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INTERCEPT CONSIDERATIONS FOR DEVISING A DIPPING SONAR SEARCH STRATEGY TO LOCATE AN APPROACHING SUBMARINE

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ABSTRACT

A hostile conventional submarine will attempt to get within close enough range to a ship so as to launch a torpedo. To counter this, an Anti-Submarine Warfare (ASW) helicopter with dipping sonar capability can be used to search for an approaching submarine. In situations where earlier contact information of the submarine by an external source is available, intercept trajectory considerations can be used to determine an intercept zone for the submarine from which it can attack the ship. This information can then be used to devise a search strategy for the helicopter to locate the submarine, after which counter measures can be taken against the submarine. This problem has been investigated using a naval combat modelling environment, with results of the methodology development, implementation and analysis reported here.

1 INTRODUCTION

A hostile conventional submarine armed with torpedoes poses a challenging and dangerous threat to naval surface forces through its ability to close undetected to within torpedo striking range of ships. A primary component of Anti-Submarine Warfare (ASW) is therefore to detect and localize a hostile submarine before it is able to enter the Torpedo Danger Zone (TDZ) centered about the ships in a naval formation. Upon detection, the submarine threat can be neutralized either through a change of course for the naval formation, when the ships have a speed advantage over the submarine, or through localization and direct attack using weapon systems. The general problem of devising ASW capabilities and tactics has therefore formed an important aspect of naval strategy since the introduction of submarines into naval combat.

This paper presents details of a new methodology to devise tactics for an ASW helicopter using dipping sonar to search for and localize a threat submarine when initial contact information is provided. The methodology has been implemented in a computer modelling environment developed by the author – the Naval Combat Evaluator (NCE, see Young 2017; Young 2019). The NCE is being employed at the Canadian Forces Maritime Warfare Centre (CFMWC) in research activities to support development of naval task group tactics, with potential pull-through involving updates for maritime tactical instructions and new concepts for tactical decision aids. Results from the implementation in NCE are presented here for illustrative examples to demonstrate the methodology.

2 PROBLEM DESCRIPTION

The problem considered here is to devise a strategy for an ASW helicopter to search for a submarine given an initial contact location. The submarine is attempting to close on a single ship so as to attack it. The helicopter, deployed from this ship, will have to first fly to the vicinity of the initial contact location and then conduct a sequence of dips at different locations to search for the submarine. Each dip involves the helicopter hovering over the sea surface, lowering its dipping sonar, listening, and then recovering the dipping sonar before moving to the next dipping location. The dipping sonar has a sensitivity enabling detection of the submarine within a given detection radius about the dipping point. The temporal nature of this search process, coupled with movement of the submarine, results in the area of possible locations of the submarine increasing as the search is conducted. The search strategy adopted by the helicopter therefore has to take into account this growing region of uncertainty for the submarine's location in deciding at what locations to conduct dips.

This general problem has been previously considered by Ben Yoash (2016) and Ben Yoash, Atkinson & Kress (2018), who devised search strategies for an ASW helicopter with dipping sensor to search for a submarine that has possible movements in any direction about its initial contact location. The area of uncertainty for the submarine's location in this work was therefore circular, expanding over time at a rate based on the submarine's speed.

For the work presented here, consideration is taken of the submarine's objective to close on the ship so as to launch a torpedo attack. We consider a conventional submarine with a maximum speed less than the ship. The region from which the submarine can achieve an intercept on the ship so as to launch a torpedo is defined by the Limiting Lines of Approach (LLA), a concept dating back to World War I for specifying ASW tactics. Contained within the LLA is the extended TDZ, a channel ahead of the ship based on its current course with width equal to the diameter of the TDZ. The objective of the submarine is therefore to get within this channel ahead of the ship, from which it can then launch a torpedo attack as the ship approaches. This region within the extended TDZ channel, which we term the submarine intercept zone, captures constraints on possible submarine movements so that it can achieve its objectives. We use this information to guide construction of an ASW helicopter search strategy. The work presented here therefore builds on earlier concepts of LLA and ASW search strategies, but incorporates the new consideration of the submarine intercept zone to provide an improved methodology for conducting submarine search using ASW helicopters with dipping sonar.

This work was undertaken by introducing LLA constructs, including representation of a threat submarine and ASW Commander (ASWC), into the NCE. Techniques were then identified to determine a submarine intercept zone given a submarine contact. The sonar dipping application was then investigated for guiding an ASW helicopter search strategy, taking into consideration possible strategies for the submarine to avoid detection and achieve intercept. Each of these areas are discussed in the following sections, followed by some concluding remarks on what has been achieved.

3 LIMITING LINES OF APPROACH

Consideration of intercept aspects for a submarine to close to within striking distance of a ship has influenced ASW tactics development since the introduction of submarines into naval combat. US Navy Department (1918) identified the key LLA relationships for ASW during World War I. Koopman (1946) provided further analysis and development during World War II. A more recent development to include endurance constraints for a sprinting submarine was provided by MacLeod (2021), this giving us the ELLA.

Both the classic LLA and ELLA can be derived from a consideration of intercept geometries for the submarine to enter a TDZ centered about the ship, as shown in Figure 1. In this figure the initial positions of the ship and submarine are denoted S and K, and their velocity vectors are $\tilde{\boldsymbol{v}}_{\boldsymbol{s}}$ and $\tilde{\boldsymbol{v}}_{\boldsymbol{k}}$, with $v_s = |\tilde{\boldsymbol{v}}_{\boldsymbol{s}}|$, $v_k = |\tilde{\boldsymbol{v}}_{\boldsymbol{k}}|$, and $v_k < v_S$. The ship is at position S_I when the submarine achieves intercept. Referring to Figure 1(a), we denote the angle between the velocity vectors as θ and define $v_{s\_close} = v_s \cos\theta$ and $d_m = (v_k - v_{s\_close})t + r_T$, for time t and TDZ radius r_T. When v_k is constant, we obtain $\theta = \cos^{-1} v_k/v_s$, this giving the classic LLA relationship $\omega = \sin^{-1} v_k/v_s$ for angle ω, shown by the the heavy solid line in Figure 1(b). MacLeod (2021) considered a functional relationship between endurance time t_e and submarine speed of the form $t_e = a v_k^{-b}$, with a and b being constants, to obtain an ELLA, an example of which is shown in Figure 1(c).

