a biogeography algorithm (BBA) that analogizes the solution search to a biological population searching for habitat. The problem considers a fleet of heterogeneous vehicles, mixed backhaul cargo, and a time window. The FSRPTTW model in this study has a nonlinear time window making the problem model complex, so a heuristic algorithm based on column generation is designed. The Danzig-Wolfe decomposition is first used to decompose the model into a set-covering master problem and the shortest path subproblem, and a label expansion heuristic algorithm is used for the subproblem to solve and generate the columns of the master problem, which can achieve a relatively high solution in a shorter time.

The main contributions of this study are as follows: firstly, the relationship between ship draft and navigation channel depth is analyzed, and the concept of the tidal time window is proposed; secondly, the feeder ship routing problem with tidal time window (FSRPTTW) model is constructed; then the model is solved by Danzig-Wolfe decomposition and heuristic column generation algorithm; finally, a case study is verified by simulation. The article will be developed in the following structure: Chapter 2 introduces the feeder ship routing problem with a tidal time window and establishes the mathematical model; Chapter 3 performs the Danzig-Wolfe decomposition of the problem, establishes the path-based model, and designs the column generation algorithm to solve it; Chapter 4 presents the simulation and Chapter 5 conclusions.

2 MATHEMATICAL MODEL

The problem studied in this paper assumes that there is a hub port and several feeder ports in a region, and the shipping company needs to arrange a fleet of feeder ships, each of which leaves from the hub and visits a sequence of feeder ports before return to the hub, during each visit, both picking-up and delivery are considered. Unlike the traditional vehicle routing problem, a vessel visiting a feeder port needs to consider the "tidal time window", the size of which depends on the vessel's load at the time.

2.1 Problem Statement

$G = (V, E)$ represents a transportation network in a region, where $V = \{0,1, \dots, n\}$ represents the set of ports, 0 is the hub port, $V^{'} = \{1, \dots, n\}$ is the set of feeder ports, each feeder port i and the hub port have two-directional cargo transportation demand, the demand that pickup from the hub and deliver to the feeder is d_i, that pickup from the feeder and deliver to the hub is p_i. $E = \{(i,j)|i,j \in V, i \neq j\}$ is the set of edges, and c_{ij} and t_{ij} are the travel cost and travel time of the edge, both of which are non-negative. K ships load cargos from the hub port at time 0, and when they visit a feeder port i, they deliver d_i cargos and pickup p_i cargos, and return to the hub port after completing the route. The latest time return to the hub port (the maximum travel time of the vessel) is Tt. The loading and unloading services at each port will consume the service time of s_i, and q_{ik} is an intermediate variable of the ship k's load when departing from port i. At any time, the load of a ship must not exceed its capacity limit C. Each feeder port i has a draft limit of function $F_i(t)$. Generally speaking, the tidal function is symmetry and periodicity, so the feasible time for a ship to visit a port can be considered as multiple time windows, and its time window is narrower when the ship is heavily loaded than when it is lightly loaded. The whole time can be divided into several periods by the low ebb of tide, making sure that there is only one continuous time window in each period. Figure 2 is an illumination of the tidal time window. In each period p, $GS_{ip}(q)$ is the earliest time when a ship with a load of q can go through the navigation channel, and $GE_{ip}(q)$ is the latest time. $(GS_{ip}(q), GE_{ip}(q))$ is the tidal time window in period p.

The time when ship k berths at port i is ta_{ik} and the time when it leaves is tl_{ik}, both needed to be within the tidal time window, if the ship arrives or finished the service before a time window, the ship needs to wait, the berthing waiting time and the leaving waiting time are wa_{ik} and wl_{ik}.

The decision variable x_{ijk} indicates whether ship k sails from port i to port j; the intermediate variable y_{ik} indicates whether ship k serves and satisfies the demand of port i; the decision variables a_{ipk} and l_{ipk} are used to indicate in which time window ship k enters or leaves port i.

