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MICROSCOPIC VEHICULAR TRAFFIC SIMULATION: COMPARISON OF CALIBRATION TECHNIQUES

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ABSTRACT

Modeling the flow of traffic is an extremely important endeavor for city planners around the world. Accurate models of traffic systems are the main tool researchers require to solve traffic problems. From traffic congestion to unsafe traffic corridors to forecasting travel times, accurate traffic models are essential to improve our experience on our roads and highways. Traffic is a highly complex and dynamic system that, nevertheless, also shows clear patterns of regular behaviors, and as such appears to be a problem that is solvable. Effective traffic models need to be constructed with real-world data informing the design and must be calibrated with the data as well. This paper presents an arrival modeling technique using online data that improves upon offline arrival modeling. The calibration of traffic simulation models is also discussed, and three techniques are compared and contrasted according to accuracy and efficiency.

1 INTRODUCTION

With populations rising around the world, and with many countries increasing the size and scope of their traffic systems, the need for accurate and efficient traffic modeling is only becoming more important. City planners need these models to decide how and where to build new roads, and how to make changes to existing roads. Citizens need these models to plan their daily routes to and from work, or how to deal with special events. Large tech companies and auto manufacturers are bringing self-driving vehicles closer to reality, and traffic flow models will make these vehicles more reliable, efficient, and safe. Governments will also need to use traffic flow models to determine how to regulate this new class of vehicles (Mordue et al. 2020; Fagnant and Kockelman 2015).

This paper is primarily focused on building good traffic models for the purpose of traffic forecasting using a microscopic traffic simulation. Macroscopic simulations, and other techniques from disciplines such as time series analysis and deep learning, do not model the interactions of vehicles in detail, and so can only really give a high-level overview of the traffic system. These models may be able to generally identify problem spots in a traffic network, but with a good microscopic model, there is the hope that very specific details of vehicle interactions can be identified and used to get a much more accurate picture of the entire traffic system. And indeed, a microscopic traffic simulation can provide more of a prescriptive model than other techniques, in that a good microscopic model can be used much more easily to play out what-if scenarios. For example, a government department of transportation which is interested in making a change to a traffic corridor can use a microscopic model to make those changes in the simulation and try them out before real-world construction.

In this work a microscopic discrete event traffic simulation system has been developed using the Scala programming language and the software suite SCALATION (Miller et al. 2010). This software was also used to create the vehicle arrival models defined and discussed in Section 3, and to conduct the calibration procedures discussed in Section 4. This work is part of an ongoing project using Scala and SCALATION to build a self-contained traffic simulation system.

The Caltrans Performance Measurement System (PeMS) is the state of California's repository for traffic data collected in the state. The data used in this project is taken from their raw data files for U.S. Highway 101 near San Jose, California. The raw data files were processed into data matrices of size 96 rows by 365 columns for the years 2017, 2018, and 2019. The data is represented in five-minute increments which were then scaled down into fifteen-minute time increments. Weekends and holidays were not desired for this work, so Tuesday data was used in the bulk of the project. The PeMS system records many traffic variables, including counts, speeds, occupancy, and the percentage of observed data to imputed data. At this time, the system described in this paper relies only on traffic counts.

The structure of the paper is as follows: First, some background on the topic of traffic simulation is presented, including arrival processes, and traffic simulation calibration. Second, the arrival model is discussed in detail with different approaches compared and contrasted, along with an analysis of results. Third, the process of model calibration is discussed, and calibration results are shown in detail. Fourth, an analysis of the contributions of this paper is given. Lastly, directions for future research are explored.

2 RELATED WORK

Microscopic traffic simulations have been studied for a long time and have a rich history of applications in traffic design (Lieberman 2014). The critical elements for microscopic traffic simulation that are addressed in this paper are modeling vehicle arrivals, and the calibration of the car-following model to the data. Related work on these topics is discussed below.

2.1 Arrival Modeling

There have been many techniques proposed for modeling traffic arrivals. Queueing models have been proposed (Van Woensel and Vandaele 2007) where vehicles arrive according to a distribution and then are served (flow along the road) according to another distribution. This approach, however, assumes the overall traffic flows are being modeled and not individual vehicles.