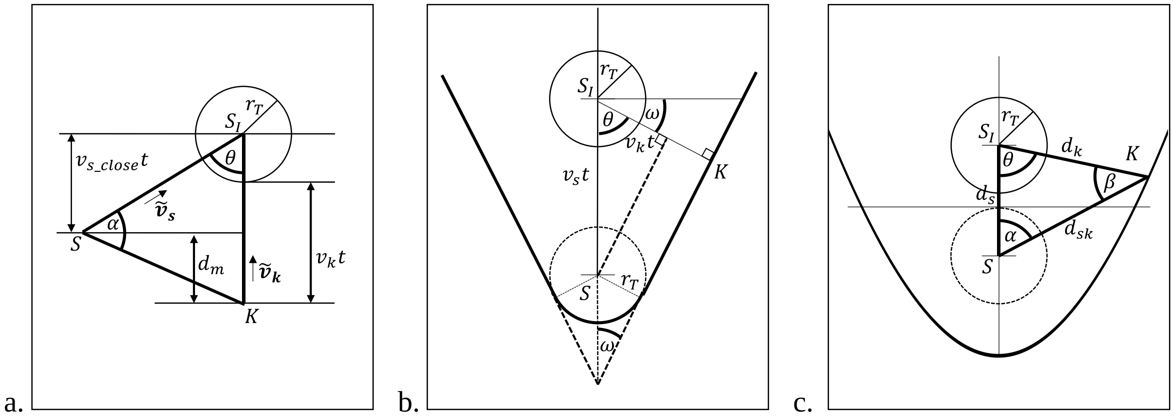

Figure 1: Geometries for determining LLA: (a) Submarine interception of a ship; (b) Classic LLA for a submarine at constant speed; (c) ELLA for a sprinting submarine.

The LLA and ELLA reflect intercept boundaries above which the submarine can achieve an intercept and below which it cannot. One simple strategy for the ship to avoid interception by the submarine is to turn away by a sufficient angle so that the submarine position rotates out of the LLA/ELLA. This illustrates one value of depicting the LLA boundary/ELLA. The boundaries also show what region ahead of a ship must be searched to counter a potential submarine threat.

4 SUBMARINE INTERCEPT ZONES

The LLA/ELLA boundaries define regions where a submarine can intercept a ship, based on the ship's current track. We now consider the situation where contact information of a submarine has been provided, this consisting of the submarine's position at some time point. The submarine is assumed to have a constant speed, i.e. the analysis corresponds to LLA assumptions. Given the submarine's position, trajectory considerations can be used to define a submarine intercept zone along the ship's track where the submarine can intercept the ship, i.e. enter the TDZ. Obviously the intercept will occur within the extended TDZ channel defined on the ship's track. The objective for the submarine is to get into this channel ahead of the ship, at which point, if needed, it can wait for the ship to drive onto itself so that it enters the TDZ.

Figure 2 presents a general intercept situation, where the notation from above is used. We define $S = (x_s, y_s)$ and $K = (x_k, y_k)$. Intercept occurs at time t_I when the ship reaches point $S_I = (x_1, y_1)$ and the submarine reaches point $K_I = (x_2, y_2)$, the distance between these two points being equal to the radius of the TDZ, r_T.

We wish to derive an expression for t_I given the initial locations, velocities and heading of the submarine. Equations of motion for the ship and submarine are, respectively, $(x_1, y_1) = (x_s, y_s) + \tilde{\boldsymbol{v}}_{\boldsymbol{s}} t_I$ and $(x_2, y_2) = (x_k, y_k) + \tilde{\boldsymbol{v}}_{\boldsymbol{k}} t_I$. Substituting these expressions into ${r_T}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$, which holds at intercept, gives the following quadratic equation for t_I after some manipulation:

$$A_I {t_I}^2 + B_I t_I + C_I = 0 \,, \tag{1}$$

where

$$\begin{aligned} A_I &= {v_s}^2 + {v_k}^2 - 2 v_k v_s \sin\theta_k \,, \\ B_I &= 2(v_k(\Delta x \cos\theta_k + \Delta y \sin\theta_k) - \Delta y v_s) \,, \\ C_I &= \Delta r^2 - {r_T}^2 \,, \end{aligned}$$

and $\Delta x = x_k - x_s$, $\Delta y = y_k - y_s$, $\Delta r^2 = \Delta x^2 + \Delta y^2$.

Equation (1) has real roots when the determinate ${B_I}^2 - 4A_I C_I$ is non-negative, noting this is dependent on the submarine heading θ_k. Defining $f(\theta_k) = {B_I}^2 - 4A_I C_I$, we find $f(\theta_k)$ is a function of quartic form in θ_k with either zero (no intercept for given θ_k), two or four real roots. Solutions of $f(\theta_k) = 0$ provide limits for submarine headings that will lead to a successful intercept. Intercept times can then be obtained using equation (1) for submarine headings varying within these limits. Two solutions of equation (1) exist for a given θ_k with $f(\theta_k) > 0$: the first at smaller time corresponding to the submarine entering the TDZ and the second at larger time corresponding to the submarine exiting the TDZ, e.g. a point opposite K_I on the edge of the TDZ shown in Figure 2.

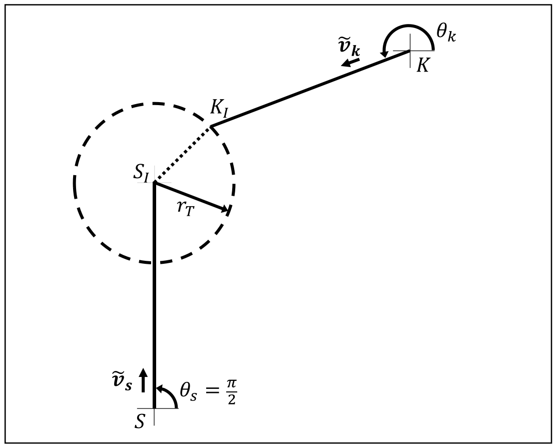

Figure 2: Submarine intercept of a traversing ship.

