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A FIXED-SAMPLE-SIZE METHOD FOR ESTIMATING STEADY-STATE QUANTILES

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ABSTRACT

We propose FQUEST, a fully automated fixed-sample-size procedure for computing confidence intervals (CIs) for steady-state quantiles. The user provides a (simulation-generated) dataset of arbitrary size and specifies the required quantile and nominal coverage probability of the anticipated CI. FQUEST incorporates the simulation analysis methods of batching, standardized time series (STS), and sectioning. Preliminary experimentation with the waiting-time process in a congested M/M/1 queueing system showed that FQUEST performed well by delivering CIs with estimated coverage probability close to the nominal level, even in unfavorable circumstances where the sample sizes were inadequate. In the latter cases and for very small samples for steady-state quantile estimation, the close conformance of the CI coverage probability typically came at the expense of loose CI precision.

1 INTRODUCTION

Steady-state simulations play a crucial role in the design and performance evaluation of complex production and service systems (Law 2015). While the steady-state mean is a measure of central tendency, quantiles of the marginal steady-state distribution are standard measures of risk (Glasserman 2004). The estimation of a steady-state quantile is a much harder problem than the estimation of the mean: while both problems are subject to effects from the potential presence of an initial transient, substantial serial correlation in the simulation output process, and departures from normality, quantile estimation is adversely affected by additional issues ranging from the inherent bias of point estimators (see Theorem 1 below) and the challenging nature of the marginal distribution such as nonexistence of a probability density function (p.d.f.), discontinuities, and multimodalities with sharp peaks (Alexopoulos et al. 2018).

Recently, Alexopoulos et al. (2019) and Lolos et al. (2022, 2023) have developed Sequest and SQSTS, respectively, two fully automated sequential methods for effective estimation of steady-state quantiles. Unfortunately, users are often constrained by simulation models that are not integrated with the underlying sequential method or by datasets that are limited due to budget constraints. The literature contains a few fixed-sample-size procedures for estimating the steady-state mean; see Law (2015). The most efficient automated method is N-Skart by Tafazzoli et al. (2011), which is based on batch means computed from dynamically reconstructed batches with intervening "spacers."

In this article, we introduce FQUEST, a fully automated fixed-sample-size procedure for computing CIs for steady-state quantiles based on a single run. To the best of our knowledge, FQUEST is the first such method that (i) uses the STS methodology; (ii) addresses the simulation initialization problem; and (iii) warns the user when the dataset is insufficient and, subject to user's approval, delivers a heuristic CI. We substantiate our claim with a synopsis of a few methods from the literature. Methods based on regenerative cycles (Iglehart 1976; Moore 1980; Seila 1982a; Seila 1982b) can address the simulation initialization problem but do not lie within our scope because the number of cycles that can be completed within a finite limit N on the sample size may be insufficient so as to ensure good performance of the point estimators and CIs for the quantile of interest. This challenge escalates for extreme quantiles (Seila 1982b).

Heidelberger and Lewis (1984) presented three procedures for estimating steady-state quantiles, the first based on the spectral method and the last two based on empirical quantiles computed from groups of nonoverlapping batches. The estimation of the p-quantile was reduced to the estimation of the p^v-quantile of a sequence composed of the maxima of v spaced observations, where $v \approx \lfloor \ln(q)/\ln(p) \rfloor$, q is a value away from 0 or 1, and $\lfloor \cdot \rfloor$ is the floor function. The authors provided no recommendations for the spacing between the observations or the number of groups. Although the experimentation was based on stationary processes, the CIs generated by all methods exhibited substantial undercoverage for waiting-time processes generated by single-server queues with traffic intensity 0.9 and large values of the associated probability p.

The indirect method of Bekki et al. (2010) also assumes that the initial transient phase has been eliminated and computes point estimators and CIs for a set of selected quantiles. This fixed-sample-size method estimates a given quantile by a four-term Cornish-Fisher expansion (Fisher and Cornish 1960) based on the respective standard normal quantile and the first four sample moments of the time series. The method has the advantage of estimating multiple quantiles simultaneously without storing or sorting data. However, a sample moment computed from strongly correlated data often requires a large sample for accurate estimation of the associated true moment, and this problem worsens for higher-order moments. The impact of this problem is evident with use of sample sizes of 30 and 60 million to estimate job cycle times in simple queueing systems with server utilizations below and above 90%, respectively. In addition, this method may yield unreliable point estimates of quantiles if the marginal density exhibits highly nonnormal behavior. This issue was partially rectified in Bekki et al. (2009) by combining the Cornish-Fisher expansion with a Box-Cox transformation. Notably, the latter three methods do not address the issues in items (ii) and (iii) above.

The proposed FQUEST method is designed to provide a CI for a selected steady-state quantile, with a user-specified error probability, based on a single time series of an arbitrary fixed length. If the sample size is deemed to be insufficient, FQUEST issues a warning and the user has the option to terminate the procedure early without obtaining a CI. In any case, the user can utilize the output of FQUEST as the first step for obtaining a conservative estimate of the sample size required to compute a CI with a certain precision (absolute or relative). FQUEST draws ideas from three procedures: (i) SQSTS (Lolos et al. 2023); (ii) Sequest (Alexopoulos et al. 2019), and (iii) N-Skart (Tafazzoli et al. 2011). Since the aforementioned methods have different objectives, FQUEST differs from all three with regard to its scope, structure, and the computation of the final CI. For instance, the Sequest and SQSTS are sequential methods, while N-Skart addresses the computation of the steady-state mean and does not use the STS methodology.

