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TOWARDS DECENTRALIZED DECISIONS FOR MANAGING PRODUCT TRANSITIONS IN SEMICONDUCTOR MANUFACTURING

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ABSTRACT

Continuous renewal of the product portfolio through product transitions is crucial to semiconductor manufacturing firms. These decisions take place in a decentralized environment, where decisions by different functional units must be coordinated to optimize corporate performance. Starting from a centralized optimization model, we obtain decentralized models using Lagrangian relaxation, and explore the challenges encountered in formulating and solving these decentralized models. Although the Lagrangian approaches yield tight upper bounds on the optimal solution value, the structure of the dual solution renders the construction of a near-optimal feasible solution difficult, and fully separable decentralized models encounter significant problems in achieving convergence due to scaling issues. We present computational experiments that illustrate the difficulties involved, and discuss directions for future work.

1 INTRODUCTION

Semiconductor firms must continuously introduce new products and retire older ones in a process known as *product transitions* or *product rollovers*. These firms are generally organized into Product Divisions (PD), each of which manages the product portfolio for a market segment. Each PD must forecast their market's demand so that the firm's Manufacturing organization (MFG) can produce the units to meet it, with revenues credited to the PD. Each PD receives an operating budget from Corporate Management (CORP), from which it compensates MFG for the production of its products for sale in the market. A product can only be manufactured for sale if its development has been completed by the firm's Product Engineering Group (PEG). The PD develops specifications for new products and a target date for its introduction into the market, by which its development must be completed, paying PEG for development work out of its

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operating budget. PEG then develops a design that is later transformed into a saleable product ready to be manufactured for sale.

A centralized approach is impractical due to the firm's decentralized structure in which each unit makes its decisions based on its own resource availability and objective. The resources available to each agent depend on other agents' decisions; a product produced for sale will compete for MFG capacity with products sold by other PDs and with other PDs' products in development. Efficient management of product transitions thus requires coordination between the different units of the firm. Due to the presence of uncertainty in the production, development and demand processes, any plan agreed to between agents is constantly evolving over time horizon. However, in this initial work we focus on deterministic problems to develop insight and solution techniques.

An initial centralized model (Leca et al. 2021) captured the complex interactions between different units of the firm pursuing different objectives but mutually dependent due to their dependence on shared resources. This paper presents a family of decentralized models derived from this initial centralized model. We first present a decentralized model based on Lagrangian relaxation, solved using a pure subgradient algorithm and, later, a deflected subgradient method. These solution approaches allow a complete separation of the agents' subproblems, allowing a completely decentralized solution using the Lagrange multipliers in a price-driven coordination scheme. However, the convergence of these methods was inconsistent. These convergence issues were solved using an Augmented Lagrangian approach, which yielded tight upper bounds on the optimal value of the centralized problem. However, the Augmented Lagrangian model is no longer completely separable, and the interpretation of the Lagrange Multipliers as transfer prices is more difficult. Computational experiments find that the Augmented Lagrangian approach, despite producing tight upper bounds, produces essentially uncoordinated, primal infeasible solutions from which it is hard to obtain near-optimal feasible solutions.

2 LITERATURE REVIEW

Several authors have addressed the problem of managing product transitions from different perspectives. Billington et al. (1998) assess two primary product transition strategies: a *solo-product rollover* where a new product is introduced only after its predecessor is taken out of the market, and a *dual-rollover* strategy where the new product shares the market with the older one. The *solo-product rollover* represents a riskier strategy since any disruption in the development of the new product will leave the firm with no product in the market. *Dual-rollover* strategies, on the other hand, require efficient coordination of the manufacturing, distribution, and marketing of both products. Erhun et al. (2007) emphasize the importance of assessing the product driver and risk factors to determine the right rollover strategy, and suggest determining a transition strategy over several generations.

A key element of a successful transition strategy is accurately predicting the new product's demand. The most common approach to modelling the demand for a new product is diffusion models Bass (1969). which use differential equation to simulate the market traction of the product based on the rate of its adoption by innovators and imitators. Norton and Bass (1987) extended the original Bass model to multiple product generations. Robinson and Lakhani (1975) incorporate prices into the diffusion model such that product prices affect the adoption rate in any period, but do not affect the potential demand. Dockner and Jorgensen (1988) introduce competition into the Bass model. Padmanabhan and Bass (1993) extend the Norton and Bass (1987) model by introducing prices, deriving the optimal pricing policy of a monopolistic firm marketing successive product generations. In this paper we use the model of Bass (1969) to generate demand scenarios in our computational experiments. However, we do not consider the effects of the pricing strategy on the transition rates.