Figure 3 presents example plots of $f(\theta_k)$ for the situations where $f(\theta_k) = 0$ has two and four real roots. In these plots the θ_k-axis has been cyclically extended to smoothly join up the two local maxima. In Figure 3(a) the submarine is located at (4, 10) nm with speed 6 knots, while in 3(b) it is located at (6.2, 10) nm with speed 10 knots. In both cases the ship is located at the origin with speed 15 knots and the TDZ radius is 1 nm. Figure 3(a) shows a single region with $f(\theta_k) > 0$, while Figure 3(b) shows two regions.

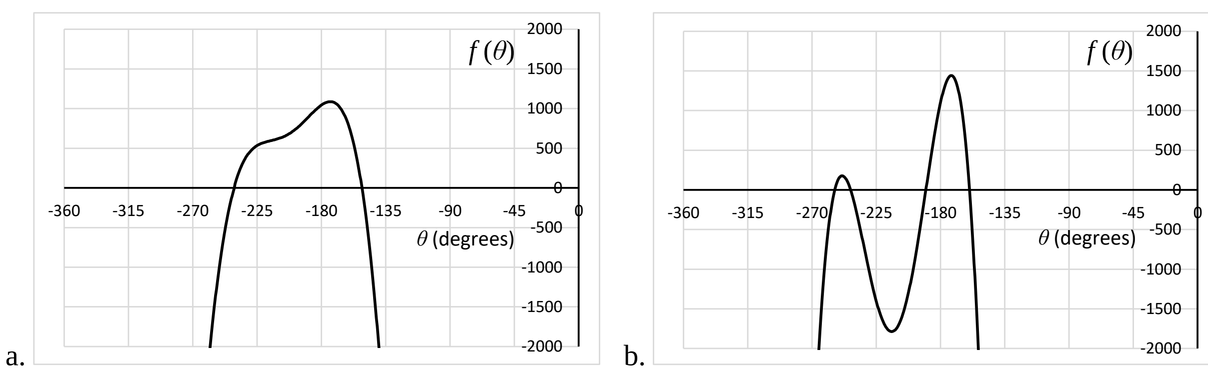

Figure 3: Plots of $f(\theta_k)$ showing two situations based on number of roots for $f(\theta_k) = 0$: (a) Two real roots for submarine with speed 6 knots; (b) Four real roots for submarine with speed 10 knots.

With the regions of $f(\theta_k) \geq 0$ from Figure 3, we can now obtain solutions of equation (1) for varying θ_k. Figure 4 show the resulting intercept zones defined by $t_I(\theta_k)$ for the two situations from Figure 3. In this figure the ship is placed at the origin with a heading along the positive y-axis and the submarine's position is at the point K. Figure 4(a) corresponds to the situation in Figure 3(a) with a single intercept zone residing within the channel of the ship, this being defined by its track with width given by the TDZ radius

r_T. The intercept zone has an oval shape with a concavity to the right, noting that a submarine that reaches this concave region and loiters can also achieve intercept. The angular limits within which the submarine must take for a successful intercept are also shown.

The intercept zone in Figure 4(b) is defined by two regions, corresponding with the two regions from Figure 3(b). The regions define exactly when the submarine, when adopting a constant heading and speed, will cross the TDZ boundary – once entering and once exiting. The actual intercept zone comprises these two regions and the area between them within the TDZ channel of the ship. The lower region reflects the earliest intercept that can be achieved by the submarine, while the upper region reflects the latest possible intercept. For both regions the right sides, with dotted lines, reflect the submarine entering the regions, and the left sides, with dashed lines, when the submarine exits them. The joining of the lower and upper regions within the TDZ channel defines the complete intercept zone for the submarine, with intercepts also being possible when the submarine gets into the TDZ channel ahead of the ship and slows down, turns and/or stops. It can then wait for the ship to sail towards it to achieve an intercept.

a. b.

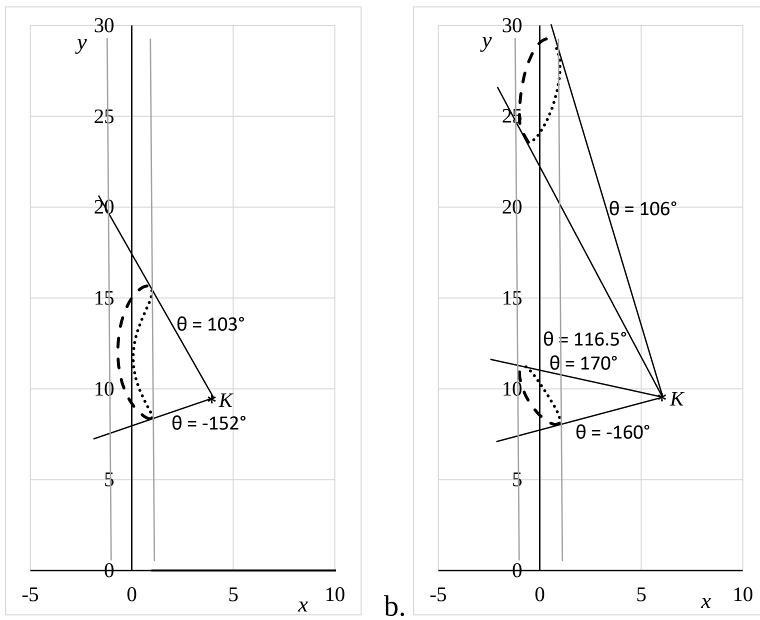

Figure 4: Submarine intercept zones for the two situations from Figure 3: (a) Zone defined by the single $f(\theta_k) > 0$ region from Figure 3(a); (b) Zone defined by the two regions from Figure 3(b).