It was long assumed that vehicle arrival rates and vehicle headway distributions were equivalent (Li and Chen 2017). Some of the earliest work was done by Robertson (1969) and explored platoon dispersion models to estimate traffic arrivals. Some recent work by Yang et al. (2020) looks at creating a vehicle arrival model in the presence of connected vehicles (CV).

Poisson distributions have been used extensively in the past to model arrivals in simulations, though it has been shown that for many contexts a simple Poisson distribution with a single arrival rate is inappropriate (Meng and Khoo 2009), and various alternative distributions have been proposed. Klein and Roberts (1984) proposed a time-varying Poisson process. Lee et al. (1991) and Leemis (1991) both proposed nonhomogeneous Poisson processes (NHPPs) for arrivals. We have chosen to use an NHPP to model the arrivals to our system, and our technique is described below.

2.2 Car-Following Models

Car-following models that are in wide use today are Gipps' Model (Gipps 1981) and the Intelligent Driver Model (IDM) (Treiber et al. 2000). A comprehensive genealogy of microscopic models is presented in (van Wageningen-Kessels et al. 2015). The IDM has been chosen as the system's current car-following model, though other car-following models can be easily integrated and analyzed in the future. The details of the IDM are presented in Equations 1 and 2, and in Table 1.

$$\dot{v}_\alpha = a\left(1 - \left(\frac{v_\alpha}{v_0}\right)^\delta - \left(\frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha}\right)\right) \tag{1}$$

$$s^*(v_\alpha, \Delta v_\alpha) = s_0 + v_\alpha T + \frac{v_\alpha \Delta v_\alpha}{2\sqrt{ab}} \tag{2}$$

Table 1: IDM Parameters.

Parameter	Definition	Parameter	Definition
α	Current vehicle	δ	IDM tuning parameter
v_0	Desired velocity of vehicles	s_0	Minimum distance headway
a	Maximum comfortable acceleration	s_α	Current vehicle's distance headway
b	Maximum comfortable deceleration	T	Minimum time headway
v_α	Current velocity of vehicle α	Δv_α	Velocity difference between cars

2.3 Calibration Methods

There have been many efforts to calibrate traffic models. Treiber and Kesting (2013) attempt to calibrate using two techniques, one using a local approach, and another using a global approach. The local approach uses local maximum-likelihood as a measurement and the Levenberg-Marquardt algorithm (Levenberg 1944) (Marquardt 1963) to optimize. In the global approach, they use global least-squared errors and again use the Levenberg-Marquardt algorithm.

Schultz and Rilett (2004) discuss traffic model calibration using a genetic algorithm (GA). Paz et al. (2012) conduct a calibration procedure using simultaneous perturbation stochastic approximation (SPSA) (Spall et al. 1992). Yu and Fan (2017) present several approaches using both Genetic Algorithms and the Tabu Search (TS) algorithm. Patwary et al. (2021) used metamodels to calibrate multimodal microscopic traffic simulation models. Once the metamodel was created gradient-based optimization schemes can be much more effective in locating optimal parameters.

Due to the long computation times that many simulations require it is desired to keep simulation runs to a minimum. Gradient-based optimization procedures tend to require a large number of function executions and therefore a large number of simulation runs. Simulation calibration will thus often use derivative-free optimization methods. The calibration methods chosen for this work are GAs, the Nelder-Mead Simplex algorithm (NM) (Nelder and Mead 1965), and SPSA.

Each of the algorithms was chosen because it essentially represents a different paradigm of optimization technique. SPSA guarantees that no matter how many parameters are chosen for the calibration, there are only two function evaluations in each iteration of the optimization, thereby vastly reducing the execution time while still employing a gradient-type of approach. The Nelder-Mead algorithm and genetic algorithms are direct-search methods that do not require gradients, and so are much more attractive than gradient-based techniques. These two methods, however, conduct their search in radically different ways. Nelder-Mead maintains a simplex that gradually encompasses the optimal value, while GAs use the principles of evolution and selection to maintain a pool of solutions that always remain as good or better than the previous generation.

3 ARRIVAL PROCESS MODELING

The physical traffic network being modeled in this work are the northbound lanes of U.S. Highway 101 between E. San Martin Ave. and Tennant Ave. This includes four traffic sensors, but no on-ramps or off-ramps. The first sensor, therefore, acts as the entry point to the simulation, and the fourth sensor acts as the exit point. The traffic network can be seen in Figure 1.