The objective of the FSRPTTW is to find the set of ship route that satisfy the loading constraint and draft limit, complete the transportation demand of all feeder ports (each feeder port is visited once), and minimize the total transportation cost, where the transportation cost includes sailing cost and waiting cost, and the weights are denoted by α and β.

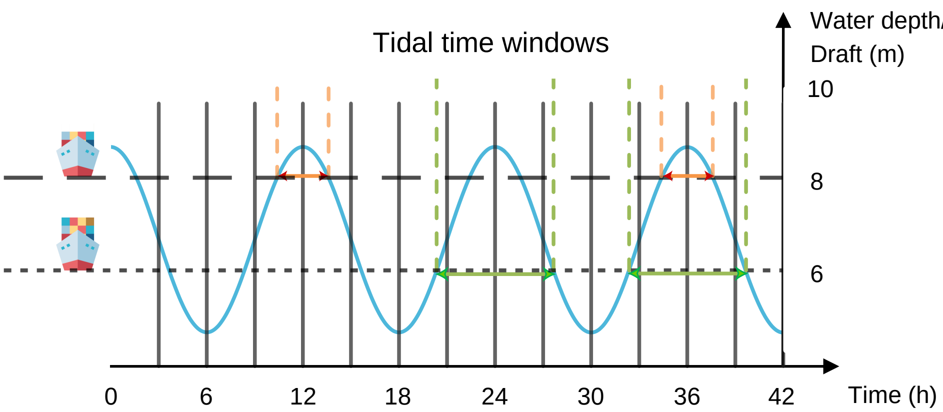

Figure 2: Tidal time window.

2.2 Notation and Model

Based on the vehicle routing problem with time window with simultaneous pickup and delivery, this study proposes a periodic tidal time window constraint to describe the tidally influenced draft limit. The sets, parameters, variables and functions involved in the model are organized as follows.

Set:

V	Set of Ports;
V'	Set of Feeder ports;
E	Set of edges;
P	Set of periods.

Parameters:

K	Number of vehicles;
C	Capacity limit;
Tt	Maximum time of time horizon;
p_i	Pickup demand at vertex i;
d_i	Delivery demand at vertex i;
c_{ij}	Travel cost of arc (i,j);
t_{ij}	Travel time of arc (i,j);
s_i	A number big enough;
M	Pickup demand at vertex i;
α	cost parameter, weight of sailing cost;
β	cost parameter, weight of sailing cost.

Functions:

$GS_{ip}(q)$	The time window front edge of a ship with load q visiting port i at period p;
$GE_{ip}(q)$	The time window rear edge of a ship with load q visiting port i at period p.

Varaibles:

q_{ik}	Load of ship k leaving port i;
TD_{ik}	Total delivery of ship k leaving vertex i;
TP_{ik}	Total pickup of ship k leaving vertex i;

ta_{ik}	Arrival time of ship k at vertex i;
tl_{ik}	Leaving time of ship k at vertex i;
wa_{ik}	Waiting time of ship k at vertex i for entering;
wl_{ik}	Waiting time of ship k at vertex i for leaving;
a_{ipk}	Binary, whether ship k arrivals at vertex i during period p;
l_{ipk}	Binary, whether ship k leaves from vertex i during period p;
x_{ijk}	Binary, whether ship k travels from vertex i to vertex j;
y_{ik}	Binary, whether ship k visits vertex i.