The approach described here has been implemented into the NCE to show the intercept zones for a submarine approaching a ship. Given a submarine position, intercept zones based on an assumption of the submarine's speed are computed and displayed alongside the LLA for a ship. Example NCE screen captures are shown in Figure 5, these corresponding to two situations similar to those from Figures 3 and 4. In this figure, simulation time is displayed to the upper left, the ship's position is shown with the surface icon labeled "FFH", for helicopter frigate, and the submarine's position with the subsurface icon labelled "SSK", for conventional submarine. A channel defined by the width of the TDZ is shown along the track of the ship. In Figure 5(a), the assumed submarine speed is 6 knots, this giving a narrow LLA fan and single intercept zone in the ship's TDZ channel. In Figure 5(b) the assumed submarine speed is 10 knots, resulting in a wider LLA and intercept zone defined by two TDZ boundary intercept regions with the area in-between them. The arc centered on the submarine icon in these figures corresponds to the distance traveled at the minimum time for intercept, with the corresponding submarine heading for minimum time intercept also shown, this occurring at a small angle just above the lower angle limit. These figures demonstrate how the submarine intercept zone can be generated when a ship is advised of a submarine location within its LLA.

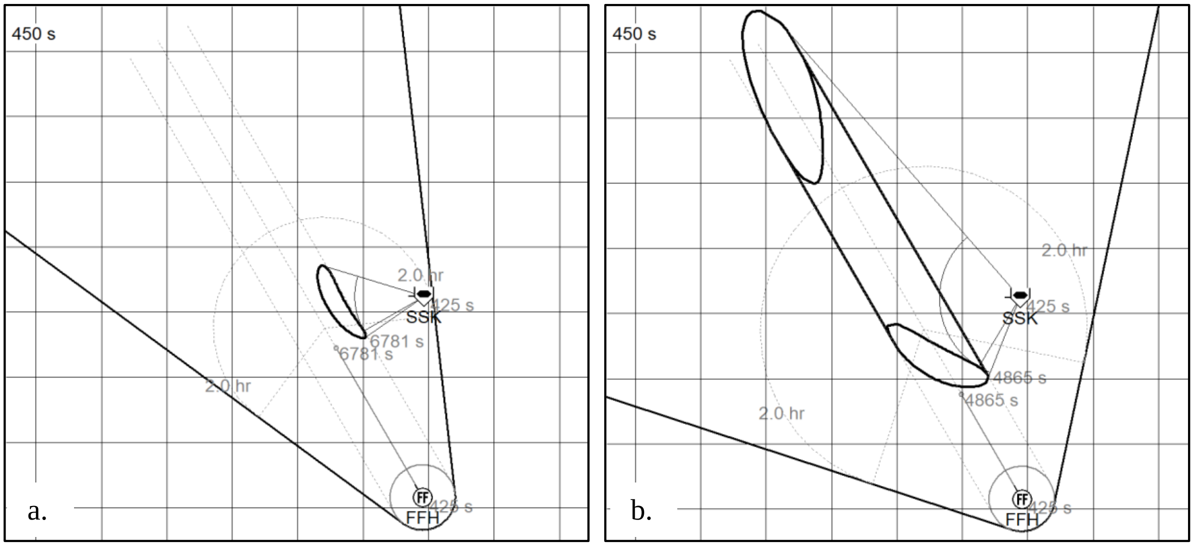

Figure 5: Submarine intercept zones generated in the NCE: (a) Zone based on solution with two real roots for submarine speed 6 knots; (b) Zone based on solution with four real roots for submarine speed 10 knots.

5 ASW SONAR DIPPING MISSION IMPLEMENTATION

A simple model of a ship-based ASW helicopter undertaking a sonar dipping mission was implemented in the NCE, an overview of which is given here. An ASW helicopter object class template was created by sub-classing off the top level unit class in NCE. The resulting ASW helicopter class inherits the NCE simulation mechanisms for time stepped and discrete event processing. Unit states and discrete events were then defined for the helicopter to conduct an ASW mission, from launch to return-to-ship. Figure 6 provides an overview of these states and events. Processing of discrete events leads to state transitions and scheduling of future events, e.g. processing of the "launch" event activates the helicopter with its location set to that of the ship at launch, its state is set to "climb to cruise altitude", and the future event "push over to cruise" is scheduled. Time stepped processing of the simulation uses the helicopter's velocity for maximum climb rate to provide location updates until the cruise altitude is attained. Processing of the discrete event "push over to cruise" will change the helicopter's state to "cruise". In this manner the helicopter's ASW mission can be described as a sequence of the discrete events with time intervals between their processing.

Further changes to the NCE included introduction of the command and control (C2) class C2_ASWC to represent the ASWC, an instance of this class being attached to the ship. This C2 object monitors track information being passed to the ship from external sources. On receipt of a threat submarine track comprising time and location information, the C2 object is triggered to plan an ASW sonar dipping mission to regain contact with the threat submarine. The planning comprises construction of the possible submarine engagement zone for an assumed submarine speed using the method described in the previous section. The C2 planner will then determine the first dipping point reachable by the ship's ASW helicopter through an helicopter intercept analysis based on the submarine assuming a minimum time intercept to the ship. With this initial dipping point, follow-on dipping points are scheduled using techniques as developed by Ben Yoash (2016) such that the full arc of possible submarine headings for it to ingress into its engagement zone are covered. The mission information is then passed to the ASW helicopter, which is then launched to undertake the mission.

A simple helicopter intercept trajectory calculation is conducted to determine the initial and follow-on dipping points. Figure 7 provides the context for this. In Figure 7(a), the geometry from launch point S_L to the first dipping point D_1 is shown, this permitting determination of the initial helicopter heading and time for cruise flight. In Figure 7(b) for two adjacent dipping points, the second dipping point is positioned relative to the first so that the edges of the sensor detection coverages for the two dipping points, described by circles of radius r_D, are tangential to a common ray originating from the submarine track location. This ensures that there are no angular gaps between the two dipping points. In both situations the submarine track location is K and the submarine is assumed to have constant speed v_k.

