Proceedings of the 2024 Winter Simulation Conference
H. Lam, E. Azar, D. Batur, S. Gao, W. Xie, S. R. Hunter, and M. D. Rossetti, eds.

TESTING FACILITY LOCATION WITH CONSTRAINED QUEUE TIME PROBLEM: A CASE STUDY IN FLORIDA, USA

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ABSTRACT

This study addresses the Testing Facility Location with Constrained Queue Time Problem. This optimization problem focuses on determining the best places to deploy testing sites and their available testers for infectious diseases, while constraining the maximum time in the queue with a given probability. An integer programming model is introduced and applied to the three biggest counties, in terms of population, of Florida, United States. Moreover, the Monte Carlo method is used to evaluate the model's output, aiming to check if the queueing time constraint is being satisfied. Through the experiments, a testing facility deployment plan can be determined for each county and further validated by the simulation. The results show that the solutions returned by the model behaved successfully when submitted to the Monte Carlo method, not exceeding the time in the queue in more than the predefined probability.

1 INTRODUCTION

In an infectious disease outbreak scenario, the identification of a contaminated individual must be done as soon as possible, aiming to break the spreading of the disease (World Health Organization 2022). Therefore, testing procedures play a vital role in the process (Ricks et al. 2021), and testing sites must be carefully planned and deployed. Failing to do so may lead to crowded places and individuals traveling long distances to take a test, risking infecting or being infected by others. However, budget constraints usually apply (Ricks et al. 2021), making it impossible to deploy testing sites in every neighborhood, for example. Thus, it is imperative to find a balance between distance and costs when planning the testing plan. This problem can be modeled as a variant of a class of optimization problems known as Facility Location, where there are candidate places to receive facilities, that in turn must cover (totally or not) the existing demand (Ahmadi-Javid et al. 2017). This class of optimization problems has been extensively researched from decades ago (Church and Revelle 1974; Hakimi 1964), and its application to the health field has increased even more in and after the Novel Coronavirus Disease (COVID-19) pandemic. From determining the places to deploy field hospitals (Hassan et al. 2021) to choosing pharmacies to be used as testing sites (Risanger et al. 2021), different approaches used the Facility Location problem as a base.

Nevertheless, the regular implementation of this optimization problem would not cover all the main issues a testing plan for infectious disease should cover. Only the total demand would be considered, which could lead to long waiting times and overcrowded testing sites, raising the risk of the spread of the disease to a non-contaminated individual waiting for a test. To approach this side of the problem, queue theory techniques can be used (Marianov and Serra 1998; Marianov and Serra 2002).

This work applies the Testing Facility Location with Constrained Queue Time Problem (Monteiro Júnior and González 2023) to the three biggest counties of Florida, USA. This problem aims to determine the best places to deploy testing facilities for infectious diseases while constraining the waiting time in the queue. In addition, the solution returned by the introduced integer programming model is used as an input to a Monte Carlo simulation to evaluate if the maximum waiting time constraint is being satisfied.

Moreover, additional experiments are executed with different maximum waiting times, to assess its impact on the results. The obtained results indicate that, in all scenarios, the probability of staying more than the defined time in the queue remained under the proposed limit, therefore validating the outputs of the integer programming model.

The remainder of this paper is structured as follows: Section 2 introduces and details the problem under investigation. Section 3 describes the methodology employed, while Section 4 shows the computational experiments and elaborates their outcomes. Finally, Section 5 concludes the work and suggests topics for future research.

2 PROBLEM DEFINITION

This section introduces the Testing Facility Location with Constrained Queue Time Problem (TFL-CQTP) and the definition of each of its components. Proposed by Monteiro Júnior and González (2023), TFL-CQTP states that, given an area of interest, like a city, composed of a set of locations, each with a demand to be satisfied, and a set of candidate places, one must find the best places to deploy testing facilities, considering that all the demands must be covered. Furthermore, every opened facility must have a service contract stating the number of testers working there. In addition, with a probability of at least α, an individual should not wait more than a predefined time in the queue to take a test.

From a mathematical point of view, the problem data can be represented as a graph $G = (V,E)$, where the set of vertexes $V = B \cup F$ is composed of two disjoint sets (which means that $B \cap F = \emptyset$). Those two sets B and F represent the set of locations to be attended and the set of candidate places to receive a facility, respectively. The set of edges, E, is defined by the cartesian product of B and F. To facilitate the understanding of the model, one may define the subset $F_i \subseteq F$, which represents the set of facilities that can be used to attend a given location $i \in B$. As every facility needs testers, one may define set C as the set of all possible team of testers contracts available. Also, let $C_j \subseteq C$ be the set of all possible team of testers contracts available for the candidate place $j \in F$, if one decides to deploy a facility there.

Having defined these elements, an integer programming model can be defined as:

$$\min \quad \sum_{j \in F} \sum_{c \in C_j} p_{jc} y_j^c + \sum_{i \in B} \sum_{j \in F_i} d_{ij} x_{ij} \tag{1}$$

$$\text{s.t.} \quad \sum_{j \in F_i} x_{ij} = 1 \qquad \forall i \in B \tag{2}$$

$$x_{ij} \leq \sum_{c \in C_j} y_j^c \qquad \forall i \in B, j \in F_i \tag{3}$$

$$\sum_{c \in C_j} y_j^c \leq 1 \qquad \forall j \in F \tag{4}$$

$$\sum_{i \in B_j} f_i x_{ij} \leq \sum_{c \in C_j} y_j^c \lambda_{m(c)}^{\alpha j} \qquad \forall j \in F \tag{5}$$

$$y_j^c \in \{0,1\} \qquad \forall j \in F, c \in C_j \tag{6}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in B, j \in F_i \tag{7}$$

where the parameters p_{jc} represent the cost to deploy facility j with contract c, while y_j^c refers to a binary variable indicating whether or not facility j is deployed with contract c. If yes, it receives the value 1, receiving 0 otherwise. The parameters d_{ij} represent the distance between location i and facility j, while x_{ij} is a binary variable indicating if the location i is being served by facility j. In a positive case, it receives 1, receiving 0 otherwise. The parameters f_i represent the demand of the location i, while $\lambda_{m(c)}^{\alpha j}$ refers to the capacity of facility j with m servers from contract c and a minimum probability α of respecting the maximum waiting time in the queue.