The mathematical model for FSRPTTW is as follows.

$$Z = \min\left(\alpha \sum_{(i,j)\in E} c_{ij} \sum_{k=1}^{K} x_{ijk} + \beta \sum_{i\in V} (wa_{ik} + wl_{ik})\right) \tag{1}$$

$$\sum_{k=1}^{K} y_{ik} = 1 \qquad \forall i \in V' \tag{2}$$

$$\sum_{k=1}^{K} y_{0k} = K \tag{3}$$

$$\sum_{j\in V} x_{ijk} = \sum_{j\in V} x_{jik} = y_{ik} \qquad \forall i \in V', \forall k = 1,\cdots,K \tag{4}$$

$$M(x_{ijk} - 1) \geq (q_{jk} - q_{ik}) - (p_j - d_j) \qquad \forall (i,j) \in E, \forall\, k = 1,\cdots,K , \tag{5}$$

$$M(x_{ijk} - 1) \geq (TD_{jk} - TD_{ik}) - d_j \qquad \forall (i,j) \in E, \forall\, k = 1,\cdots,K , \tag{6}$$

$$M(x_{ijk} - 1) \geq (TP_{jk} - TP_{ik}) - p_j \qquad \forall (i,j) \in E, \forall\, k = 1,\cdots,K , \tag{7}$$

$$q_{0k} = \sum_{j\in V'} x_{j0k} * TD_{jk} \qquad \forall\, k = 1,\cdots,K \tag{8}$$

$$\sum_{i\in V'} x_{i0k} * (q_{ik} - TP_{ik}) = 0 \qquad \forall\, k = 1,\cdots,K \tag{9}$$

$$0 \leq q_{ik} \leq C \qquad \forall i \in V, \forall k = 1,\cdots,K \tag{10}$$

$$tl_{0k} = 0 \qquad \forall\, k = 1,\cdots,K \tag{11}$$

$$ta_{0k} \leq Tt \qquad \forall\, k = 1,\cdots,K \tag{12}$$

$$M(1 - x_{ijk}) \geq (ta_{jk} - tl_{ik}) - (t_{ij}) \qquad \forall i,j \in V', \forall (i,j) \in E, \forall\, k = 1,\cdots,K \tag{13}$$

$$tl_{ik} = ta_{ik} + wa_{ik} + s_i + wl_{ik} \qquad i \in V', \forall\, k = 1,\cdots,K \tag{14}$$

$$ta_{ik} + wa_{ik} \geq GS_{ip}(q_{ik} - p_i + d_i) + M(y_{ik} - 1) + M(a_{ipk} - 1) \qquad \forall i \in V', p \in P, \forall\, k = 1,\cdots,K \tag{15}$$

$$ta_{ik} + wa_{ik} \leq GE_{ip}(q_{ik} - p_i + d_i) - M(y_{ik} - 1) - M(a_{ipk} - 1) \qquad \forall i \in V', , p \in P, \forall\, k = 1,\cdots,K \tag{16}$$

$$tl_{ik} \geq GS_{ip}(q_{ik}) + M(y_{ik} - 1) + M(l_{ipk} - 1) \qquad \forall i \in V', p \in P, \forall\, k = 1,\cdots,K \tag{17}$$

$$tl_{ik} \leq GE_{ip}(q_{ik}) - M(y_{ik} - 1) - M(l_{ipk} - 1) \qquad \forall i \in V', p \in P, \forall\, k = 1,\cdots,K \tag{18}$$

$$\sum_{p \in P} a_{ipk} = \sum_{p \in P} l_{ipk} = y_{ik} \qquad \forall i \in V', \forall\, k = 1, \cdots, K \tag{19}$$

$$x_{ijk} \in \{0,1\} \qquad \forall (i,j) \in E, \forall k = 1, \cdots, K \tag{20}$$

$$y_{ik} \in \{0,1\} \qquad \forall i \in V', \forall k = 1, \cdots, K \tag{21}$$

Where the objective function of the model (1) is the minimization of the total ship cost, α and β are the coefficients of sailing cost and waiting cost. Constraints (2)-(4) are the degree balance constraints at the hub port as well as the feeder port. Constraint (5) establishes the relationship between visiting a port and the change of the ship, constraints (6)-(9) limit the total pickup amount and total delivery amount, and constraint (10) is the load constraint of the ship. Constraints (11) and (12) define the start time and end time, and constraints (13) and (14) establish the relationship between arrival time at a port, departure time from a port, service time and waiting time. Constraints (15)-(19) ensure that berthing and unberthing time are within the time window: when a feeder ship k visits port i in period p, its berthing time (arrival time + waiting time) needs to be later than the front edge of the tidal time window of that period and earlier than the rear edge of the tidal time window of that period, so is the unberthing time (service completion time + waiting time). The constraint (20) is a binary constraint of the decision variables.