The core theoretical background for the CIs used in FQUEST is laid out in Alexopoulos et al.(2020, 2023) and in Lolos et al. (2023), who established asymptotic properties for a variety of variance-parameter estimators for the sample-quantile process computed from nonoverlapping batches, showed that as the batch size grows while the batch count remains constant the vector of the signed weighted areas of the STSs computed from the nonoverlapping batches converges in law to a vector of independent and identically distributed (i.i.d.) random variables (r.v.'s) from the normal distribution (see Theorem 2 below), and closed various theoretical gaps related to STS-based variance-parameter estimation dating back to the 1980s.

Section 2 includes the necessary background information, the main assumptions, and the theorems on which we build our fixed-sample-size method. Section 3 contains a description of the FQUEST algorithm. In Section 4 we conduct a preliminary evaluation of the performance of FQUEST using the waiting-time process in an M/M/1 system. In Section 5 we summarize our work and discuss future extensions.

2 FOUNDATIONS

For $p \in (0,1)$, the p-quantile of a r.v. X with c.d.f. $F(y)$ is defined as

$$y_p \equiv F^{-1}(p) \equiv \inf\{y : F(y) \geq p\}.$$

Our goal is the computation of a point estimate and a CI for y_p based on a stationary sample path $\{Y_k : k \geq 1\}$, which is a warmed-up version of the original sequence of simulation outputs. Let $\{Y_k : k = 1, \ldots, n\}$ denote a time series of length n, and let $Y_{(1)} \leq \cdots \leq Y_{(n)}$ be the respective order statistics. The classical point estimator of y_p is the empirical p-quantile $\widetilde{y}_p(n) \equiv Y_{(\lceil np \rceil)}$, where $\lceil \cdot \rceil$ denotes the ceiling function. For each $x \in \mathbb{R}$ and $k \geq 1$, we define the indicator r.v. as $I_k(x) \equiv 1$ if $Y_k \leq x$, and $I_k(x) \equiv 0$ otherwise; hence $\mathrm{E}[I_k(y_p)] = p$. Assuming $n \geq 1$, we let $\overline{I}(y_p, n) \equiv n^{-1} \sum_{k=1}^{n} I_k(y_p)$; and for each $\ell \in \mathbb{Z}$, we let $\rho_I(\ell; y_p) \equiv \mathrm{Corr}[I_k(y_p), I_{k+\ell}(y_p)]$ denote the autocorrelation function of the indicator process $\{I_k(y_p) : k \geq 1\}$ at lag ℓ. Below we also use the following notation: Z denotes an r.v. from $N(0,1)$, the standard normal distribution; $\boldsymbol{Z}_\nu \equiv \left[Z_1, \ldots, Z_\nu\right]^\mathsf{T}$ denotes a $\nu \times 1$ vector whose components are i.i.d. $N(0,1)$; χ^2_ν denotes a chi-squared r.v. with ν degrees of freedom (d.f.); t_ν denotes an r.v. having Student's t distribution with ν d.f.; $t_{\delta,\nu}$ denotes the δ-quantile of t_ν; and $D \equiv D[0,1]$ denotes the space of real-valued functions on $[0,1]$ that are right continuous with left-hand limits.

The assumptions and key results that are outlined below form the skeleton for variance cancellation methods used to develop $100(1-\alpha)\%$ CIs for y_p. The basic (unadjusted) CIs for y_p have form

$$\widetilde{y}_p(n) \pm t_{1-\alpha/2,\nu} \widehat{\sigma}_p / \sqrt{n},$$

where $\widehat{\sigma}^2_p$ is an estimator of the variance parameter $\sigma^2_p \equiv \lim_{n\to\infty} n \mathrm{Var}\big[\widetilde{y}_p(n)\big]$ of the quantile process $\{\widetilde{y}_p(n) : n \geq 1\}$, and the d.f. ν depends on the underlying quantile-estimation method.

2.1 Assumptions

In this subsection we present the main assumptions for the processes $\{Y_k : k \geq 1\}$ and $\{I_k(y_p) : k \geq 1\}$.

Geometric-Moment Contraction (GMC) Condition (Wu 2005). The process $\{Y_k : k \geq 1\}$ is defined by a function $\xi(\cdot)$ of a sequence of i.i.d. r.v.'s $\{\varepsilon_j : j \in \mathbb{Z}\}$ such that $Y_k = \xi(\ldots, \varepsilon_{k-1}, \varepsilon_k)$ for $k \geq 0$. Moreover, there exist constants $\psi > 0$, $C^* > 0$, and $r \in (0,1)$ such that for two independent sequences $\{\varepsilon_j : j \in \mathbb{Z}\}$ and $\{\varepsilon'_j : j \in \mathbb{Z}\}$ each consisting of i.i.d. r.v.'s with the same distribution as ε_0, we have

$$\mathrm{E}[|\xi(\ldots, \varepsilon_{-1}, \varepsilon_0, \varepsilon_1, \ldots, \varepsilon_k) - \xi(\ldots, \varepsilon'_{-1}, \varepsilon'_0, \varepsilon_1, \ldots, \varepsilon_k)|^\psi] \leq C^* r^k, \quad \text{for } k \geq 0.$$

The GMC condition holds for a plethora of random processes including the autoregressive–moving average time series, a rich set of linear and nonlinear processes with short-range dependence, and a broad class of Markov chains. Alexopoulos et al. (2019, 2023) provide an extensive list of these processes and empirical methods for verifying the GMC assumption. Recently, Dingeç et al. (2022) established the validity of the GMC condition for the customer waiting-time process (prior to service) in an M/M/1 queueing system and a G/G/1 system with non-heavy-tailed service-time distributions.