A typical highway traffic system will require a dynamic arrival process because most days will see dramatic shifts in vehicle counts throughout the day. Figure 2 shows the average traffic flow for all Tuesdays from 2017 through 2019 using 15-minute time intervals, as well as the spread of the data by showing lines dividing the quartiles. The complex nature of the traffic can be seen in the multiple busy periods evident in the shape of the graph.

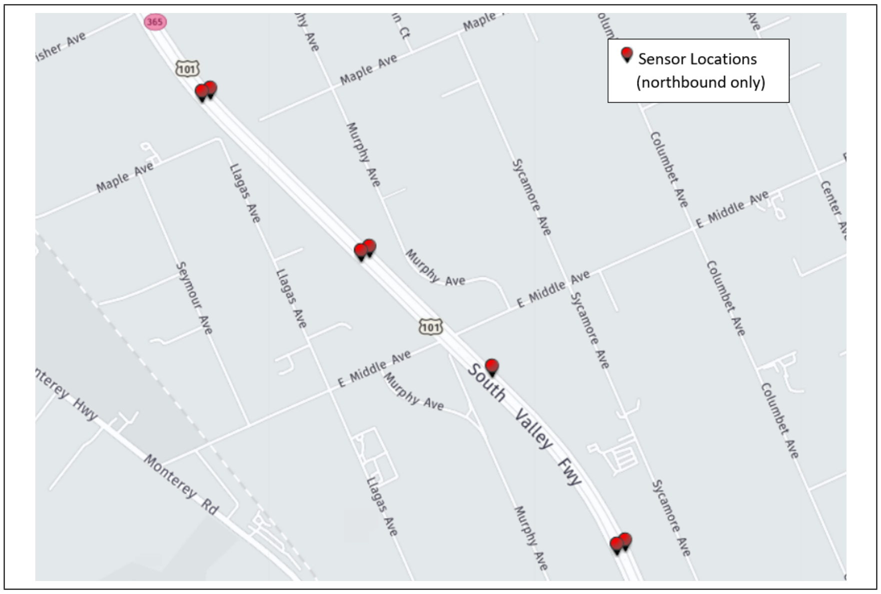

Figure 1: U.S. Highway 101 Network.

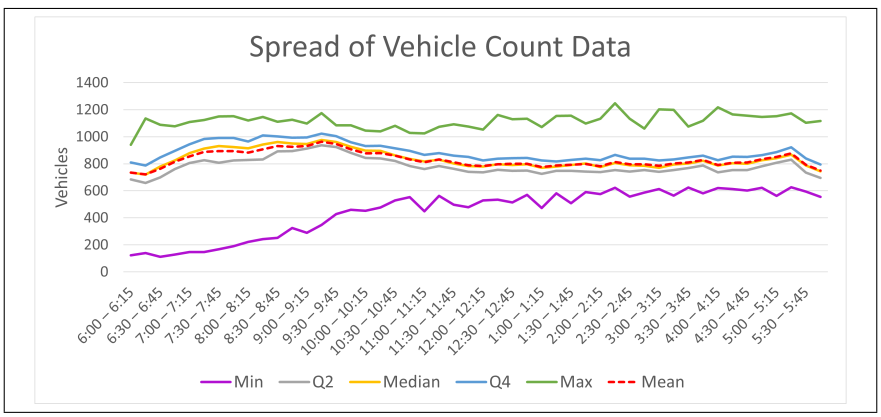

Figure 2: Average Traffic Flow.

A microscopic simulation needs a way to convert vehicle count data into individual vehicle arrival times. This process first requires a forecasting technique to generate counts for the time interval of interest. These counts are then used to generate the arrival times using some form of probability distribution.

In this paper five methods of forecasting the counts are presented. Two of the methods use a seasonal autoregressive integrated moving average model (SARIMA) (Box et al. 1967) which is a standard time series forecasting technique which extends the idea of ARIMA models. ARIMA methods (Whittle 1951), (Box and Jenkins 1962) have a long history of use for time series forecasting and remain an extremely popular forecasting method. The other three forecasting methods, which are defined below, are much simpler, and require very little training time. The five forecasting methods are defined below.