The problem studied in this paper differs from the traditional VRPTW in that constraints (15)-(19) construct a set of multiple time windows which change with the ship's load and are nonlinear, which make the solution extremely difficult.

3 ALGORITHM DESIGN

Since the VRPTW is NP-hard, and the model proposed in this study includes nonlinear constraints (tidal time window), the problem complexity is greater, so this study first simplifies the model by Dantzig-Wolfe decomposition, and then designs a column generation algorithm to solve it.

It can be observed that only constraints (2) and (3) consider all ships at the same time; all others are constraints on the routes of every single ship. This means that the model can be decomposed into two parts: assuming that all possible ship routes are known, the problem can be transformed into finding the set of minimum-cost routes satisfying the constraint that all feeder ports should be visited once, which is referred to the set-covering master problem model. While the relaxation problem of the master problem is solved by the simplex method, the process of finding an entering variable is to obtain a route that satisfies constraint (4)-(19) with a minimum reduced cost, which is called the pricing subproblem here. The decomposition process is known as Dantzig-Wolfe decomposition.

Because the number of columns of the master problem is large, if all of them are considered, the complexity of solving the master problem is high. Therefore, during the solution, we can start from a feasible initial set of routes, obtain dual variables by solving the relaxed master problem and solve the sub-problem to generate new routes and add into the set of routes so that the cost of the master problem may drop, repeat by iterations until the master problem gets a satisfying result. This solving process is called column generation.

3.1 Model of the master problem

Assuming that the set of all routes satisfying the (4)-(19) problem is S, the original problem can be transformed into a simple set-covering problem to find a subset of S with minimum cost and all feeder ports are visited all and only once. The mathematical model is as follows.

$$Z_{\text{master}} = \min \sum_{r \in S} c_r \lambda_r \tag{22}$$

$$\sum_{r\in S} \delta_{ir}\lambda_r = 1 \qquad \forall \mathrm{i} \in V' \quad (23)$$

$$\lambda_\mathrm{r} \in \{0,1\} \qquad \forall \mathrm{r} \in S \quad (24)$$

Where, c_r denotes the cost of route r, λ_r is a decision variable with a value of 1 indicating that route r is selected and a value of 0 indicating that it is not selected, and δ_{ir} is a binary constant with a value of 1 indicating that route r visits port i and a value of 0 indicating otherwise. The objective function (22) of the master problem is to minimize the total cost and the constraint (23) restricts all feeder ports to be visited once and only once. The main problem model no longer contains the nonlinear "tidal time window" constraint, which makes it less difficult to solve.

3.2 Column generation

When the problem size is large and there are too many elements in S, it is difficult to solve the master problem directly. However, it can be solved considering a feasible initial routes subset S', which is called the restricted master problem, and the solution of the restricted master problem is the upper bound of the master problem solution, and can be optimized by adding routes to S' by column generation. The column generation requires relaxing the integer constraints (constraints (23) and constraints (24)) of the master problem, known as the relaxed restricted master problem. The model of the relaxation-constrained master problem is as follows.

$$\mathrm{Z}_{RMP} = \min \sum_{r\in S} c_r\lambda_r \quad (25)$$

$$\sum_{r\in S'} \delta_{ir}\lambda_r \geq 1 \qquad \forall \mathrm{i} \in V' \quad (26)$$

$$\lambda_\mathrm{r} \in (0,1) \qquad \forall \mathrm{r} \in S' \quad (27)$$

According to the principle of duality, constraint (26) gives a dual variable π_i, whose economic significance is the cost paid to satisfy the demand of feeder port i in the present solution. For each ship route r, define the reduced cost $\overline{c_r}$, which is calculated as follows.