Product transitions often lead to increased uncertainty in the firm's supply chain due to the possibility of severe demand and supply mismatches, resulting in capacity shortages. From the production side, the increased frequency of adverse events induced by the new product can negatively affect the delivery of both the newly developed products and others that share capacity with it (Manda and Uzsoy 2020).

Therefore, the firm must decide how much production capacity to allocate to each product at different points in its life cycle, specifically when to initiate production and when to terminate it. Carrillo and Franza (2006) study the relation between time-to-market and ramp-up time decisions. Shen et al. (2014) consider how a capacity-constrained firm prices products during new product introductions using a control-theory framework to model integrated optimal pricing, production and inventory decisions. Schwarz and Tan (2021) explored how limited production capacity affects the optimal unconstrained decision modifying the rollover strategy. They examined the decision between single and dual rollover strategies proposing two levels of recourse in case of capacity shortage: 1) increasing the prices, 2) changing the sales and/or production rollover strategies and preproduction and adjusting the prices accordingly.

The papers discussed above treat the product transition using a centralized framework. However, the information and operational capabilities required for its execution lie within different units of the firm such as PDs, MFG and PEG and are not easily available outside the unit. Hence the agents (units) make decisions individually, potentially emphasizing their local objectives over corporate profit and raising the question of how to best coordinate independent agents to best serve corporate objectives. Karabuk and Wu (2002) developed a coordination scheme for a semiconductor firm involving two different agents, marketing and manufacturing, where the firm's objective is the sum of the agents' objectives. They propose a decentralized coordination scheme based on transfer prices that are updated iteratively until the decisions made by the different agents are consistent. Karabuk and Wu (2005) design an incentive scheme using bonus payments and participation charges that elicits private demand information from the agents without requiring external transfers to reach equilibrium. The mechanism also guarantees voluntary participation by the agents. Kutanoglu and Wu (2006) present a similar approach for a production scheduling problem that arises when schedulers must coordinate their schedules with internal or external customers. They design a schedule selection auction where all participating agents state their preferences via a valuation scheme, and the mechanism selects a final schedule based on the collective input. They show that this scheme is a direct revelation mechanism that implements the optimal schedule selection under agents' dominant strategies.

These papers treat one agent as an auctioneer who tries to coordinate operations by allocating the resources among the other agents, the bidders, who reveal their valuation of the resource bundle that is being offered. In our problem agents can act as buyers of some resources and sellers of others. Bansal et al. (2020) approach the problem of effective coordination between manufacturing and product development activities, with MFG acting as auctioneer. Since the product development teams request production capacity from MFG for capacity allocation, MFG is not a simple auctioneer, since it also requires the use of some of the resource that is offering. The product development groups, in turn, offer MFG dates by which new products will complete development and be ready for production. They propose an iterative combinatorial auction that seeks to maximize the firm's profit while motivating all units to share information truthfully. In a subsequent paper (Bansal et al. 2022) they consider a single PD as auctioneer coordinating the activities of MFG and several PDGs, proposing a decentralized approach based on subadditive duality.

Most of the studies cited above approach the problem of managing product transitions from a strategic perspective, using simplified models that do not consider the complex technological and resource constraints affecting the production and development processes. Our model expands the number of agent types by including the budget allocation decisions that allow CORP to subsidize a PD that is temporarily unprofitable. We also allow more complex relationships among the agents, instead of treating one agent as an auctioneer coordinating resource allocation among other agents. The inclusion of separate PDs for different market segments, each of which must interact with MFG, CORP and PEG to fulfill their demand, represents a significant extension of previous work.

3 CENTRALIZED MODEL

We first review the centralized model which allows us to describe the constraints and decision variables involved in the different agents' decisions, and provides a mathematical point of departure from which alternative decentralized models can be derived. The centralized formulation, whose notation is defined in Appendix A, seeks to maximize corporate profit shown in (1) subject to constraints describing the capabilities of each agent (CORP, PD, MFG or PE) and can be stated as follows:

$$\textbf{maximize} \ : \ \left\{ \sum_{\forall p\in\mathbb{P}}\sum_{\forall i\in\mathbb{I}}\sum_{\forall t\in\mathbb{T}} \rho_{pit}x_{pit} \ - \ h^{mtl}_{pit}I_{pit} \ - \ h^{tr}_{pit}w_{pit} \ - \ c^{M}_{pit}m_{pit} \ - \ c^{D}_{pit}(z^{tr}_{pit} \ + \ z^{mtl}_{pit} \ + \ z^{dbg}_{pit}) \right\} \quad (1)$$

subject to

Corporate Constraints:

$$B^{corp}_{t} = B^{corp}_{t-1} - \sum_{\forall i\in\mathbb{I}} OC_{it} + \sum_{\forall i\in\mathbb{I}}\sum_{\forall p\in\mathbb{P}} \rho_{pit}x_{pit} \qquad \forall t\in\mathbb{T} \qquad (2)$$

$$B^{corp}_{t} \geq 0 \quad ; \quad 0 \leq x_{pit} \leq d_{pit} \qquad \forall i\in\mathbb{I};\forall p\in\mathbb{P};\forall t\in\mathbb{T} \qquad (3)$$