Density-Regularity (DR) Condition. The p.d.f. $f(\cdot) \equiv F'(\cdot)$, exists, and is bounded on $\mathbb{R}$ and continuous almost everywhere (a.e.) on $\mathbb{R}$. Moreover, $f(y_p) > 0$, and the derivative $f'(y_p)$ exists.

Short-Range Dependence (SRD) of the Indicator Process. The indicator process $\{I_k(y_p) : k \geq 1\}$ has the SRD property so that

$$0 < \sum_{\ell \in \mathbb{Z}} \rho_I(\ell; y_p) \leq \sum_{\ell \in \mathbb{Z}} |\rho_I(\ell; y_p)| < \infty.$$

Thus the variance parameters for the processes $\{\overline{I}(y_p, n)\}$ and $\{\widetilde{y}_p(n)\}$ obey

$$\left.\begin{aligned} \sigma^2_{I(y_p)} &\equiv \lim_{n\to\infty} n\mathrm{Var}\big[\overline{I}(y_p,n)\big] = p(1-p)\sum_{\ell\in\mathbb{Z}} \rho_I(\ell; y_p) \in (0,\infty), \\ \sigma^2_p &= \lim_{n\to\infty} n\mathrm{Var}\big[\widetilde{y}_p(n)\big] = \frac{\sigma^2_{I(y_p)}}{f^2(y_p)} \in (0,\infty). \end{aligned}\right\}$$

Functional Central Limit Theorem (FCLT) for the Indicator Process. The sequence of random functions $\{\mathscr{I}_n : n \geq 1\}$ in D defined by

$$\mathscr{I}_n(t; y_p) \equiv \frac{\lfloor nt \rfloor}{\sigma_{I(y_p)} n^{1/2}} [\overline{I}(y_p, \lfloor nt \rfloor) - p], \quad \text{for } t \in [0,1] \text{ and } n \geq 1,$$

where $\lfloor \cdot \rfloor$ denotes the floor function, satisfies the FCLT $\mathscr{I}_n \underset{n\to\infty}{\Longrightarrow} \mathscr{W}$ in D with the appropriate metric. Herein $\mathscr{W}$ denotes standard Brownian motion on $[0,1]$ and $\underset{n\to\infty}{\Longrightarrow}$ denotes weak convergence as $n \to \infty$ (Billingsley 1999, pp. 1–6 and Theorem 2.1). Below, the argument y_p is omitted from the notation for random functions unless it is needed to avoid ambiguity.

2.2 Asymptotic Properties Based on Nonoverlapping Batches

The FQUEST procedure relies on nonoverlapping batches. Given a fixed batch count $b \geq 2$, for $j = 1, \ldots, b$, the jth nonoverlapping batch of size $m \geq 1$ consists of the subsequence $\{Y_{(j-1)m+1}, \ldots, Y_{jm}\}$, where we assume $n = bm$. The batch mean of the associated indicator r.v.'s from the jth batch is $\overline{I}_j(y_p, m) \equiv m^{-1}\sum_{\ell=1}^{m} I_{(j-1)m+\ell}(y_p)$. Similarly to the full-sample case, we define the order statistics $Y_{j,(1)} \leq \cdots \leq Y_{j,(m)}$ corresponding to the jth batch. Then the jth batched quantile estimator (BQE) of y_p is $\widehat{y}_p(j,m) \equiv Y_{j,(\lceil mp \rceil)}$.

Theorem 1 (Alexopoulos et al. 2019) If the output process $\{Y_k : k \geq 1\}$ satisfies the GMC and DR conditions, and the indicator process $\{I_k(y_p) : k \geq 1\}$ satisfies the SRD and FCLT conditions, then we obtain the Bahadur representation

$$\widehat{y}_p(j,m) = y_p - \frac{\overline{I}_j(y_p, m) - p}{f(y_p)} + O_{\text{a.s.}}\left[\frac{(\log m)^{3/2}}{m^{3/4}}\right] \quad \text{as } m \to \infty$$

for $j = 1, \ldots, b$, where the big-$O_{\text{a.s.}}$ notation for the remainder $Q_{j,m} \equiv \widehat{y}_p(j,m) - y_p + \big[\overline{I}_j(y_p,m) - p\big]/f(y_p)$ means that there exist associated r.v.'s $\mathscr{U}_j$ and $\mathscr{R}_j$ that are bounded almost surely (a.s.) and satisfy $|Q_{j,m}| \leq \mathscr{U}_j \frac{(\log m)^{3/2}}{m^{3/4}}$ for $m \geq \mathscr{R}_j$ and $j = 1, \ldots, b$ a.s. Further,

$$m^{1/2}\big[\widehat{y}_p(1,m) - y_p, \ldots, \widehat{y}_p(b,m) - y_p\big]^{\mathsf{T}} \underset{m\to\infty}{\Longrightarrow} \sigma_p \boldsymbol{Z}_b \tag{1}$$

in $\mathbb{R}^b$ with the standard Euclidean metric.

2.3 Confidence Intervals for Quantiles

The CIs employed by FQUEST are computed from STSs based on nonoverlapping batches, the BQEs $\widehat{y}_p(j,m)$, and the full-sample empirical quantile $\widetilde{y}_p(n)$.