Once the forecasts are made, the arrival times are generated using an NHPP, which is a form of Poisson process where there is an arrival rate function $\lambda(t)$ instead of a single arrival rate λ. In this project the rate function is defined

$$\lambda(t) = \hat{c}_t$$

where $\hat{c}_t$ is the forecasted vehicle count for the time interval represented by t. The NHPP is created using the procedure described by Leemis (1991).

3.1 Offline Methods

Offline forecasting methods only use data from past days, and are trained ahead of the day being forecasted. The first offline method discussed uses a SARIMA time series model to create the forecasts.

SARIMA models require order and differencing parameters for both the non-seasonal and seasonal components of the model. In this project these parameters were chosen using a brute-force optimization technique. The parameter set with the best average accuracy value was chosen for the model. The time series consists of some number n of past days' data over the entire 12-hour period and used to create a forecast for the next 12 hours, where the count at each time point is the value $\hat{c}_t$ used to generate the NHPP.

The second offline method calculates a simple average of n previous Tuesdays, and uses that value as the predicted vehicle count $\hat{c}_t$ for that time interval on the day being forecasted. In Equation 3, and for Equations 4, 5, 6, and 7 below, j represents the day index within the data matrix. This calculation is made for all time intervals to give a vector of vehicle counts for the day, and this is then used as the basis for the rate function $\lambda(t)$.

$$\hat{c}_{t,j} = \frac{1}{n} \sum_{i=j-n}^{j-1} c_{t,i} \tag{3}$$

3.2 Online Methods

Online forecasting techniques assume that new data is available as time passes, which can be incorporated into the model to improve accuracy. The methods below all assume that for each 15-minute interval, the count of vehicles for the *previous* 15 minute interval is already known in the system.

The first online method is a SARIMA model where each time series consists of $96n$ time points where n again represents the number of days in the past to use. Once again the order and differencing parameters were determined using a brute-force optimization technique. These parameters are the same throughout the 12-hour period being examined here, but the model is retrained for each 15-minute interval with the new data value that is available.

Both of the other methods use a simple average rate of change of past data for each time interval that can then be applied to the new data as it arrives. The first of these methods uses a ratio-based calculation, presented in Equations 4 and 5. This will yield a percentage of vehicles either gained or lost in that time interval for each of the previous n weeks. Take the average of these percentages, and apply them, starting at the last real data value available for earlier that same day to construct a new forecasted count of vehicles on which to base the arrival rate function.

$$r_t = \frac{1}{n} \sum_{i=j-n}^{j-1} \frac{c_{t,i}}{c_{t-1,i}} \tag{4}$$

$$\hat{c}_{t,j} = c_{t-1,j} \cdot r_t \tag{5}$$

The difference-based approach is similar to the ratio-based approach, however, instead of taking the ratio of the counts, differences are used. Equations 6 and 7 show the calculations.

$$d_t = \frac{1}{n} \sum_{i=j-n}^{j-1} [c_{t,i} - c_{t-1,i}] \tag{6}$$

$$\hat{c}_{t,j} = c_{t-1,j} + d_t \tag{7}$$

To increase the flexibility of these models, a weighted average of the new methods with the offline method can be utilized.

3.3 Arrival Process Comparisons

Each of the methods uses some number n of past days' data. Once again using a brute-force optimization procedure, it was determined that all of the models optimized their accuracy when $n = 6$.

The accuracy metric used throughout this work is the symmetric mean absolute percent error (sMAPE) and is defined in the Equation 8.

$$\text{sMAPE} = \frac{200}{n}\sum_{t=1}^{n}\frac{|F_t - A_t|}{(|F_t| + |A_t|)} \tag{8}$$

where A_t is the actual value and F_t is the forecast value. sMAPE is a commonly used accuracy metric that was chosen as one of the accuracy metrics in both the M3 and M4 Competitions (Makridakis et al. 2020) due to the fact that it is "scale-independent and intuitive to understand". With traffic flow changing throughout the day it is important to have a relative error metric. An absolute error metric can be difficult to interpret when the target values change so much over the course of the day.

The performance of the methods was tested for 150 Tuesdays from 2017 through 2019. For each day the arrival forecasts were produced using each of the procedures described above for each fifteen-minute interval from 6:00 am until 6:00 pm. This time period encompasses most of the heavy traffic parts of the day. Each of the forecasts was compared to the actual data and sMAPE values were calculated.