CORP assigns an operating budget to each PD at the start of each planning period, and receives the revenue they generate in the market place at its end. This allows CORP to subsidize temporarily unprofitable PDs to obtain increased revenues in the future. Constraints (2) ensure that the firm's expenditures do not exceed its revenues, capturing CORP's principal role of allocating resources among the different units. Constraints (3) guarantee that the corporation budget must remain positive at all periods.

Product Division Constraints:

$$B^{div}_{it} = OC_{it} - [\sum_{\forall p\in\mathbb{P}} c^{M}_{pit}m_{pit} + c^{D}_{pit}(z^{tr}_{pit} + z^{mtl}_{pit} + z^{dbg}_{pit})] \qquad \forall i\in\mathbb{I};\forall t\in\mathbb{T} \qquad (4)$$

$$I_{pit} = I_{p,i,t-1} + m_{pit} - x_{pit} \qquad \forall t\in\mathbb{T} \qquad (5)$$

$$B^{div}_{it} \geq \quad ; \quad I_{pit} \geq 0 \qquad \forall i\in\mathbb{I};\forall p\in\mathbb{P};\forall t\in\mathbb{T} \qquad (6)$$

At the beginning of each period each PD receives its operating budget from CORP. From this budget it must pay MFG for its products produced for sale, and PEG for any development work requested per the constraints (4). Each PD is responsible for its products' finished goods inventory holding costs, since it provides demand forecasts to MFG. Constraints (5) define each PD's finished goods inventory. Constraints (6) prohibits a deficitary budget in any period for all divisions.

Manufacturing Constraints:

$$\sum_{\forall p\in\mathbb{P}}\sum_{\forall i\in\mathbb{I}} \tau^{tr}_{pit}w_{pit} + \sum_{\forall p\in\mathbb{P}}\sum_{\forall i\in\mathbb{I}} n^{tr}_{pi}\omega^{tr}_{pit}z^{tr}_{pit} \leq C^{tr}_{t} \qquad \forall t\in\mathbb{T} \qquad (7)$$

$$\sum_{\forall p\in\mathbb{P}}\sum_{\forall i\in\mathbb{I}} \tau^{mtl}_{pit}m_{pit} + \sum_{\forall p\in\mathbb{P}}\sum_{\forall i\in\mathbb{I}} n^{mtl}_{pi}\omega^{mtl}_{pit}z^{mtl}_{pit} \leq C^{mtl}_{t} \qquad \forall t\in\mathbb{T} \qquad (8)$$

$$m_{pit} \leq I^{mtl}_{pit} \qquad \forall i\in\mathbb{I};\forall p\in\mathbb{P};\forall t\in\mathbb{T} \qquad (9)$$

$$I^{mtl}_{p,i,t-1} + w_{pit} - m_{pit} = I^{mtl}_{pit} \qquad \forall i\in\mathbb{I};\forall p\in\mathbb{P};\forall t\in\mathbb{T} \qquad (10)$$

$$M[\sum_{t_p=0}^{t_p=t} R_{pit_p}] \geq w_{pit} \qquad \forall i\in\mathbb{I};\forall p\in\mathbb{P};\forall t\in\mathbb{T} \qquad (11)$$

At the beginning of each period MFG receives payment for production activities, which is used to pay for their own operations. The production process for high-volume units comprises two stages, transistor fabrication and metal fabrication, each requiring one planning period to complete. The primary constraints for MFG are capacity constraints for each processing stage (7) and (8), consistency of shipments with available material (9), and material balance constraints across planning periods defined by (10). The constraints (11), ensure that MFG can only produce a product for sale if it has completed development.