For $j = 1,\dots,b$, we define $\widehat{y}_p(j,\lfloor mt \rfloor)$ as the empirical p-quantile (i.e., the $\lceil p\lfloor mt \rfloor \rceil$-th order statistic) computed from the partial sample $\{Y_{(j-1)m+k} : k = 1,\dots,\lfloor mt \rfloor\}$, and the STS-based quantile-estimation process formed from batch j as

$$T_{j,m}(t) \equiv \frac{\lfloor mt \rfloor}{m^{1/2}}\big[\widehat{y}_p(j,m) - \widehat{y}_p(j,\lfloor mt \rfloor)\big], \quad \text{for } t \in [0,1] \text{ and } m \geq 1.$$

Under the assumptions of Theorem 1, Theorem 2 of Alexopoulos et al. (2023) implies

$$\big[m^{1/2}(\widehat{y}_p(j,m) - y_p), T_{j,m}\big] \underset{m\to\infty}{\Longrightarrow} \sigma_p\big[\mathscr{W}(1), \mathscr{B}\big], \quad \text{for } j = 1,\dots,b,$$

where $\mathscr{B}(t) \equiv \mathscr{W}(t) - t\mathscr{W}(1)$ for $t \in [0,1]$ is a standard Brownian bridge process that is independent of $\mathscr{W}(1)$. We define the signed area computed from batch j as

$$A_p(w;j,m) \equiv m^{-1}\sum_{k=1}^{m} w(k/m) T_{j,m}(k/m).$$

where $w(\cdot)$ is a deterministic weight function that is bounded and continuous a.e. on $[0,1]$ (so that $w(t)\mathscr{B}(t)$ is Riemann integrable on $[0,1]$); and

$$Z(w) \equiv \int_0^1 w(t)\mathscr{B}(t)\,dt \sim N(0,1). \tag{2}$$

There are many weight functions that satisfy condition (2), including the constant $w_0(\cdot) \equiv \sqrt{12}$. Preliminary experimentation in Lolos et al. (2023) did not reveal any compelling reasons for replacing the constant weight function $w_0(\cdot)$ with other weight functions from the literature; see Lolos et al. (2022).

The batched STS-area estimator is the average of the squared signed areas,

$$\mathscr{A}_p(w;b,m) \equiv b^{-1}\sum_{j=1}^{b} A_p^2(w;j,m).$$

Theorems 1 (above) and 2 (below) constitute the basis for the statistical tests in FQUEST.

Theorem 2 (Lolos et al. 2023) If $\{Y_k : k \geq 1\}$ satisfies the assumptions of Theorem 1, then as $m \to \infty$, the $b \times 1$ vector of the signed areas $\big[A_p(w;1,m),\dots,A_p(w;b,m)\big]^\top$ converges weakly to the same distributional limit as the (scaled) vector of BQEs in Theorem 1:

$$\big[A_p(w;1,m),\dots,A_p(w;b,m)\big]^\top \underset{m\to\infty}{\Longrightarrow} \sigma_p \mathbf{Z}_b. \tag{3}$$

Further,

$$\mathscr{A}_p(w;b,m) \underset{m\to\infty}{\Longrightarrow} \sigma_p^2 \chi_b^2/b. \tag{4}$$

We also define the nonoverlapping batched quantile (NBQ) variance-parameter estimator

$$\widetilde{\mathscr{N}_p}(b,m) \equiv (b-1)^{-1} m \sum_{j=1}^{b}\big[\widehat{y}_p(j,m) - \widetilde{y}_p(n)\big]^2, \tag{5}$$

and the combined variance estimator

$$\widetilde{\mathscr{V}_p}(w;b,m) \equiv \frac{b\mathscr{A}_p(w;b,m) + (b-1)\widetilde{\mathscr{N}_p}(b,m)}{2b-1}. \tag{6}$$

Theorem 3 (Alexopoulos et al. 2023) If $\{Y_k : k \geq 1\}$ satisfies the assumptions of Theorem 1, then

$$n^{1/2}\big[\widetilde{y}_p(n) - y_p\big] \underset{m\to\infty}{\Longrightarrow} \sigma_p Z, \tag{7}$$

$$\widetilde{\mathscr{N}_p}(b,m) \underset{m\to\infty}{\Longrightarrow} \sigma_p^2 \chi^2_{b-1}/(b-1), \tag{8}$$

$$\widetilde{\mathscr{V}_p}(w;b,m) \underset{m\to\infty}{\Longrightarrow} \sigma_p^2 \chi^2_{2b-1}\big/(2b-1), \tag{9}$$

the limiting r.v.'s in Equations (4), (7), and (8) are independent, and the limiting r.v.'s in Equations (7) and (9) are also independent. Further, for fixed $b \geq 2$,

$$\begin{aligned} &\widetilde{y}_p(n) \pm t_{1-\alpha/2,b}\big(\mathscr{A}_p(w;b,m)/n\big)^{1/2},\\ &\widetilde{y}_p(n) \pm t_{1-\alpha/2,b-1}\big(\widetilde{\mathscr{N}_p}(b,m)/n\big)^{1/2}, \quad \text{and}\\ &\widetilde{y}_p(n) \pm t_{1-\alpha/2,2b-1}\big(\widetilde{\mathscr{V}_p}(w;b,m)/n\big)^{1/2} \end{aligned} \tag{10}$$

are asymptotically valid $100(1-\alpha)\%$ CIs for y_p as $m \to \infty$ (their coverage probabilities converge to the nominal value $1-\alpha$ as $m \to \infty$).