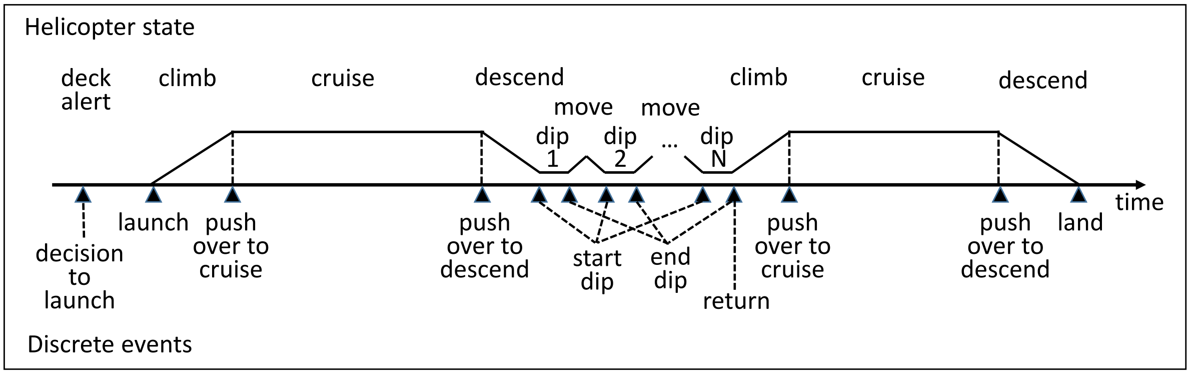

Figure 6: ASW helicopter states and discrete events for undertaking a sonar dipping mission.

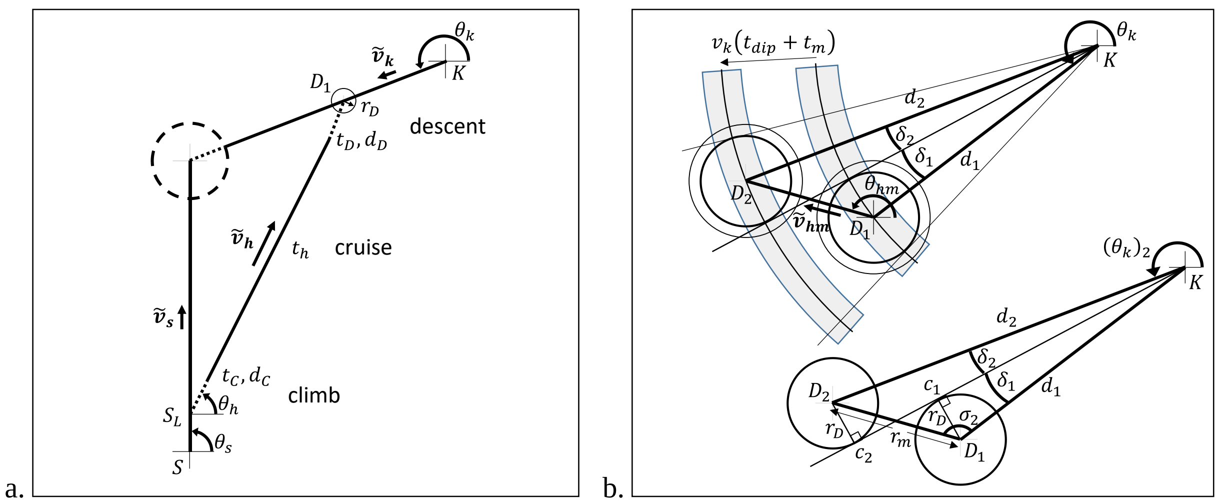

Figure 7: ASW helicopter movement to sonar dipping locations: (a) Helicopter launch and movement to first dipping location; (b) Helicopter movement to follow-on dipping locations.

Referring to Figure 7(a), the helicopter's heading after launch θ_h and time for cruise flight t_h are unknowns that must be determined. The helicopter is assumed to have a constant cruise speed denoted v_h, with associated velocity vector $\tilde{\boldsymbol{v}}_{\boldsymbol{h}} = v_h(\hat{\imath}\cos\theta_h + \hat{\jmath}\cos\theta_h)$, $\hat{\imath}$ and $\hat{\jmath}$ being unit vectors for the x and y axes. The ship's location and velocity vector at the time of the submarine track are (x_s, y_s) and $\tilde{\boldsymbol{v}}_{\boldsymbol{s}} = v_s(\hat{\imath}\cos\theta_s + \hat{\jmath}\cos\theta_s)$. The C2 planner anticipates a time delay of t_L for the launch to occur, at which time the ship's location is S_L: $(x_1, y_1) = (x_s, y_s) + \tilde{\boldsymbol{v}}_{\boldsymbol{s}} t_L$, this becoming the initial location for the helicopter. The helicopter's flight to the first dipping point D_1: (x_2, y_2), comprises a climb to cruise altitude, cruise at constant altitude and descent to the dipping point. Times associated for each of these flight phases are denoted using, respectively, t_C, t_h, and t_D, with horizontal distances traveled by the helicopter during climb and descent denoted using d_C and d_D. The dipping point is then related to the launch point by $(x_2, y_2) = (x_1, y_1) + (d_{CD} + v_h t_h)(\hat{\imath}\cos\theta_h + \hat{\jmath}\cos\theta_h)$, where $d_{CD} = d_C + d_D$.

The submarine is assumed to have constant speed v_k and to take heading θ_k for the first dipping point, this corresponding to the lower angular bound for possible submarine headings to achieve intercept with the ship. The submarine's velocity vector is then given by $\tilde{\boldsymbol{v}}_{\boldsymbol{k}} = v_k(\hat{\imath}\cos\theta_k + \hat{\jmath}\cos\theta_k)$. The variables t_h

and θ_h have to be determined so that the dipping point is positioned correctly for the submarine speed v_k and heading θ_k. The dipping point is positioned $\frac{1}{2}v_k t_{dip}$ ahead of the submarine's anticipated position at the time of the helicopter's arrival, where t_{dip} denotes the helicopter's dipping time. Relative to the submarine's track position, the dipping point is given by $(x_2, y_2) = (x_k, y_k) + (t_S + t_h)\widetilde{\boldsymbol{v}}_{\boldsymbol{k}}$, where $t_S = t_L + t_C + t_D + \frac{1}{2}t_{dip}$. Relating the two expressions for (x_2, y_2) permits the following expressions to be obtained for t_h and θ_h:

$$A_h {t_h}^2 + B_h t_h + C_h = 0\,, \tag{2}$$

$$\theta_h = \tan^{-1}\left(\frac{c_y + v_k t_h \sin\theta_k}{c_x + v_k t_h \cos\theta_k}\right), \tag{3}$$

where

$$\begin{aligned}
A_h &= {v_h}^2 - {v_k}^2\,,\\
B_h &= 2\left(d_{CD} v_h - v_k\big(c_x \cos\theta_k + c_y \sin\theta_k\big)\right),\\
C_h &= {d_{CD}}^2 - {c_x}^2 - {c_y}^2\,,\\
c_x &= x_k - x_1 + v_k t_S \cos\theta_k\,,\\
c_y &= y_k - y_1 + v_k t_S \sin\theta_k\,.
\end{aligned}$$