Product Engineering Constraints:

$$\sum_{\forall p\in\mathbb{P}}\sum_{\forall i\in\mathbb{I}} \varepsilon^{tr}_{pit} z^{tr}_{pit} + \sum_{\forall p\in\mathbb{P}}\sum_{\forall i\in\mathbb{I}} \varepsilon^{mtl}_{pit} z^{mtl}_{pit} + \sum_{\forall p\in\mathbb{P}}\sum_{\forall i\in\mathbb{I}} \varepsilon^{dbg}_{pit} z^{dbg}_{pit} \leq E_t \qquad \forall t \in \mathbb{T} \tag{12}$$

$$\sum_{t_p=0}^{t_p=t} z^{dbg}_{pit_p} \geq r_{pi} R_{pit} \qquad \forall i \in \mathbb{I}; \forall p \in \mathbb{P}; \forall t \in \mathbb{T} \tag{13}$$

$$z^{tr}_{pit} + z^{mtl}_{pit} + z^{dbg}_{pit} \leq 1 \qquad \forall i \in \mathbb{I}; \forall p \in \mathbb{P}; \forall t \in \mathbb{T} \tag{14}$$

$$z^{tr}_{pit} + z^{tr}_{p,i,t+1} + z^{mtl}_{p,i,t+1} + z^{mtl}_{p,i,t+2} + z^{dbg}_{p,i,t+2} + z^{dbg}_{p,i,t+3} = 3 \qquad \forall i \in \mathbb{I}; \forall p \in \mathbb{P}; \forall t \in \mathbb{T} \tag{15}$$

$$z^{mtl}_{p,i,t+1} + z^{mtl}_{p,i,t+2} \geq z^{tr}_{pit} \qquad \forall i \in \mathbb{I}; \forall p \in \mathbb{P}; \forall t \in \mathbb{T} \tag{16}$$

$$z^{dbg}_{p,i,t+1} + z^{dbg}_{p,i,t+2} \geq z^{mtl}_{pit} \qquad \forall i \in \mathbb{I}; \forall p \in \mathbb{P}; \forall t \in \mathbb{T} \tag{17}$$

$$\sum_{t_p=0}^{t_p=t} z^{tr}_{pit_p} \geq z^{mtl}_{pit} \qquad \forall i \in \mathbb{I}; \forall p \in \mathbb{P}; \forall t \in \mathbb{T} \tag{18}$$

$$\sum_{t_p=0}^{t_p=t} z^{mtl}_{pit_p} \geq z^{dbg}_{pit} \qquad \forall i \in \mathbb{I}; \forall p \in \mathbb{P}; \forall t \in \mathbb{T} \tag{19}$$

$$\sum_{t_p=0}^{t_p=t} R_{p-1,i,t_p} \geq z^{tr}_{pit} \qquad \forall i \in \mathbb{I}; \forall p \in \mathbb{P}; \forall t \in \mathbb{T} \tag{20}$$

At the beginning of each period PEG receives payment from each PD for the development work on its products in the current period. The development process for a semiconductor comprises three design-test-fabricate cycles, each involving three different steps. The first step is transistor design and fabrication, which requires capacity from both MFG and PEG for one planning period. The second step, metal design and fabrication, also consumes MFG and PEG capacity for one planning period. The development cycle ends with a debugging process requiring only PEG resources for one planning period. Constraints (13) ensure that a new product can only be produced for sale after r_{pi} cycles, while (12) represents resource capacity. (14) ensures that PEG can only work on one type of development stage per product during a given period. Constraints (15) forces the PEG to keep working on the product development project on the next consecutive cycles, allowing only one period break between the transistors and the metal process, or between the metal and the debugging process. Constraints (16) - (19) define precedence between development stages of each product, while (20) allows PEG to develop a product only if the previous generation has been developed (although it may not have been introduced into the market).

4 DECENTRALIZED MODELS

The centralized formulation has two key elements: the objective function is completely separable between agents. However, some variables appear in constraints related to different agents. For example, OC_{it} appears in constraints 2 which is associated with CORP, and also in 4 which involves the PDs. Our first step towards decentralization was to identify all variables shared by more than one agent and create a copy of each variable for each of the agents involved, adding constraints requiring their values to be consistent. For example, the variable OC_{it} in the centralized model is replaced by OC^{Corp}_{it} in constraint set 2, and by OC^{Div}_{it} in 4, together with a linking constraint $OC^{Div}_{it} \leq OC^{Corp}_{it} \quad \forall i \in \mathbb{I}; \forall t \in \mathbb{T}$ ensuring that PD i cannot exceed the operating budget allotted by CORP. The same procedure was performed on the other eight

Table 1: Variable separation, linking constraints and Lagrange Multipliers.