Equations (4), (8), and (9) illustrate the potential benefits of the combined estimator $\widetilde{\mathscr{V}_p}(w;b,m)$ of σ_p^2: since the asymptotic distribution of the latter estimator has nearly twice the d.f. than the limiting distributions of its components, the CI in Equation (10) will typically be less variable (by a factor of about $\sqrt{2}$) than the CIs based solely on either $\widetilde{\mathscr{N}_p}(b,m)$ or $\mathscr{A}_p(w;b,m)$.

3 THE FQUEST ALGORITHM

In this section we present the proposed fixed-sample-size procedure for estimating a steady-state quantile. The formal algorithmic statement of FQUEST is given in Figure 1 below. FQUEST draws elements from other procedures with different goals including the sequential methods Sequest (Alexopoulos et al. 2019) and SQSTS (Lolos et al. 2023) for estimating steady-state quantiles, and the fixed-sample-size N-Skart method of Tafazzoli et al. (2011) for estimating the steady-state mean.

In Step **[0]**, the simulation model or user provides a sample path $\{Y_1,\ldots,Y_N\}$ of fixed size N, the probability associated with the quantile p, and the nominal error probability $\alpha \in (0,1)$ for the CI for y_p. Step **[1]** initializes the experimental parameters. The initial number of batches is set at $b = 50$ to enhance the power of von Neumann's randomness test in Step **[3]**, and the initial batch size is set at $m = 500$. We also define the array of batch counts $\boldsymbol{s} = [32,24,16,10]$ for Steps **[5]**–**[9]**. Further, we initialize the counters $l = 1$ and $v = 1$, and set `flag = false`. At this point the algorithm sets the weight function that will be used for the calculation of the signed areas and the STS variance-parameter estimator. The level of significance for the statistical test in Step **[3]** is set according to the sequence $\{\beta\psi(\ell) : \ell = 1,2,\ldots\}$, where $\beta = 0.3$, $\psi(\ell) \equiv \exp\big[-\eta(\ell-1)^\theta\big]$, $\eta = 0.2$, and $\theta = 2.3$. For the statistical tests in Steps **[6]**–**[9]** we fix the significance level at β. The values of the parameters β, η, and θ were chosen after careful experimentation to control the growth of the batch size and to avoid excessive truncation during Step **[5]** which can be detrimental given the sample-size limitation. Notice that on a potential fourth iteration within Step **[3]** one has $\beta\psi(4) = 0.025$, which makes passing the test easier.

Since the sample size N is fixed, it is possible that it is less than the initial assignment $bm = 25{,}000$. In this case, Step **[2]** sets $m = \lfloor N/b \rfloor$, which is the largest allowable value for the current batch count b. Step **[3]** consists of a loop that tests for the randomness of the signed areas $\{A_p(w;j,m) : j = 1,\ldots,b\}$ computed from the first bm observations (with the trailing $N - bm$ observations ignored, but not discarded) using a two-sided test based on von Neumann's ratio (von Neumann 1941, Young 1941) with progressively decreasing significance level $\beta\psi(\ell)$ on iteration ℓ. If the randomness test fails, we increase the batch size to $[\![m\sqrt{2}]\!]$, where $[\![\cdot]\!]$ denotes rounding to the nearest integer. If the updated sample size exceeds N, we set $m = \lfloor N/b \rfloor$, which is the largest allowable value given the current batch count b. If the randomness test fails with the largest allowable batch size $\lfloor N/b \rfloor$, FQUEST exits Step **[3]** and moves to Step **[4]**, where it issues a warning to the user regarding the insufficiency of the sample. Then it seeks permission to continue with the construction of a CI.

If the randomness test in Step **[3]** is passed or the user decides to proceed despite the failure of the randomness test, Step **[5]** removes the first batch, sets the new sample size to $N^* = N - m$, and reindexes the truncated dataset. Assuming the successful completion of Step **[3]**, the (approximate) independence between $A_p(w;1,m)$ and the remaining signed areas $\{A_p(w;j,m) : j = 2,\ldots,b\}$ indicates that any initialization bias due to warmup effects is mostly confined to the first batch. Step **[5]** restarts with $b = s[1] = 32$ and $m = \lfloor N^*/b \rfloor$. We chose the entries of the vector $\boldsymbol{s} = [32,24,16,10]$ after extensive experimentation. Notice that 32 batches typically suffice for effective estimation of the variance parameter σ_p^2, while fewer than 10 batches may result in unreliable CIs.

In Steps **[6]**–**[9]** we conduct the two-sided randomness test of von Neumann (1941) and the one-sided test of Shapiro and Wilk (1965) for univariate normality to assess whether the signed areas $\{A_p(w;j,m) : j = 1,\ldots,b\}$ and the BQEs $\{\widehat{y}_p(j,m) : j = 1,\ldots,b\}$ satisfy the asymptotic properties in Equations (3) and (1), respectively. Each of the Steps **[6]**–**[9]** has a very similar structure. First we compute the signed areas $\{A_p(w;j,m) : j = 1,\ldots,b\}$ or the BQEs $\{\widehat{y}_p(j,m) : j = 1,\ldots,b\}$ and conduct the pertinent statistical test using the fixed significance level of $\beta = 0.3$. The significance level is kept constant and high to avoid passing a test with an inadequately small batch size leading to unreliable CIs. If the test is passed, FQUEST proceeds to the next step; otherwise, the batch count decreases to the next element of the array $\boldsymbol{s}$. Since $\boldsymbol{s}$ contains only four values, we can have up to four failed attempts to pass any of the statistical tests in Steps **[6]**–**[9]**. If at any point a statistical test fails with $b = 10$, then FQUEST advances to Step **[10]**.