Figure 3 shows a comparison of the accuracy of the arrival techniques in terms of forecasting the actual counts. As can be seen, the offline approaches are inferior to the online methods. Figure 4 compares the various methods once their forecasts have been used to generate arrival times for the simulation using an NHPP. Figures 3 and 4 look very similar but there is a slight accuracy loss through the process of turning the forecasts into arrival times, and then back into vehicle counts.

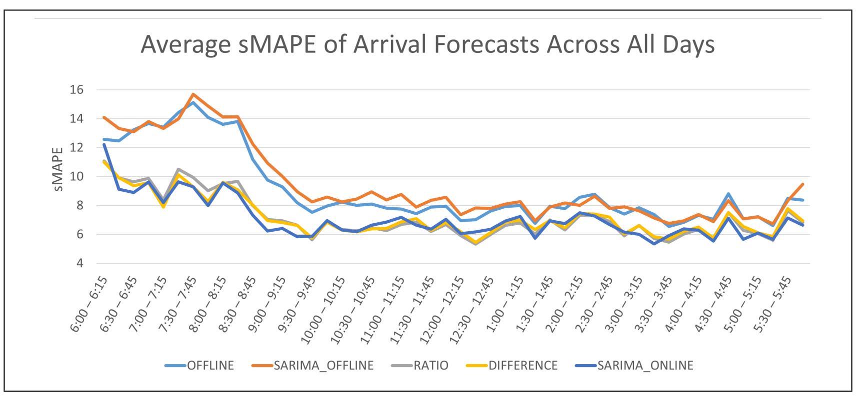

Figure 3: Comparison of Arrival Methods Before Application of NHPPs.

A comparison of the average performance of the different arrival process models is given in Table 2. Across several statistics it can be seen that the online models perform better than the offline models. The worst performer was the offline SARIMA model, while the best performer was the online SARIMA model, but with only a slight improvement over the other two online techniques. With the amount of time it can take to optimize the various parameters of a SARIMA model, and recognizing the desire for time-efficient forecasting systems, the fact that the two proposed online methods are actually quite competitive with the SARIMA model, and execute very quickly due to their simplicity, it is reasonable to investigate these methods further.

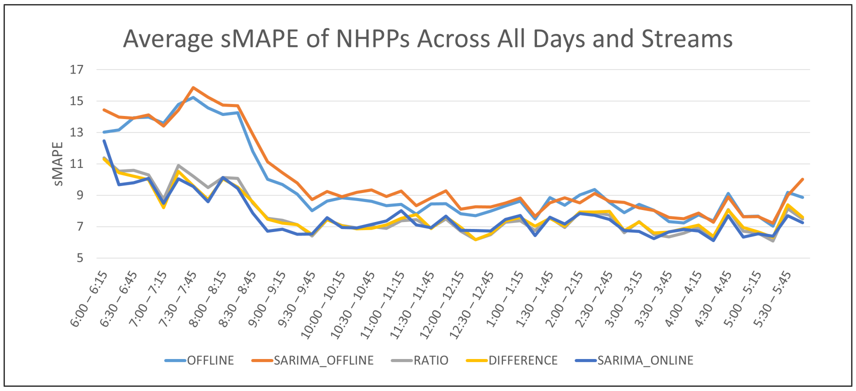

Figure 4: Comparison of Arrival Methods After Application of NHPPs.

Table 2: Arrival Model Comparisons.

sMAPE	Offline	Offline SARIMA	Ratio	Difference	Online SARIMA
Mean	9.6094757	9.952413997	7.75967613	7.75723732	7.62439966
Median	8.62071215	8.895266305	7.373682648	7.400417794	7.213051117
Min	7.025413449	7.220760702	6.087008921	6.165682312	6.112353622
Max	15.23084061	15.85398104	11.38148564	11.29757662	12.46071667

The process of using online data implies a form of ongoing calibration where new data is used to keep the simulation model informed. An event, such as an accident, can greatly change traffic from what drivers are accustomed to, and methods that incorporate the latest data will necessarily have a better chance of capturing these changes and maintaining the ability to produce accurate results in these instances.