Original Variables	**New Variables**	**Linking Constraints**	**Lagrange Multipliers**
OC_{it}	OC_{it}^{Corp} ; OC_{it}^{Div}	$OC_{it}^{Div} \leq OC_{it}^{Corp}$	α_{it}
x_{pit}	x_{pit}^{Corp} ; x_{pit}^{Div}	$x_{pit}^{Corp} \leq x_{pit}^{Div}$	β_{pit}
m_{pit}	m_{pit}^{Mfg} ; m_{pit}^{Div}	$m_{pit}^{Div} \leq m_{pit}^{Mfg}$	γ_{pit}
z_{pit}^{tr}	$z_{pit}^{tr}(Div)$; $z_{pit}^{tr}(Eng)$	$z_{pit}^{tr}(Div) \leq z_{pit}^{tr}(Eng)$	θ_{pit}
z_{pit}^{tr}	$z_{pit}^{tr}(Mfg)$; $z_{pit}^{tr}(Eng)$	$z_{pit}^{tr}(Eng) \leq z_{pit}^{tr}(Mfg)$	η_{pit}
z_{pit}^{mtl}	$z_{pit}^{mtl}(Div)$; $z_{pit}^{mtl}(Eng)$	$z_{pit}^{mtl}(Div) \leq z_{pit}^{mtl}(Eng)$	λ_{pi}
z_{pit}^{mtl}	$z_{pit}^{mtl}(Mfg)$; $z_{pit}^{mtl}(Eng)$	$z_{pit}^{mtl}(Eng) \leq z_{pit}^{mtl}(Mfg)$	σ_{pit}
z_{pit}^{dbg}	$z_{pit}^{dbg}(Div)$; $z_{pit}^{dbg}(Eng)$	$z_{pit}^{dbg}(Div) \leq z_{pit}^{dbg}(Eng)$	δ_{pit}
R_{pit}	R_{pit}^{Eng} ; R_{pit}^{Mfg}	$R_{pit}^{Mfg} \leq R_{pit}^{Eng}$	ψ_{pit}

variables shared by two or more agents as shown in Table 1, rendering each agent's problem completely independent of all others once the linking constraints are relaxed. The duplication of variables and the corresponding linking constraints are summarized in Table 1. This variable duplication approach is known as Lagrangian Decomposition (Guignard and Kim 1987).

Relaxing the linking constraints with associated Lagrange multipliers yields the Lagrangian function

$$
\begin{aligned}
max: \Bigg\{ & \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \rho_{pit} x_{pit}^{Corp} - h_{pit}^{mtl} I_{pit} - h_{pit}^{tr} w_{pit} - c_{pit}^{M} m_{pit}^{Mfg} \\
& - c_{pit}^{D}(z_{pit}^{tr}(Mfg) + z_{pit}^{mtl}(Mfg) + z_{pit}^{dbg}(Eng)) - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \alpha_{it}(OC_{it}^{Div} - OC_{it}^{Corp}) \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \beta_{pit}(x_{pit}^{Corp} - x_{pit}^{Div}) - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \gamma_{pit}(m_{pit}^{Div} - m_{pit}^{Mfg}) \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \theta_{pit}(z_{pit}^{tr}(Div) - z_{pit}^{tr}(Eng)) - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \eta_{pit}(z_{pit}^{tr}(Eng) - z_{pit}^{tr}(Mfg)) \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \lambda_{pit}(z_{pit}^{mtl}(Div) - z_{pit}^{mtl}(Eng)) - \sigma_{pit}(z_{pit}^{mtl}(Eng) - z_{pit}^{mtl}(Mfg)) \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \delta_{pit}(z_{pit}^{dbg}(Div) - z_{pit}^{dbg}(Eng)) - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \psi_{pit}(R_{pit}^{Mfct} - R_{pit}^{Eng}) \Bigg\}
\end{aligned} \quad (21)
$$

The first five terms of (21) inside the triple summation represent the total revenue, minus the finished goods inventory cost, the inventory cost of intermediate inventory held between the transistor and metal stages, the manufacturing cost, and the cost of development. The remaining elements of the objective function are the Lagrange multipliers penalizing violations of the linking constraints.

We attempted to solve this formulation using a pure subgradient algorithm and a deflected subgradient method (Guta 2003) that reduces the zigzagging of the objective value from iteration to iteration, without achieving satisfactory convergence. These observations revealed the significant scaling problem embedded in this model. The Lagrange multipliers are updated using the subgradients. However, the magnitude of the subgradient element linking two continuous variables can be of the order of the thousands or the millions. Hence the Lagrange multipliers related to the continuous variables will be adjusted relatively rapidly across iterations. However, the magnitude of the subgradient elements relating to two binary variables is at most one, causing the associated Lagrange multipliers to be updated slowly, requiring many iterations to reach values high enough to prevent violation of the linking constraints.