$$\overline{c_r} = c_r - \sum_{i\in V'} \delta_{ir}\pi_i \quad (28)$$

If there exists a route with a negative reduced cost, it means that the cost of that route is lower than the cost of satisfying the same set of feeder port demands in the current solution. Adding it to S' can drop the objective function of the relaxation restricted master problem. The subproblem is to find a route with negative and minimum reduced cost, and optimize the current solution. When the route with negative reduced cost cannot be found, the restricted master problem is considered to reach the same lower bound as the master problem, and the column generation process ends. The relaxations are removed in order to obtain an integer solution.

3.3 Subproblem model

Choosing x_{ij} as the decision variable indicating whether the route r contains the edge (i,j), the subproblem model is established as follows.

$$Z_{sub} = \min\left(\alpha \sum_{(i,j)\in E}\left(c_{ij} - \frac{\pi_i}{\alpha}\right)x_{ij} + \beta \sum_{i\in V}(wa_i + wl_i)\right) \tag{29}$$

(4)-(20)

The objective function of the subproblem (29) is the minimization of the reduced cost, and the constraints are similar to the original problem constraints (4)-(20), because the subproblem solves the route of each ship, so there is no ship index k.

3.4 Labeling algorithm

The subproblem is a shortest path problem with resource constraints, and it contains negative cost because the dual variable π_i needs to be considered, a multi-label algorithm is designed to solve it. In the multi-label labeling algorithm, each label represents a feasible path, and it contains the sequence of nodes and the reduced cost. Initially, there is only one empty label, and the label is extended to traverse the solution domain. All labels are stored in $List$ for arranging the label extension order.

Label extension: select a label in $List,$ and select a feeder port that has not been included in the label, the label is extended by inserting the port into the path before returning to the hub and gets a new label. The feasibility and reduced cost of the new label is calculated. If the new label is feasible, the new label will be added to the end of $List$; if all ports are extended, select the next label in $List$ and repeat the iteration. When all the labels in $List$ are extended, the multi-label algorithm ends and returns the routes with the negative reduced cost.

Tidal time window: when solving the subproblem, the feasibility of each label needs to be calculated. The problem in this paper differs from other VRPs in that the time window for ship berthing and unberthing is affected by tides and ship's loads.

Taking berthing as an example, when a ship arrives at the port, the arrival time as well as the ship's load and draft can be obtained based on the ship's previous path. According to the port's tide function and the arrival time, we can get the port draft limit at arrival. If the draft is less than the draft limit, the ship can berth directly without waiting. If the draft is greater than the draft limit, further judgment is needed: the maximum draft limit can be known according to the port's tide function, and if the maximum draft limit is greater than the ship's draft, the waiting time can be calculated by solving an equation of when the draft limit equals the draft, otherwise, it means that the ship cannot berth at any time and the label is not feasible.

The label feasibility and waiting time are calculated in the same way when the ship leaves the port.

Domination rule: The multi-label algorithm for the subproblem is able to search all feasible labels, which also means that it takes time. To speed up the multi-label algorithm, we use a heuristic domination rule to optimize it. For two labels a and b with the same set of visited nodes and the same last feeder port, if the cost of label a is lower than that of label b, then label a is considered " dominates" label b, i.e., for any new label extended by label b there must exist a label extended by label a with a lower cost by extending the same sequence of ports. If label b has not yet been extended, then label b is removed from $List$, and if label b has been extended, then label b and all its successor labels are removed from $List$. This domination rule is heuristic because it does not consider the tidal time window

4 CASE STUDY AND SIMULATION

To verify the effectiveness of the algorithm and compare the economic benefit of considering tidal draft limit, in this section, numerical cases will be generated and solved, and finally verified by simulation. The running environment is an AMD Ryzen 5 1400 Quad-Core Processor @ 3.20GHz processor with 8GB RAM and Windows 10 Education Edition system. Our data and model are organized and developed under an open-source software framework, MicroCity (http://microcity.github.io), the programming language is lua, and linear programming and mixed integer programming are solved by CPLEX.