Again we see the intercept time is given by a quadratic equation, which has real solutions when its determinate ${B_h}^2 - 4A_h C_h$ is non-negative.

Figure 7(b) shows the geometry for follow-on dipping points so that the points align with the expanding radius of movement for a submarine traveling with constant speed v_k. The approach adopted for this is taken from Ben Yoash (2016), who showed that a "tangential" dipping pattern is optimal for searching for a submarine travelling at constant speed and heading when the speed is known but the heading is unknown. In this figure the distance from the submarine's track position to the first dipping point D_1 is given by $d_1 = (t_S + t_h)v_k$. We denote the time it takes for the helicopter to move from the first to the second dipping point D_2 by t_m, with corresponding distance between the points, denoted r_m, being given by $r_m = v_{hm} t_m$, where v_{hm} is the average helicopter speed when transiting between the dipping points. The helicopter's speed is related to its velocity vector by $\widetilde{\boldsymbol{v}}_{\boldsymbol{hm}} = v_{hm}(\hat{\imath}\cos\theta_{hm} + \hat{\jmath}\cos\theta_{hm})$, where θ_{hm} is the helicopter's heading from D_1 to D_2. The variables t_m and θ_{hm} therefore permit the second dipping point's location to be specified relative to the first. We now outline a procedure for determining these variables for a given situation.

Referring to the lower right diagram of Figure 7(b), we denote the distances along the common tangent from the tracking point to the intersections of the normals from the dipping points with c_1 and c_2. For the first dipping point c_1 is known and is given by $c_1 = d_1 \cos\delta_1$, where $\delta_1 = \sin^{-1}(r_D/d_1)$. An inspection of the right triangle with hypotenuse formed by the line segment $\overline{D_1 D_2}$ shows the following equation relating c_2 to t_m holds:

$$(v_{hm} t_m)^2 = (c_2 - c_1)^2 + 4{r_D}^2\,. \tag{4}$$

As t_m and c_2 are unknown, we need an additional equation to enable their determination. This may be obtained by considering the distance traveled by the submarine during the time of the first dip, followed by the time required for the helicopter to move to the second dipping point. This distance, which is equivalent to $d_2 - d_1$, is given by $v_k\big(t_{dip} + t_m\big)$, i.e., $d_2 = v_k\big(t_{dip} + t_m\big) + d_1$. Noting ${c_2}^2 + {r_D}^2 = {d_2}^2$, we therefore have

$$c_2{}^2 = \left(v_k\left(t_{dip} + t_m\right) + d_1\right)^2 + r_D{}^2\,. \tag{5}$$

These coupled, nonlinear equations may be solved in an iterative manner to provide t_m. Setting $f(t_m) = (v_{hm}t_m)^2 - (c_2 - c_1)^2 + 4r_D{}^2$, we solve $f(t_m) = 0$ using Newton's method to obtain t_m. In the examples provided below, solutions of t_m to accuracy 0.00001 were typically obtained in 4 to 5 iterations.

The law of cosines provides the following relationships for calculating the distances shown in Figure 7(b) from the angular displacements in dipping point positions relative to the initial track position:

$$r_m{}^2 = d_1{}^2 + d_2{}^2 - 2d_1 d_2 \cos(\delta_1 + \delta_2)\,, \tag{6}$$

$$d_2{}^2 = d_1{}^2 + r_m{}^2 - 2d_1 r_m \cos\sigma_2\,. \tag{7}$$

The updated heading for the helicopter to fly from the first dipping point to the second is then given by

$$\theta_{hm} = \theta_k - \pi + \sigma_2\,. \tag{8}$$

The helicopter will fly along this heading a distance of r_m to reach the second dipping point. The updated heading for the submarine that goes through the second dipping point is given by

$$(\theta_k)_2 = \theta_k - \delta_2\,. \tag{9}$$

The procedure described here can be applied in a sequential fashion to determine the set of dipping points required to sweep across the arc of possible submarine headings that define the submarine intercept zone. This set contributes to the mission data passed to the helicopter and is used to schedule dipping and move events until either the submarine is detected, the set of points comprising the sweep is completed, or continuation with further points would result in range/endurance limitations of the helicopter being exceeded. Submarine detection would result in the ASW helicopter attacking the submarine through launch of a lightweight torpedo.

The helicopter will return to ship at completion of the scheduled dipping points or upon meeting its range/endurance limitations. This is achieved by determining the helicopter heading and cruise flight time required to "intercept" the ship for landing. A modified form of equations (2) and (3) is used for this.

An additional option for the helicopter at completion of a sweep of dipping points, subject to its range and endurance not yet being exceeded, is to conduct follow-on sweeps using different speed assumptions for the submarine. The ASW C2 engagement planner is able to schedule multiple sweeps for varying assumptions of the submarine speed, each sweep covering the submarine engagement zone angular arc for the speed assumption. In this manner more complex search strategies can be considered by the engagement planner that take into account uncertainties in knowledge of the speed that will actually be adopted by the submarine. A useful feature of the NCE is that it is able to consider such search strategies and then test them through simulation in a systematic manner by varying submarine behavior across a range of speeds and headings.