4 MODEL CALIBRATION

The essential problem of calibration is to minimize the error between the computed outputs of the simulation and the observed real-world data. The problem can be formulated as

$$\min_{\mathbf{x}} \left[f\left(\mathbf{y}_{obs}, \mathbf{y}_{sim}\right) \right] \tag{9}$$

where $f(\cdot)$ is the loss function. The observed output values $\mathbf{y}_{sim}$ are calculated as

$$\mathbf{y}_{sim} = g(\mathbf{x}) \tag{10}$$

where $\mathbf{x}$ is the vector of parameters being calibrated, and $g(\cdot)$ is, in this case, the simulation itself.

The model parameters that have been chosen for calibration are given in Table 3 along with their domains. These domains have been based partially using previous calibration efforts (Kurtc and Treiber 2016), (Rahman, Ismail, and Ali 2020), and partially on the modeler's own experience.

Table 3: Model Parameters and Domains.

Definition	Symbol	Domains	Units	Definition	Symbol	Domains	Units
Acceleration	a	$[0.5, 10.0]$	m/sec^2	Time Headway	T	$[1.0, 5.0]$	sec
Deceleration	b	$[-10.0, -0.5]$	m/sec^2	Dist. Headway	s	$[3.0, 12.0]$	m
Reaction Time	τ	$[1.0, 3.0]$	sec	IDM Parameter	δ	$[3.0, 8.0]$	–

It should be noted that parameter δ is a tuning parameter which, according to the authors, is used to control how the acceleration decreases as the vehicle approaches the desired velocity v_0. Therefore it has no units on its own and is used simply to aid in calibrating the model to a specific data set.

The domains are important to the process of calibration because the optimization algorithms must stay within them when searching for the optimum. In order to create random values for the initial solution pool, the GA is given a uniform random variable based on the feasible domain for each parameter. This ensures that every solution makes sense in the real world. NM and SPSA must also stay within the feasible domains of the parameters, and this is achieved using penalties in the objective function if the algorithms veer outside the acceptable region.

GAs have one advantage over the other two types of search methods which is that the creation of random solutions and random mutations will more robustly cover the surface. The NM and SPSA approaches have a higher chance of finding local optima rather than global optima in a very bumpy surface. Both NM and SPSA require a starting point $\mathbf{x}_0$, which in a very noisy surface can have a great effect on the optimization results. The two algorithms were tested on four starting points, which are shown in Table 4. These starting points differ only in the acceleration and deceleration parameters, where these values are chosen to divide their respective domains into thirds. Parameter δ is usually set to 4.0 according to the original authors. Reaction time was chosen as a small value just inside the bottom boundary of its domain. Time headway and distance headway were both set at values halfway through their respective domains.

Table 4: Initial Starting Points for Calibration.

Point	a	b	τ	T	s	δ
$\mathbf{x}_1$	7.0	-3.5	1.5	3.0	7.5	4.0
$\mathbf{x}_2$	3.5	-7.0	1.5	3.0	7.5	4.0
$\mathbf{x}_3$	7.0	-7.0	1.5	3.0	7.5	4.0
$\mathbf{x}_4$	3.5	-3.5	1.5	3.0	7.5	4.0

To test the three different calibration approaches data from four Tuesdays in 2018 was used to generate arrivals for the simulation using the Offline, Ratio, Difference, and Online SARIMA arrival models defined in Section 3. The Offline SARIMA model was left out due to its having the worst performance for modeling simulation arrivals. The simulations were then calibrated over each fifteen-minute period from 6:00 am until 6:00 pm on the four chosen Tuesdays with each of the chosen algorithms, with NM and SPSA also being calibrated across the four given starting points. Fifteen-minute intervals were chosen both to avoid long simulation execution times, and because intervals that are too long will not have the flexibility to adapt to changes in traffic patterns. Each simulation execution counts the number of vehicles that pass through the sensor locations and then compares those counts to the actual data for that day using the sMAPE calculation in Equation 8. The calibrated parameters, full list of optimal values over epochs, and execution times were saved for each calibration run.

It turns out that these four starting points yield the same results in the uncalibrated model, which suggests that acceleration and deceleration are not actually the most consequential parameters for this case study. Average results of calibrating the model can be seen in Table 5 and show that in general the best results came from the GA. Note that the GA does not depend on an initial starting point so the differences in this table are the random number streams used in the evolutionary process to create new candidate solutions.