In the simulation case design, the feeder port locations and demands are randomly generated. The feeder ports are randomly distributed in a square area with a length of 100 km, the hub port is located in the center of the area, and any two ports are accessible to each other. The distance is set as Euclidean distance. The hydrological environment and ship data are assumed based on the actual situation of the eastern and northeastern coast of China. The draft limit of the hub port is considered sufficient, and the draft limit of the feeder port at the lowest tide is 8m~10m, which differs for different ports. The tide is semi-diurnal, the function of which is a triangular function, the peak occurs at 0:00 and 12:00, while 6:00 and 18:00 are the low ebb of tide, the difference between which is 3 m. The sailing speed of the ship is 10 km/h, the maximum load is 3000 TEUs, the ship draft is 6m at no load, 12m at full load. According to Sun et al.(2016), the sailing cost coefficient is $\alpha = 900\ USD/km$, waiting cost coefficient is $\beta = 1200\ \mathrm{USD/hour}$. The cases are named as Case_n_m, where n is the case index, m is the number of vertexes.

4.1 Computational results

Five cases are generated, including 3 small-scale cases and 2 large-scale cases. The computing time and objective function values are shown in the Table 1. Additionally, in order to illustrated the benefits of tidal draft limit, an analysis of the static draft limit is applied. The static draft limit means that the depth of the navigation channel only considered the minimum draft limit.

Table 1: Case Study.

Case Name	Static draft limit	Tidal draft limit				GAP
		Enumeration Algorithm		CG Algorithm		
	Obj.	Time	Obj.	Time	Obj.	
Case_1_21	857938.1744	26.637	556180.7119	19.283	588432.3715	5.80%
Case _2_21	945327.7169	18.439	604028.7465	17.412	606291.1169	0.37%
Case _3_21	702260.0632	39.82	496511.1251	24.27	496511.1251	0.00%
Case _4_26	925134.5793	---	---	160.748	659001.4017	---
Case _5_26	1062123.926	212.051	663408.1955	81.255	681443.5874	2.72%
Average						2.22%

Enumeration algorithm is a direct mixed-integer programming to solve the master problem after enumerate all feasible routes. This algorithm is guaranteed to obtain the exact solution, but it will occupy a large amount of memory because of the lack of domination rules. It can be observed from Table 1 that our purposed algorithm can obtain a satisfying result in shorter time and can solve large-scale problems.

The analysis of the static draft limit indicates that considering tidal draft can expand the solution space and achieve better results. An overall 33.10% transportation cost is reduced, which indicates that it is economically meaningful to consider the tidal time window.

4.2 Simulation design

At the beginning of the simulation, each ship is located at the hub and loaded with cargo which needs to be distributed to the ports along the route, and the sequence of ports is obtained by the algorithm described in this paper. The ships have five states, which switches under different conditions. At the beginning of the simulation, the ship's state is sailing from the hub port to its first port.

The **sailing state** refers to the state in which the ship sails from one port to another, and the origin and destination ports are known. In this state, the ship's position will be changed as time goes by, and the position change is calculated as follows.

$$y_k' = y_k + \frac{(y_j - y_i)}{\sqrt{(y_j - y_i)^2 + (x_j - x_i)^2}} * \Delta t * speed \quad (30)$$

$$x_k' = x_k + \frac{(x_j - x_i)}{\sqrt{(y_j - y_i)^2 + (x_j - x_i)^2}} * \Delta t * speed \quad (31)$$

When the ship arrives at the destination port, the ship state is changed. The draft and the draft limit of the navigation channel of the destination port is calculated at this time, and if the draft meets the limit, the ship's state is changed to the service state, otherwise it is changed to the anchorage waiting state.