We implemented an Augmented Lagrangian approach to improve convergence using a decentralized procedure. For this, we augmented 21 with the sum of the squared subgradients multiplied by a step size μ that starts with a value between 1 and 2 and increases by 10% on each iteration as proposed by Nocedal and Wright (2006). The resulting objective function is shown in (22). Even though consistency between the binary variables was not easily achieved due to scaling challenges, this solution approach yielded duality gaps of between 0.6% and 17%. While in this solution approach all the constraints are directly related to one type of agent, the objective function is no longer separable due to the quadratic subgradient terms. Nevertheless, this problem can be solved by developing equivalent objective functions through the Auxiliary Problem principle (Cohen 1980).

$$
\begin{aligned}
maximize: \Bigg\{ & \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \rho_{pit} x_{pit}^{Corp} - h_{pit}^{mtl} I_{pit} - h_{pit}^{tr} w_{pit} - c_{pit}^{M} m_{pit}^{Mfg} \\
& - c_{pit}^{D} (z_{pit}^{tr}(Mfg) + z_{pit}^{mtl}(Mfg) + z_{pit}^{dbg}(Eng)) - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \alpha_{it} (OC_{it}^{Div} - OC_{it}^{Corp}) \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \beta_{pit} (x_{pit}^{Corp} - x_{pit}^{Div}) - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \gamma_{pit} (m_{pit}^{Div} - m_{pit}^{Mfg}) \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \theta_{pit} (z_{pit}^{tr}(Div) - z_{pit}^{tr}(Eng)) - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \eta_{pit} (z_{pit}^{tr}(Eng) - z_{pit}^{tr}(Mfg)) \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \lambda_{pit} (z_{pit}^{mtl}(Div) - z_{pit}^{mtl}(Eng)) - \sigma_{pit} (z_{pit}^{mtl}(Eng) - z_{pit}^{mtl}(Mfg)) \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \delta_{pit} (z_{pit}^{dbg} - z^{Divdbg}_{pit}) - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \psi_{pit} (R_{pit} - R_{pit}^{Eng}) \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \mu (OC_{it}^{Div} - OC_{it}^{Corp})^2 \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \mu (x_{pit}^{Corp} - x_{pit}^{Div})^2 - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \mu (m_{pit}^{Div} - m_{pit}^{Mfg})^2 \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \mu (z_{pit}^{tr}(Div) - z_{pit}^{tr}(Eng))^2 - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \mu (z_{pit}^{tr}(Eng) - z_{pit}^{tr}(Mfg))^2 \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \mu (z_{pit}^{mtl}(Div) - z_{pit}^{mtl}(Eng))^2 - \mu (z_{pit}^{mtl}(Eng) - z_{pit}^{mtl}(Mfg))^2 \\
& - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \mu (z_{pit}^{dbg}(Div) - z_{pit}^{dbg}(Eng))^2 - \sum_{\forall p \in \mathbb{P}} \sum_{\forall i \in \mathbb{I}} \sum_{\forall t \in \mathbb{T}} \mu (R_{pit}^{Mfct} - R_{pit}^{Eng})^2 \Bigg\}
\end{aligned} \tag{22}
$$

5 EXPERIMENTAL RESULTS

We compare the results of a set of experiments using three solution approaches; the centralized model, the dual-feasible solution obtained by the augmented Lagrangian approach, and a primal feasible solution constructed from that of the augmented Lagrangian. We created two demand patterns for a firm with three divisions, each launching three generations, over 63 periods. Under the first demand pattern the life cycle events of the products (introduction and retirement from the market) of the same generation happen simultaneously; this is the synchronous demand pattern. In the second demand pattern, one of the product life cycle events was designed to occur between the introduction and retirement of the other products of the same generation, giving an asynchronous demand pattern. For each demand class, we randomly generate five independent realizations and solve the problem by varying the manufacturing capacity, the product engineering capacity, and the product development complexity, as shown in Table 2, for a total of

16 instance configurations and 80 total instances. For the Augmented Lagrangian procedures we allow a maximum of 100 iterations and 400 seconds per iteration. The results are summarized in Table 1.

Table 2: Experimental design.