The model was also compared across the different types of arrival models. These results are presented in Table 6. It is again clear that the GA produced better calibration results than NM and SPSA. It is also clear that the online arrival modeling techniques are an improvement over the offline arrival model, though it that is not as clear for the SARIMA process. It is unclear why the online SARIMA arrival method has performed more poorly under calibration than the other online methods, and more investigation is warranted.

Table 5: Average Calibration Results Across Initial Points.

Point	NM	GA	SPSA	Uncalibrated
$\mathbf{x}_1$	7.10192	6.54251	7.11636	9.61395
$\mathbf{x}_2$	6.85883	6.44766	7.12737	9.61395
$\mathbf{x}_3$	6.57336	6.52649	6.93121	9.61395
$\mathbf{x}_4$	6.77225	6.45791	7.0992	9.61395

Table 6: Average Calibration Results Across Arrival Models.

Method	Offline	Ratio	Difference	SARIMA
NM	7.30434	6.54926	6.54413	7.14751
GA	6.78898	6.13006	6.30251	6.75303
SPSA	7.32823	6.72383	6.75111	7.47098

Figure 5 shows the average calibration results in terms of sMAPE values for the three methods throughout the 12-hour period across all other characteristics.

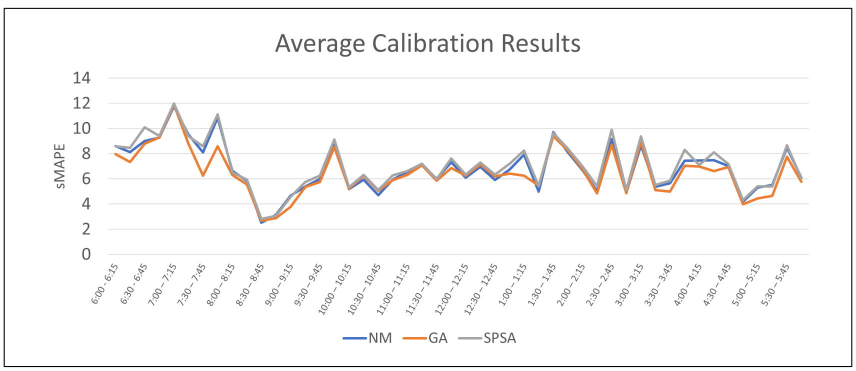

Figure 5: Results of Calibration Across Times and Starting Points.

In terms of calibration accuracy it appears that the GA has outperformed the other two methods, NM and SPSA. However, the picture is very different when comparing the efficiency of the approaches, as can be seen in both Figures 6 and 7, which clearly show that the GA takes much longer to reach optima than do the other methods. Both the means and medians were displayed because a few outliers in the many calibration executions that were performed have skewed some of the results.

It is not surprising that SPSA performs more efficiently than the other methods as it only requires two function executions in each iteration of the algorithm. Combining the two perspectives reveals that neither the Nelder-Mead, nor SPSA should be overlooked for the task of calibrating traffic models. Indeed, for an online simulation system, where recalibrations of the model will have to be done from time to time, a time-efficient algorithm that can still produce ”pretty good” results might be the method best suited for the purpose.

The calibration methods were also compared by analyzing the improvement obtained by each process over epochs of their respective algorithms. The average results can be seen in Figure 8, and show that the GA had the most dramatic improvement of the three methods. SPSA does not actually improve much over the course of the optimization procedures. It is interesting that on average the Nelder-Mead algorithm shows faster improvement than the other two methods, which could suggest it as a good technique when the number of iterations of the calibration needs to be kept very low.

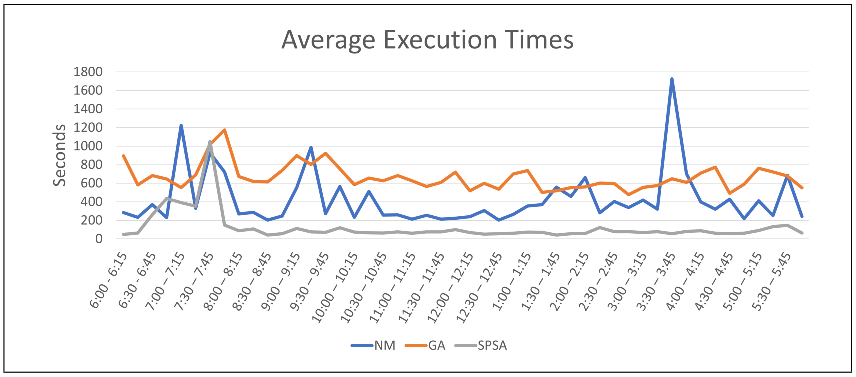

Figure 6: Efficiency Comparison Using Means.