The **anchorage waiting state** refers to the state where a ship arrives at the port and waits for the tide. In this state, the port draft limit changes with time, and the ship state changes to the service state when the draft limit is greater than the ship's draft.

The **service state** refers to the state in which the ship is in port for loading and unloading. In this state, the ship's load will change with time, and the change is as follows.

$$q_k' = q_k + (p_i - d_i) * \frac{\Delta t}{s_i} \quad (32)$$

When the operation is completed, the ship's state is changed. The draft of the ship at this time and the draft limit of the current port navigation channel are calculated. If the draft meets the limit, the ship's state is changed to sailing state from current port to the next, otherwise it is changed to the berth waiting state.

The **berth waiting state** is known as the state when the ship finishes the service and waits for the tide at the berth before leaves the port. In this state, the port draft limit will change as time goes by. When the draft limit is greater than the ship draft, the ship's state changes to sailing state from current port to the next.

The **end state** is when the ship position returns to the hub port again, the ship's state changes to end state. In this state the ship status and properties will not be changed anymore. The simulation ends when all ships' states are changed to end state.

4.3 Simulation result

Figure 3 shows the ship route of Case_5_26 under four equally divided time slices. From the final results, it can be seen that the ship visits are mostly concentrated at noon or midnight, when the tide is at its highest, and the ship draft limit is easily to be satisfied. The video and codes of the computer simulation in this paper are available online（https://github.com/NemoChina/FSRPTTW_Simulation.git).

5 CONCLUSION AND FUTURE RESEARCH

In this paper, we study the routing problem of feeder ships, and combine with the influence of tide on the feasibility of berthing and unberthing. Establish the feeder ship routing problem with tidal time window (FSRPTTW). Unlike the traditional vehicle routing problem with time window (VRPTW), the "tidal time window" is periodic, and the size of the time window changes with the increase or decrease of the ship's load, which is more suitable for describing the actual situation of shipping.

Since the problem proposed in this study is nonlinear, the Dantzig-Wolfe decomposition is carried out to simplify the model, and a column generation algorithm is applied; then a heuristic dominant labeling algorithm is designed for the sub-problem. Our proposed algorithm framework achieves a balance in solution accuracy and speed.

Finally, case simulation is applied to prove the effectiveness of the algorithm and the economic significance of considering the tidal time window.

The column generation algorithm used in this paper is a heuristic algorithm, and we intend to develop an exact algorithm to solve this problem in future research. In addition, the assumptions for case generation in this paper are idealistic, and we would make them more realistic in future studies, such as using reality-based ship and cargo assumptions and considering the impact of the COVID epidemic.