In Step **[10]**, if all the statistical tests have been passed, FQUEST computes the combined variance estimator $\widetilde{\mathscr{V}}_p(w;b,m)$ and returns the CI in Equation (10). Otherwise, it issues a warning mentioning that some of the statistical tests failed (with the significance level of $\beta = 0.3$) and asks the user for permission to continue with the calculation of a point estimate and a heuristic CI for y_p.

Figure 1: Algorithm FQUEST

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[0] User-Initialization: Provide a sample of fixed size N, the probability p corresponding to the quantile, and the error probability $\alpha \in (0,1)$.

[1] Parameter-Initialization: Set the number of batches $b = 50$, batch size $m = 500$, $\ell = 1$, $v = 1$, and `flag` = `false`. Also set $\beta = 0.30$ and $\boldsymbol{s} = [32,24,16,10]$. Let $w(t)$, $t \in [0,1]$, be the weight function and define the initial significance level for the first hypothesis test in Step **[3]** as $\beta\psi(\ell) \equiv \exp\left[-\eta(\ell-1)^{\theta}\right]$, $\ell = 1,2,\ldots$, with $\eta = 0.2$ and $\theta = 2.3$.

[2] **If** $N < bm$: Set $m \leftarrow \lfloor N/b \rfloor$;

[3] **Until** von Neumann's test fails to reject randomness **or** `flag` = `true`:

- Compute the signed areas $\{A_p(w;j,m) : j = 1,\ldots,b\}$ from the initial bm observations;
- Assess the randomness of $\{A_p(w;j,m) : j = 1,\ldots,b\}$ using von Neumann's two-sided randomness test with significance level $\beta\psi(\ell)$;
- Set $\ell \leftarrow \ell + 1$ and $m \leftarrow \llbracket m\sqrt{2} \rrbracket$;
- **If** $N < bm$ **and** $m \neq \lfloor N/b \rfloor$: Set $m \leftarrow \lfloor N/b \rfloor$; **Else** Set `flag` ← `true`;

[4] If the randomness test in Step **[3]** failed, then issue a warning that the randomness test failed due to insufficient size of the dataset and seek permission from the user to continue with the construction of a CI. If the user declines, then exit without delivering a CI.

[5] Remove the first batch, reindex the truncated dataset, and set N^* equal to the size of the truncated sample. Set the number of batches $b \leftarrow s[v]$ and calculate the batch size as $m \leftarrow \lfloor N^*/b \rfloor$. Ignore the initial $N^* - bm$ observations.

[6] **Until** von Neumann's test fails to reject randomness **or** $v = 5$ (a test has failed with $b = 10$):
- Compute the signed areas $\{A_p(w; j,m) : j = 1,\ldots,b\}$ and assess their randomness using von Neumann's two-sided randomness test with significance level β;
- Set $v \leftarrow v+1$. Update $b \leftarrow s[v]$ and $m \leftarrow \lfloor N^*/b \rfloor$. Ignore the initial $N^* - bm$ observations.

[7] **Until** the Shapiro-Wilk test fails to reject normality **or** $v = 5$ (a test has failed with $b = 10$):
- Compute the signed areas $\{A_p(w; j,m) : j = 1,\ldots,b\}$ and assess their univariate normality using the Shapiro–Wilk test with significance level β;
- Set $v \leftarrow v+1$. Update $b \leftarrow s[j]$ and $m \leftarrow \lfloor N^*/b \rfloor$. Ignore the initial $N^* - bm$ observations.

[8] **Until** von Neumann's test fails to reject randomness **or** $v = 5$ (a test has failed with $b = 10$):
- Compute the BQEs $\{\widehat{y}_p(j,m) : j = 1,\ldots,b\}$ and assess their randomness using von Neumann's two-sided randomness test with significance level β;
- Set $v \leftarrow v+1$. Update $b \leftarrow s[v]$ and $m \leftarrow \lfloor N^*/b \rfloor$. Ignore the initial $N^* - bm$ observations.

[9] **Until** the Shapiro–Wilk test fails to reject normality **or** $v = 5$ (a test has failed with $b = 10$):
- Compute the BQEs $\{\widehat{y}_p(j,m) : j = 1,\ldots,b\}$ and assess their univariate normality using the Shapiro–Wilk test with significance level β;
- Set $v \leftarrow v+1$. Update $b \leftarrow s[v]$ and $m \leftarrow \lfloor N^*/b \rfloor$. Ignore the initial $N^* - bm$ observations.

[10] Set $n^* \leftarrow bm$.

If $v < 5$ (no statistical test in Steps **[6]**–**[9]** failed), then
- Compute the combined variance estimator $\widetilde{\mathscr{V}}_p(w; b,m)$ in Equation (6), deliver the respective $100(1-\alpha)\%$ CI $\widetilde{y}_p(n^*) \pm t_{1-\alpha/2,2b-1}\big(\widetilde{\mathscr{V}}_p(w;b,m)/n^*\big)^{1/2}$, and exit;