Factors	**Levels**
Solution	Centralized; Augmented Lagrangian
Product Life Cycle Timing	Synchronous ; Asynchronous
Manufacturing Capacity	80% ; 100%
Product Development Teams	2; 6
Product Development Complexity	[3 cycles; 4 cycles]

Our first observation is that the Augmented Lagrangian procedure consistently obtains dual-feasible solutions whose objective function value is close to those of the Centralized model, with an average gap of 6.01%, a maximum gap of 16.67%, and a minimum gap of 0.29%. The solutions' gaps are mainly explained by the violations of the linking constraints involving the binary variables in the augmented Lagrangian. However, it is here that the consequences of the scaling issues are seen. As seen in Table 1, the decentralized approach produces solutions where no development takes place, and MFG produces as much as it can given its capacity constraints. This is clearly unrealistic, and results from the violation of the linking constraints involving binary variables. Unless the values of the Lagrange multipliers associated with these constraints are extremely high, they can be violated with little impact on the optimal value of the Lagrangian function. For the case of the Lagrange multipliers relating the binary variables that dictate if a product is ready to be manufactured in high volumes because the whole development process finishes (ψ_{pit}), the values should go higher. We found consistent solution structures when the initial value of ψ_{pit} was around one-third of the total cost of the development project. Thus the task of obtaining a feasible solution from that generated by the Augmented Lagrangian, a common approach known as a Lagrangian heuristic, appears to be difficult for this problem.

The computation times for the centralized and decentralized models were roughly comparable, with an average of 216 seconds for the former and 168 seconds for the latter. Instances with asynchronous rollover and plentiful capacity could be solved faster on average.

6 CONCLUSION

This paper discusses our attempts to develop decentralized solution approaches from a previously developed centralized model (Leca, Kempf, and Uzsoy 2021). Our Augmented Lagrangian procedure always yields an upper bound on the value of the centralized solution with a gap ranging from 0.29% to 16.67%. Nevertheless, the structure of these solutions renders it difficult to construct a near-optimal feasible solution using these solutions. This is because the Augmented Lagrangian solutions consistently violate the constraints coordinating development and manufacturing activities, precluding any simple procedures that allow decisions from one agent to dictate the actions of others.

In order to find appropriate solution structures, the initial values of the Lagrange multipliers must be set accordingly to the constraint they are associated with. In this regard, the penalty for violating the linking constraints between the binaries that define if a product is ready for high-volume production becomes critical. When the solution structure resulting from the augmented Lagrangian does not show consistency between MFG and PEG, there is a conflict of interest within the agents. In this case, finding a feasible solution by only taking into account one agent's decisions may result in very poor solutions. Our future work will be directed towards addressing the scaling issues leading to these difficulties, and using this solution to develop both a fully decentralized solution approach and a consistent, systematic approach to

				Centralized			Decentralized			
				Average # of Mfct Tight Periods	Average # of Dev Tight Periods	Average # of Not Introduced Products	Average of GAP with Cent	Average # of Mfct Tight Periods	Average # of Dev Tight Periods	Average # of Not Introduced Products
Asynchronous Demand	**Asynchronous Demand**			**21.30**	**7.00**	**0.28**	**5.82%**	**0.00**	**0.00**	**0.00**
	80%	**Manufacturing Capacity at 80%**		**26.95**	**6.25**	**0.25**	**4.99%**	**0.00**	**0.00**	**0.00**
		3 Cycles	3 Cycles	27.80	8.30	0.00	3.53%	0.00	0.00	0.00
			2 Engineering Teams	26.80	15.40	0.00	5.97%	0.00	0.00	0.00
			6 Engineering Teams	28.80	1.20	0.00	1.09%	0.00	0.00	0.00
		4 Cycles	4 Cycles	26.10	4.20	0.50	6.45%	0.00	0.00	0.00
			2 Engineering Teams	24.60	7.60	1.00	9.36%	0.00	0.00	0.00
			6 Engineering Teams	27.60	0.80	0.00	3.53%	0.00	0.00	0.00
	100%	**Manufacturing Capacity at 100%**		**15.65**	**7.75**	**0.30**	**6.64%**	**0.00**	**0.00**	**0.00**
		3 Cycles	3 Cycles	16.80	10.60	0.00	4.45%	0.00	0.00	0.00
			2 Engineering Teams	16.80	19.80	0.00	7.63%	0.00	0.00	0.00
			6 Engineering Teams	16.80	1.40	0.00	1.28%	0.00	0.00	0.00
		4 Cycles	4 Cycles	14.50	4.90	0.60	8.83%	0.00	0.00	0.00
			2 Engineering Teams	16.00	8.00	1.00	8.78%	0.00	0.00	0.00
			6 Engineering Teams	13.00	1.80	0.20	8.88%	0.00	0.00	0.00
Synchronous Demand	**Synchronous Demand**			**24.13**	**4.68**	**0.35**	**6.23%**	**0.00**	**0.18**	**0.00**
	80%	**Manufacturing Capacity at 80%**		**28.30**	**5.35**	**0.35**	**5.41%**	**0.00**	**0.00**	**0.00**
		3 Cycles	3 Cycles	29.70	6.10	0.20	4.43%	0.00	0.00	0.00
			2 Engineering Teams	27.40	12.20	0.40	7.59%	0.00	0.00	0.00
			6 Engineering Teams	32.00	0.00	0.00	1.26%	0.00	0.00	0.00
		4 Cycles	4 Cycles	26.90	4.60	0.50	6.39%	0.00	0.00	0.00
			2 Engineering Teams	26.00	8.40	1.00	9.32%	0.00	0.00	0.00
			6 Engineering Teams	27.80	0.80	0.00	3.45%	0.00	0.00	0.00
	100%	**Manufacturing Capacity at 100%**		**19.95**	**4.00**	**0.35**	**7.06%**	**0.00**	**0.35**	**0.00**
		3 Cycles	3 Cycles	20.00	3.70	0.00	4.79%	0.00	0.70	0.00
			2 Engineering Teams	19.40	6.80	0.00	8.01%	0.00	0.00	0.00
			6 Engineering Teams	20.60	0.60	0.00	1.56%	0.00	1.40	0.00
		4 Cycles	4 Cycles	19.90	4.30	0.70	9.34%	0.00	0.00	0.00
			2 Engineering Teams	17.20	6.80	1.40	17.62%	0.00	0.00	0.00
			6 Engineering Teams	22.60	1.80	0.00	1.05%	0.00	0.00	0.00