6 DIPPING SONAR SEARCH STRATEGY

This section provides some illustrative results using notional data for the ASW helicopter and submarine. The example from Figure 5 is used, where a ship is traversing at speed 15 knots and heading 120° relative to the x-axis. A TDZ of 5 nm is adopted. Data assumptions for the ASW helicopter include climb rate of 10 m/s, cruising altitude of 1000 m, cruise speed of 115 knots, dipping time of 5 min and dipping sonar detection radius of 1 nm.

At simulation time 425 s, a track inject occurs notifying the ship that a submarine is located directly to the north at approximately 33 nm. The ship's ASWC assumes the submarine is travelling at 10 knots and formulates the submarine engagement zone shown in Figure 5(b). With a launch delay of five minutes, an ASW sonar dipping mission consisting of eight dipping points is formulated. The ASW helicopter is launched at simulation time 725 s and commences the mission.

Two situations are considered with results shown in Figure 8. In this figure the helicopter is depicted with an air rotary icon labeled “ASWH”, light gray filled circles represent search zones centered on dipping points for which no detection occurred, and a dark gray filled circle represents a search zone with detection. In Figure 8(a) the submarine attempts to intercept the ship by travelling directly to the west at a speed of 10 knots. In this situation the ship’s ASWC speed assumption is correct and the ASW helicopter detects the submarine on the fourth sonar dip. The submarine track position provided to the ship is shown using the subsurface icon with time stamp 425 s, while the actual submarine location is shown by the icon labelled “SSK”. The arc centered on the submarine track position reflects the advancing front of possible positions for a submarine travelling from the track position to the intercept zone for the constant speed assumed by the ASWC. It is the goal of the ASW sweep to cover this arc as it expands over the time for the sweep to occur. The resulting spiral of dipping positions is characteristic of this form of search and was observed and further analyzed by Ben Yoash (2016) and Ben Yoash et al. (2018).

In the second situation shown in Figure 8(b), the submarine travels to the west at speed 6 knots. The actual intercept zone available to the submarine, shown in Figure 5(a), is smaller but still sufficient for an intercept if the ship does not change heading. In this situation the ASWC’s speed assumption is too large and the helicopter is not detected by the ASW helicopter. The sonar dipping positions are too far ahead of the submarine as the helicopter sweeps by the heading angle being taken by the submarine. The submarine therefore avoids detection and is able to achieve an intercept with the ship.

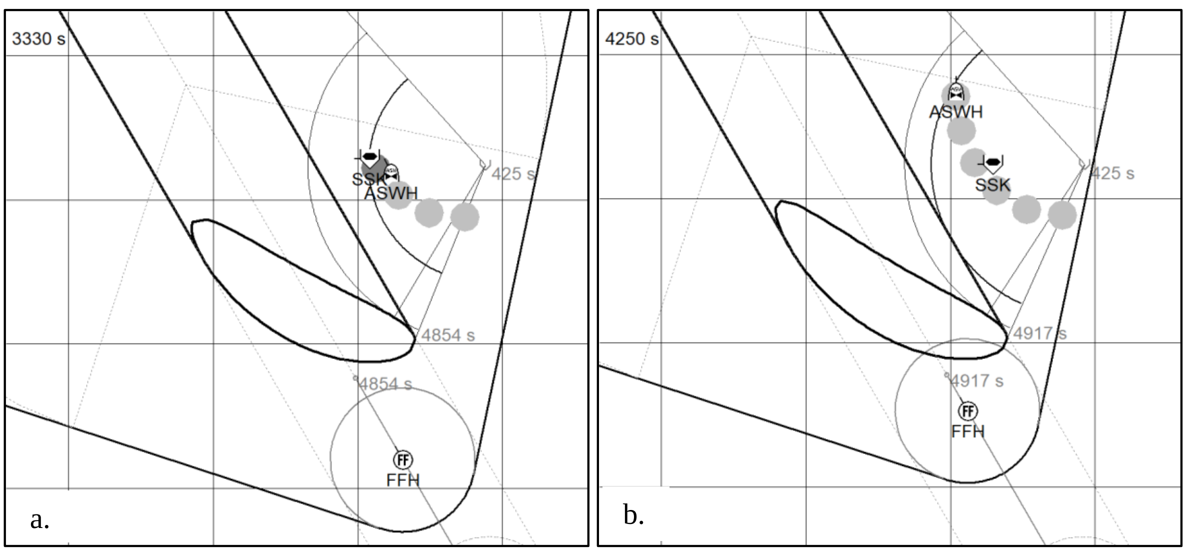

Figure 8: ASW helicopter dipping positions for a submarine speed assumption of 10 knots: (a) Submarine travelling at 10 knots – detection on the fourth dip; (b) Submarine travelling at 6 knots – no detection.

A second set of results are shown in Figure 9 for an alternative search strategy which comprises a ship heading change of 2° to the port, an ASW helicopter search sweep of 7 dips for a submarine speed assumption of 10 knots, and a second sweep of three dips for a submarine speed assumption of 6 knots. In Figure 9(a) the submarine, with an actual speed of 6 knots, is not detected by the first sweep. In Figure 9(b) the submarine is detected during the third dip of the second sweep. For this situation the possible intercept has collapsed from Figure 8(a), a risk the submarine is taking in order to evade detection. A further run was conducted using an actual submarine speed of 10 knots, for which it was detected on the fourth dip of the first sweep, in a situation similar to Figure 8(a).