Else
- Issue a warning that a statistical test failed due to insufficient size of the dataset and seek permission from the user to continue with the construction of a CI. If the user declines, then exit without delivering a CI;
- Compute the sample mean and sample variance of the BQEs

$$\overline{\overline{y}}_p(b,m) \equiv \frac{1}{b}\sum_{j=1}^{b}\widehat{y}_p(j,m) \quad \text{and} \quad S_p^2(b,m) \equiv \frac{1}{b-1}\sum_{j=1}^{b}\big[\widehat{y}_p(j,m) - \overline{\overline{y}}_p(b,m)\big]^2,$$

the quantity

$$h_{\alpha,b,m} = \max\Big\{t_{1-\alpha/2,b}\big(\mathscr{A}_p(w;b,m)/n^*\big)^{1/2},\ t_{1-\alpha/2,b-1}\big(\widetilde{\mathscr{N}_p}(b,m)/n^*\big)^{1/2}\Big\},$$

and construct the following CIs for y_p with half-length $h_{\alpha,b,m}$:

$$\widetilde{y}_p(n^*) \pm h_{\alpha,b,m} \quad \text{and} \quad \overline{\overline{y}}_p(b,m) \pm h_{\alpha,b,m}. \tag{11}$$

- Calculate the sample skewness of the BQEs

$$\widehat{B}_{\widehat{y}_p}(b,m) \equiv \frac{b}{(b-1)(b-2)}\sum_{j=1}^{b}\left[\frac{\widehat{y}_p(j,m) - \overline{\overline{y}}_p(b,m)}{S_p(b,m)}\right]^3,$$

compute the skewness-adjustment parameter $\gamma \equiv \widehat{B}_{\widehat{y}_p}(b,m)/\big[6\sqrt{b}\big]$, and define the skewness-adjustment function $G(\zeta) \equiv \zeta$ if $|\gamma| \le 0.001$ or $\frac{[1+6\gamma(\zeta-\gamma)]^{1/3}-1}{2\gamma}$ if $|\gamma| > 0.001$. Estimate the sample lag-1 autocorrelation of the BQEs by

$$\widehat{\phi}_{\widehat{y}_p}(b,m) \equiv \frac{1}{b-1}\sum_{j=1}^{b-1}\frac{[\widehat{y}_p(j,m) - \overline{\overline{y}}_p(b,m)][\widehat{y}_p(j+1,m) - \overline{\overline{y}}_p(b,m)]}{S_p^2(b,m)},$$

and compute the correlation-adjustment factor from

$$\varphi = \max\left(\frac{1+\widehat{\phi}_{\widehat{y}_p}(b,m)}{1-\widehat{\phi}_{\widehat{y}_p}(b,m)}, 1\right).$$

Set

$$G_1 \equiv G(t_{1-\alpha/2,b-1})\sqrt{\varphi \widetilde{S}^2_p(b,m)/b}, \quad \text{and} \quad G_2 \equiv G(t_{\alpha/2,b-1})\sqrt{\varphi \widetilde{S}^2_p(b,m)/b},$$

where

$$\widetilde{S}^2_p(b,m) \equiv \frac{1}{b-1}\sum_{j=1}^{b}\left[\widehat{y}_p(j,m) - \widetilde{y}_p(b,m)\right]^2$$

and construct the (asymmetric) correlation- and skewness-adjusted CI (Willink 2005; Alexopoulos et al. 2019)

$$\left[\min\left(\widetilde{y}_p(n^*) - G_1, \widetilde{y}_p(n^*) - G_2\right), \max\left(\widetilde{y}_p(n^*) - G_1, \widetilde{y}_p(n^*) - G_2\right)\right]. \tag{12}$$

- Deliver the full-sample point estimator $\widetilde{y}_p(n^*)$ and the smallest interval containing the CIs in Equations (11) and (12), and exit.

End If

4 EXPERIMENTAL RESULTS

This section contains a precursory empirical evaluation of FQUEST using the waiting-time sequence in an M/M/1 queueing system with arrival rate $\lambda = 0.8$, service rate $\omega = 1$ (traffic intensity $\rho = 0.8$), and FIFO service discipline. To assess the ability of the FQUEST method to deal with excessive initialization bias, we initialized the system with one entity beginning service and 112 entities in queue. The steady-state probability of this initial state is $(1-\rho)\rho^{113} \approx 2.240\times 10^{-12}$, implying a high probability for a prolonged transient phase. As we mentioned earlier, we used only the constant weight function $w_0(\cdot)$.

Table 1 contains experimental results for FQUEST using five different sample sizes $N \in \mathscr{S} \equiv \{50{,}000, 100{,}000, 200{,}000, 500{,}000, 1{,}000{,}000\}$ and a nominal 95% ($\alpha = 0.05$) CI coverage probability with all estimates being averages computed from 1,000 independent trials. Specifically, column 1 lists selected values of p and column 2 contains the exact value of the associated quantile y_p. Column 3 lists the fixed-sample size N. Columns 4 and 5 contain the average value of the point estimate $\widetilde{y}_p(n^*)$ and the average value of the absolute error $|\widetilde{y}_p(n^*) - y_p|$, respectively. Columns 6–8 contain the average value of the half-length (HL) of the 95% CI for y_p, the average value of the CI's relative precision expressed as a percentage, and the estimated coverage of the CI as a percentage, respectively. We report the average CI half-length and average relative precision despite the fact that the final CI delivered in Step **[10]** of FQUEST may be asymmetric for small samples (when a statistical test in Steps **[6]**–**[9]** fails with $b = 10$ batches). The standard errors of the estimated coverage probabilities are approximately $\sqrt{(0.95\times 0.05)/1000} = 0.007$. Columns 9 and 10 display the average final batch size ($\overline{m}$) and average final batch count ($\overline{b}$), respectively, after the truncation of the initial subset of observations in Step **[5]**. Finally, Columns 11 and 12 list the standard deviation of the CI HL and the average number of truncated observations ($N - n^*$), respectively.