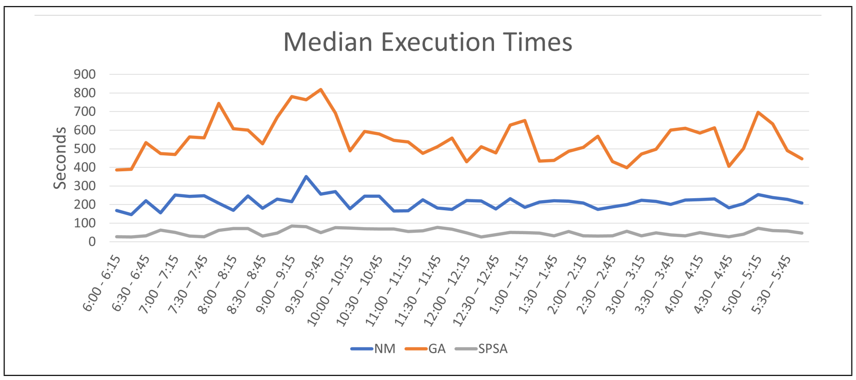

Figure 7: Efficiency Comparison Using Medians.

5 CONCLUSIONS AND FUTURE WORK

Several different techniques for generating simulation arrival times were compared including three methods which utilize fresh data to create an online arrival model. Two of these methods are quite simple but show comparable results to a more complex and time intensive SARIMA model, which in an online system might require more time than is available.

Calibrations of the system were also conducted on the data using several different optimization techniques, as well as the various arrival models. These calibration techniques were compared through three different perspectives, calibration efficiency, optimization improvement, and overall optimization execution time.

There are calibration approaches that were not used in this study, but they do bear investigation, such as using Tabu Search for optimization, and using metamodels to represent the search space. If good metamodels can be produced then gradient-based techniques for optimization are much more suitable.

The traffic network represented here is a short stretch of a limited-access highway, and thus it was determined that lane changing was not essential to this project. However, for longer networks, and other forms of analysis, integrating lane-changing behavior into the simulation would be desirable.

Currently, the system designed in this project uses only traffic count data to produce the simulation model. It is very likely that using the speed data available from PeMS in some manner would increase the accuracy of the models. Beyond being used as data to validate the model outputs, the speed data can also be used to more accurately drive the car-following model.

It was shown that incorporating online data into the arrival model greatly improved the accuracy of the random generation of arrivals to the system. This type of process leans in the direction of data assimilation, though is not quite at that point yet. Data assimilation would lead to more accurate models, but would

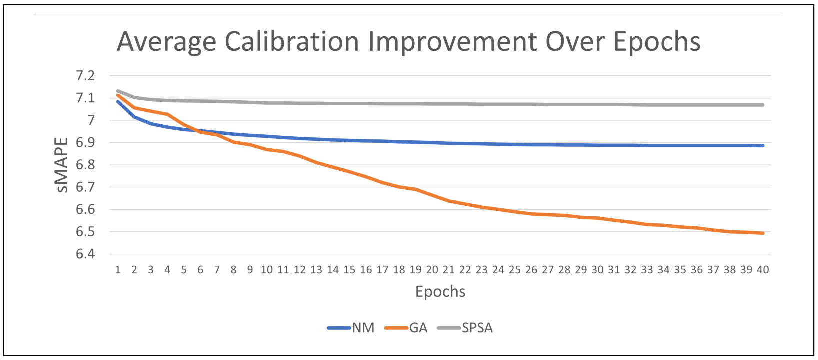

Figure 8: Change in Objective Function Value by Epoch.

likely require occasional recalibration of the system. This kind of continuous calibration of the traffic system is also an interesting topic for future work.

Finally, real traffic is composed of not just individual vehicles, but individual *drivers*, each of which has varying skill, knowledge, and temperament. An agent-based simulation approach could generate more areas of study and allow for a deeper form of analysis. With agent-based modeling each vehicle could be more unique in its set of movement-based parameters, allowing for the study of interactions between different types of drivers.

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