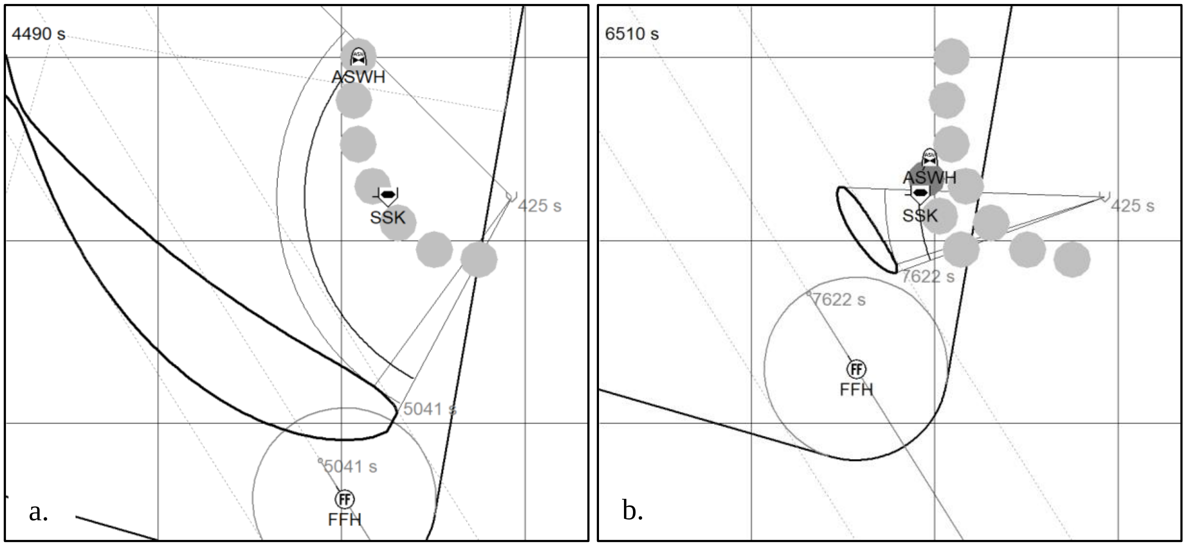

Figure 9: Dipping positions for two sweeps, submarine travelling 6 knots: (a) First sweep using speed assumption of 10 knots – no detection; (b) Second sweep using speed assumption of 6 knots – detection.

7 SUBMARINE STRATEGY TO AVOID DETECTION

Results of the previous section demonstrate how ASW search success is dependent on the submarine speed assumptions. One strategy for the submarine to avoid detection is therefore to vary its movement speed. This is complemented by a further strategy of the submarine taking an indirect route to achieve intercept, if the situation permits. The latter is demonstrated in Figure 10, where the submarine initially adopts a heading in the direction of the ship's track – Figure 10(a), or opposed to the ship's track – Figure 10(b). In both cases the resulting submarine intercept zone decreases from that for a direct intercept, but if the submarine times the initial trajectory leg correctly, it may avoid detection for the ASW search strategies described above. The ASW engagement planner may need to take into account these submarine behaviors for devising a more robust search strategy.

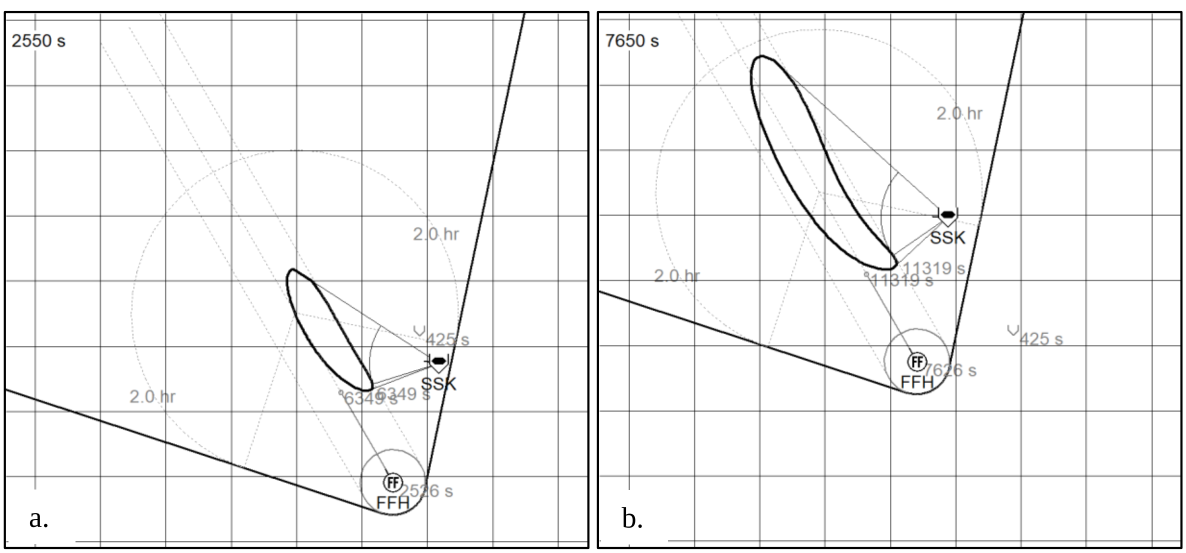

Figure 10: Modified submarine engagement zones: (a) Submarine heading opposite ship's track for 35 minutes; (b) Submarine heading parallel to ship's track for 2 hours.

8 CONCLUDING REMARKS

This paper presents a new method for devising ASW helicopter search strategies based on focusing the search to those areas which the submarine must move through so that it can achieve an intercept with its desired target. The basis for this is the determination of a submarine intercept zone, a mathematical derivation of which has been presented. The methodology has been implemented in the NCE, a modelling environment providing discrete event simulation. Illustrative results have been provided which demonstrate how improved search strategies can be developed through simulation studies using NCE.

Work is now ongoing to expand the ASW engagement planner to consider overlapping LLAs for multiple ships in a task group, and to permit more sophisticated search strategies to be devised involving multiple ASW helicopters for frigates defending a high value unit in a task group. Other ASW aspects, such as ship stationing, patrols, sector screening, other detection systems, and detection probability metrics, as discussed in Bertsche, Guffarth & Karg (2001) and Van Veldhoven & Fitski (2017), are avenues for further work. An additional area is to extend the simple cookie-cutter detection model employed for the dipping sonar to include more realistic detection, drawing on aspects as discussed in Ryder & Johnson (2004).

The work presented here is helping to achieve two aspects for improving analysis capability. Firstly, the modelling capabilities of NCE in the areas of engagement planning, simulation, and model execution on a computer cluster for high throughput computing (Young 2017) can be applied to investigate ASW search strategies with validation through simulation. Secondly, the introduction of underwater aspects into NCE, which until now has focused on above water warfare, permits interdependencies between the two battlespaces to be investigated through a common modelling environment.

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