The experimental results are displayed in Table 1. FQUEST managed to provide satisfactory estimated CI coverage probabilities, with the worst one being 93.3% for $p = 0.995$ and $N = 50{,}000$. There were a few cases with noticeable CI overcoverage for $p \le 0.7$. Although the estimated CI relative precision was a bit excessive for $p \ge 0.95$ and $N \le 100{,}000$, it dropped as the provided sample size increased. The value of FQUEST is evident from its ability to provide usable CIs for fixed sample sizes N that are smaller than those required by state-of-the-art sequential procedures. For example, the sequential SQSTS method of Lolos et al. (2023) required an average sample size near 4 million to compute a 95% CI for the 0.99-quantile of this waiting-time process (see Table 5.10 in Lolos 2023) under no CI precision requirement. Overall, FQUEST performed well in this experimental setting.

Table 1: Experimental results for FQUEST with regard to point and 95% CI estimation of y_p for the M/M/1 waiting-time process with traffic intensity 0.8 based on 1000 independent replications

p	y_p	N	$\widetilde{y}_p(n)$	Avg. |Bias|	Avg. 95% CI HL	Avg. 95% CI rel. prec. (%)	Avg. 95% CI cov. (%)	$\overline{m}$	$\overline{b}$	St. Dev. HL	Avg. Trunc. Point
0.3	0.668	50,000	0.667	0.044	0.160	24.030	97.3	4,098	13.49	0.080	1,002
		100,000	0.669	0.030	0.105	15.774	96.8	7,431	15.45	0.051	1,950
		200,000	0.669	0.021	0.071	10.582	97.3	13,665	17.37	0.031	2,460
		500,000	0.669	0.013	0.042	6.348	97.1	31,526	19.25	0.015	2,461
		1,000,000	0.668	0.010	0.030	4.429	96.9	62,711	19.48	0.009	2,463
0.5	2.350	50,000	2.348	0.090	0.335	14.223	96.9	4,099	13.37	0.180	986
		100,000	2.352	0.062	0.215	9.149	96.9	7,388	15.52	0.100	1,897
		200,000	2.352	0.044	0.143	6.070	97.3	13,807	17.20	0.059	2,336
		500,000	2.352	0.028	0.085	3.626	97.2	31,159	19.32	0.026	2,338
		1,000,000	2.350	0.020	0.060	2.545	96.6	61,917	19.72	0.019	2,340
0.7	4.904	50,000	4.905	0.173	0.658	13.368	97.1	4,170	13.08	0.401	880
		100,000	4.910	0.120	0.418	8.497	97.2	7,548	15.13	0.208	1,553
		200,000	4.909	0.083	0.276	5.623	97.9	14,031	16.93	0.120	1,881
		500,000	4.908	0.052	0.166	3.378	97.7	32,139	18.77	0.063	1,883
		1,000,000	4.905	0.038	0.113	2.304	97.8	62,625	19.47	0.033	1,885
0.9	10.397	50,000	10.431	0.416	1.784	17.005	96.5	4,437	11.95	1.289	628
		100,000	10.428	0.288	1.108	10.592	96.8	8,369	13.37	0.667	667
		200,000	10.415	0.206	0.702	6.730	96.7	15,466	15.04	0.367	673
		500,000	10.408	0.129	0.404	3.880	96.6	34,019	17.68	0.162	676
		1,000,000	10.400	0.094	0.277	2.661	96.7	64,571	18.84	0.099	677
0.95	13.863	50,000	13.922	0.638	3.064	21.803	96.6	4,585	11.37	2.302	604
		100,000	13.914	0.442	1.916	13.691	97.0	8,806	12.35	1.306	612
		200,000	13.886	0.321	1.140	8.186	96.8	16,442	13.82	0.650	613
		500,000	13.879	0.199	0.641	4.615	96.4	35,872	16.70	0.293	616
		1,000,000	13.868	0.146	0.435	3.137	96.3	66,674	18.22	0.163	617
0.99	21.910	50,000	22.107	1.607	6.700	29.648	94.9	4,827	10.43	4.864	602
		100,000	22.061	1.129	5.151	23.043	95.4	9,537	10.74	3.801	607
		200,000	21.972	0.792	3.546	16.019	95.7	18,488	11.45	2.708	608
		500,000	21.949	0.498	1.794	8.152	96.1	42,812	12.93	1.103	610
		1,000,000	21.918	0.344	1.209	5.509	96.3	79,373	14.65	0.670	611
0.995	25.376	50,000	25.630	2.317	8.272	31.061	93.3	4,888	10.20	6.109	599
		100,000	25.581	1.614	6.503	24.989	93.7	9,711	10.43	4.507	603
		200,000	25.470	1.143	5.137	19.937	95.2	19,066	10.80	3.833	604
		500,000	25.435	0.714	2.946	11.532	95.4	45,271	11.88	2.147	605
		1,000,000	25.388	0.492	1.895	7.441	95.6	85,392	13.04	1.212	607

5 CONCLUSIONS

In this article, we presented FQUEST, a fully automated fixed-sample-size procedure for computing point estimators and CIs for steady-state quantiles. Initial experimentation based on the process generated by successive customer waiting times in a heavily initialized M/M/1 system revealed that FQUEST provided CI coverage probabilities very close to the nominal level. This feat is remarkable, considering that the state-of-the-art sequential methods Sequest and SQSTS typically required substantial sample sizes for the same processes under no CI precision requirement (Alexopoulos et al. 2019; Lolos et al. 2023). Future work includes: (i) fixed-sample-size methods for simultaneous estimation of multiple quantiles; (ii) expansion of the experimental test bed with additional processes; and (iii) identification of alternative weight functions for computing STS area variance estimators.

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