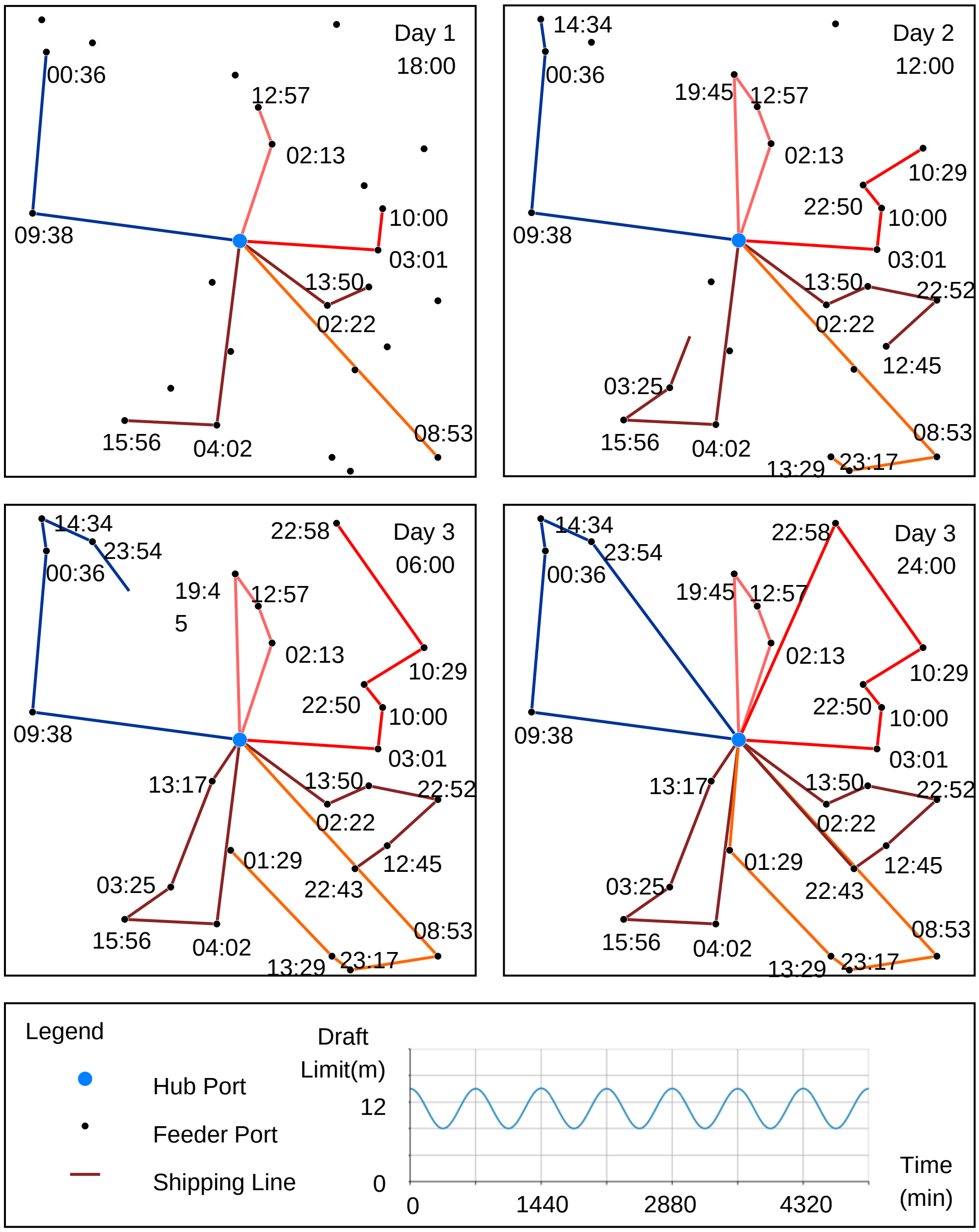

Figure 3: Simulation result.

ACKNOWLEDGMENTS

This work is supported in part by the National Natural Science Foundation of China (61304179, 71431001, 71831002); the Humanity and Social Science Youth Foundation of the Ministry of Education (19YJC630151); the International Association of Maritime Universities (20200205_AMC); the Natural Science Foundation of Liaoning Province (2020-HYLH-32); the Dalian Science and Technology

Innovation Fund (2020JJ26GX023); the Social Science Planning Fund of Liaoning Province (L19BGL011); the National Social Science Fund (21BGJ073).

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AUTHOR BIOGRAPHIES

YUAN GAO is an Doctoral Student in Management Science and Engineering at Dalian Maritime University. His research interests include Logistics Optimization and System Simulation. His e-mail address is gaoyaun0912@126.com.

ZHOU SUN is an Professor in Departure of Logistics at Dalian Maritime University. He earned his Ph.D. in Nagoya University. His research interests include Logistics Optimization and System Simulation. His e-mail address is mixwind@gmail.com.