Figure 1: Summary of the experimental results.

constructing feasible solutions from the results of the decentralized approaches that can be interpreted in terms of information sharing and decision hierarchies between different agents.

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A Appendix: Set, parameter and variables definitions

Sets:

$\mathbb{T}$: Set of periods in the planning horizon of hour problem.
$\mathbb{P}$: Set of all generations that each product PD will introduce in the problem time horizon.
$\mathbb{I}$: Set of all Product Divisions, each one in charge of managing one product type line.

Parameters:

ρ_{pit}: Unit selling price of product p of PD i in period t.
c^M_{pit}: Unit production cost of product p of PD i in period t.
c^D_{pit}: Cost of one development stage of product p from PD i at period t.
τ^{tr}_{pit}: Transistor production capacity required to process one unit of product p for PD i in period t.

τ^{mtl}_{pit}: Metal capacity require to process one unit of product p for PD i in period t.
ω^{tr}_{pit}: Transistor capacity required to process one unit of product (p,i,t) in the transistor's development stage.
ω^{mtl}_{pit}: Capacity require to process one unit of product (p,i,t) in the metal's development stage.
C^{tr}_{t}: Total capacity of transistor manufacturing process.
C^{mtl}_{t}: Total capacity of the metal manufacturing process.
ε^{tr}_{pit}: Total engineering capacity that product (p,i,t) needs in the transistor's development stage.
ε^{mtl}_{pit}: Total engineering capacity that product (p,i,t) needs in the metal's development stage.
ε^{dbg}_{pit}: Total engineering capacity that product (p,i,t) needs in the debugging development stage.
n^{tr}_{pi}: Number of units used in the transistor development stage.
n^{mtl}_{pi}: Number of units used in the metal development stage.
E_t: Total amount of engineering capacity to work on any of the development stages.
r_{pi}: Number of development cycles require by product p for PD i to complete development.
S: Initial budget available for the corporation at the beginning of the time-horizon.
d_{pit}: Demand for product p of PDi in period t.
h^{mtl}_{pit}: Inventory holding cost of one unit of final product.
h^{mtl}_{pit}: Inventory holding cost of one work in process unit.

Decision Variables:

OC_{it} : Operation budget allocated by the corporation to PD i in period t.
B^{corp}_{t}: Available budget at the corporation at period t.
B^{div}_{it}: Available budget at PD i in period t.
x_{pit}: Number of units of final product p from PD i sold to the market at period t.
m_{pit}: Number of units of final product p from PD i produced at period t.
w^{tr}_{pit}: Number of units of product p for PD i processed at the transistor process in period t.
I_{pit}: Inventory of final product (p,i,t) on inventory at the beginning of period t.
I^{mtl}_{pit}: Number of WIP units of product (p,i,t) between transistors and metal process.
z^{tr}_{pit}: Binary variable that takes the value of one if the transistor development stage of product (p,i) was performed during period t.
z^{mtl}_{pit}: Binary variable that takes the value of one if the metal development stage of product (p,i) was performed during period t.
z^{dbg}_{pit}: Binary variable that takes the value of one if the debugging development stage of product (p,i) was performed during period t.
R_{pit}: Binary variable that takes the value of one indicating that one entire cycle of the development process of product p from PD i has finished at the end of period t.

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