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# FAST APPROXIMATION TO DISCRETE-EVENT SIMULATION OF QUEUEING NETWORKS

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## ABSTRACT

The simulation of queueing system is generally carried out by discrete-event simulation, which updates the simulation time with the occurrence time of the next event. However, for large-scale queueing network, especially when the network is very busy, keeping track of all the events takes a long time and it is difficult to harness the capability of parallel computing. In this paper, we propose an innovative fast simulation approximation framework for large-scale queueing network, where time is broken up into small time intervals and the system state is updated according to the events happening in each time interval. The computational complexity analysis demonstrates that our method is much more efficient for large-scale networks, compared with discrete-event simulation. In addition, we theoretically deduce the approximation error bound of the proposed algorithms. The experimental results show that our framework can be thousands times faster than the-state-of-the-art discrete-event simulation tools.

## 1 INTRODUCTION

The simulation of queueing networks is a fundamental approach to studying many real-world systems, such as call centers, telecommunication networks and manufacturing systems. Discrete event simulation (DES) (Banks et al. 2000) has always been a popular method for queueing network simulation. Basically, DES models the operation of a system as a sequence of events in time and it is assumed that there are no change in the system between consecutive events. Therefore, the simulation time can directly jump to the occurrence time of the next event.

Historically, besides this next-event time progression method, another approach has been suggested for updating the simulation time — fixed-increment time progression (Law and Kelton 2000), where time is broken up into small time intervals and the system state is updated according to the events happening in each time interval. Since not every time interval need to be simulated, a discrete-event simulation can typically run faster than a fixed-increment time simulation for small-scale network and when the system is not busy. However, for large-scale queueing network, especially when the network is very busy, there may be lots of events in each time interval. Thus simulating the system with next-event time progression and keeping track of all the events can be quite time consuming.

In this paper, we propose an innovative fast approximation framework for large-scale queueing network which adopts the fixed-increment time progression approach. We design parallel simulation algorithms. The computational complexity analysis demonstrates that our method is much more efficient for large-scale networks, compared with discrete-event simulation. In addition, the proposed framework is an approximation method which can trade accuracy for efficiency. We prove that the approximation error will not blow up as simulation time increasing and we theoretically deduce the error bound of the proposed algorithms. The experimental results show that our framework can be thousands times faster than the-state-of-the-art discrete-event simulation tools and the approximation performance is acceptable when the time interval is small.

## 2 PROBLEM FORMULATION

We consider a queueing network with $n$ nodes, and each node represents a service station consisting of $m$ servers. When a customer is served at one node, it can join another node or leave the network according to the routing matrix $\boldsymbol{P}$. Each element $p_{ij}$ in the routing matrix denotes the probability of joining node $j$ after completing service at node $i$. It is assumed that first-come, first-served discipline (Bhat 2008) is adopted. Furthermore, we assume that the external arrivals at each node $i$ during each interval $[t, t+h)$ ($a_{t,i}^{out}$) and the service capability of server $j$ at node $i$ during each interval ($s_{t,i,j}$) follow known distributions (or, at least, may be generated through a simulation algorithm). And fixed-increment time progression is adopted in our simulation algorithms.

## 3 COMPUTATIONAL COMPLEXITY

We analyze the computational complexity of our framework and DES for simulation of queueing networks, and the results are as follows:

Table 1: Complexity comparison.

| Method | Complexity |
|---|---|
| Fast-simulation | $O(Tn_l^{max}/h)$ |
| DES | $O\left(T\lambda\eta_{n,L}log(nm)\right)$ |

where $T$ is the simulation time horizon, $\lambda$ denotes the arrival rate of the network, and $\eta_{n,L}$ and $n_l^{max}$denote the average layer number of the routing chains in the network and the maximum number of nodes in each layer.

## 4 NUMERICAL EXPERIMENTS

We test the run time of our framework and one of the-state-of-the-art tools of DES (Palmer et al. 2019) on queueing networks with different parameters, and the results are listed below:

Table 2: Run time comparison.

| Parameters | Fast Approximation | DES |
|---|---|---|
| $T = 1000, \mathrm{n} = 100, \mathrm{m} = 20$ | 1.2s | 10.8s |
| $T = 1000, \mathrm{n} = 100, \mathrm{m} = 200$ | 1.3s | 162.9s |
| $T = 1000, \mathrm{n} = 100, \mathrm{m} = 1000$ | 1.3s | 1353.5s |
| $T = 1000, \mathrm{n} = 1000, \mathrm{m} = 20$ | 3.7s | 153.9s |
| $T = 1000, \mathrm{n} = 1000, \mathrm{m} = 200$ | 3.7s | 3041.4s |
| $T = 1000, \mathrm{n} = 1000, \mathrm{m} = 1000$ | 3.7s | 29263.6s |

## REFERENCES

Banks, J., J. S. Carson, B. L. Nelson, and D. M. Nicol. 2000. *Discrete-Event System Simulation*. 3rd ed. Upper Saddle River, New Jersey: Prentice-Hall, Inc.

Bhat, U. N. 2008. An introduction to queueing theory: modeling and analysis in applications, Volume 36. New York: Springer.

Law, A. M., and W. D. Kelton. 2000. Simulation Modeling & Analysis. 3rd ed. New York: McGraw-Hill, Inc.

Palmer, G. I., V. A. Knight, P. R. Harper, and A. L. Hawa. 2019. “Ciw: An open-source discrete event simulation library”. Journal of Simulation 13(1):68-82.