Its Objective Function aims to minimize the sum of the total costs of the opened facilities and also the sum of the distances individuals should travel to take a test. In the family of Constraints (2), we ensure that each location will be served by only one facility, while the family of Constraints (3) states that a location can only be linked to a facility if the facility is open, with an active contract. In turn, the family of Constraints (4) ensures that a facility must be deployed with at most one active contract, while the family of Constraints (5) states that the sum of the demands related to a facility can be at most equal to the total service capacity of the given facility. Finally, the families of Constraints (6) and (7) define the domain of the decision variables, where both of them can only receive binary values.

The total service capacity of a facility is related to the number of testers hired by it, which is represented in the model as a service contract. Each contract has several testers, varying from one to a maximum predefined limit. This number composes the facility costs, represented by the parameters p_{jc} in Expression (1). The other element, besides personnel costs, is an opening cost if some preparation is needed before operating. Regarding the calculation of the demand, represented by f_i in the family of Constraints (5), the value is a predefined fraction of the total population of each location, like a neighborhood. This represents an expected arrival rate from that location in the testing facility.

2.1 Queue Calculation

The family of Constraints (5) refers to the maximum waiting time in the queue. These constraints state that an individual can not wait in the queue for more than τ units of time, with a probability of at least α. Therefore, the probability of waiting in a queue at a facility j for at most τ minutes ($P(w_j \leq \tau)$) must be at least α, or $P[w_j \leq \tau] \geq \alpha$. This approach is based on the method proposed by Marianov and Serra (2002), which introduces a mathematical model to locate facilities using queue constraints with a variable number of servers. Their work transforms a probabilistic constraint, like the one defined by Expression (8), based on the quality of service, denoted by α. As the authors indicate, this is important because the usual capacity constraints, like restricting the number of individuals linked to a facility, can not be used to control the quality of the service. This happens because they are not created considering the probabilistic nature of the problem.

$$P[w_j \leq \tau] \geq \alpha \qquad \forall j \in F \tag{8}$$

However, this constraint can not be used in a model in this current form. The authors then derive a constraint based on the quality of service in each facility, adjusting the quality by varying the parameters τ and α. Giving that the demands are defined by a Poisson process with intensity f_i, and the process in each facility is also a Poisson process with intensity λ_i (referring to the sum of the demands of all locations served by that facility), at each facility we have a M/M/m queuing system (Marianov and Serra 2002). Then, one can use the cumulative distribution function of the waiting time of a M/M/m queue, as denoted by Expression (9):

$$P(w_j \geq \tau) = e^{-\mu\tau}\left[1 + \frac{p_0\rho^m}{m!(1-\rho/m)}\left(\frac{1-e^{-\mu\tau(m-1-\rho)}}{m-1-\rho}\right)\right] \forall j \in F \tag{9}$$

As said before, τ is the maximum waiting time in the queue, that can not be exceeded in at least α percent of the time, and p_0 is the probability that there are no individuals in the system. The variable ρ, which can also be represented as $\lambda/\mu m$, represents the utilization factor of the facility, being μ its service rate. If the expression for p_0 is plugged into the expression, the resulting expression will only depend on m, ρ, μ and τ. Then, $1-P(w_j \geq \tau)$ must be greater than or equal α, or $P(w_j \geq \tau) \leq 1-\alpha$, resulting in Expression (10):

$$e^{-\mu\tau}\left[1 + \frac{1}{1+\sum_{j=0}^{m-1}(m!/j!)(1-\rho/m)1/\rho^{m-j}}\left(\frac{1-e^{-\mu\tau(m-1-\rho)}}{m-1-\rho}\right)\right] \leq 1-\alpha \tag{10}$$

Despite this inequality being nonlinear, its left-hand side represents the cumulative waiting time. Then, considering that $\lambda/\mu m$ must be less than or equal to 1, the cumulative waiting time must be a strictly increasing function of λ. Therefore, one can compute a parameter λ_α, so for values of $\lambda \leq \lambda_\alpha$ the equation holds (Marianov and Serra 2002). To obtain the value of λ that makes the presented inequality hold as an equation, any numeric root-finding technique can be used, like Newton methods (Marianov and Serra 1998).

3 METHODOLOGY

This section outlines the methodology used in this study. A set of instances representing the regions under evaluation was created and the TFL-CQTP was applied to each of them using a branch-and-cut algorithm. This allows an analysis of which candidate location must receive a testing facility and which demand each facility must serve. Moreover, the total cost of the operation, the number of testers hired, as well as information about the travel distance can be evaluated. This data can then be compared, to point out similarities and differences between each region under investigation. In addition, the outcomes of the integer programming model were used as the input to a simulation using the Monte Carlo method. This procedure aims to evaluate the results and their behavior, to verify if the waiting time constraint presented in the model is being satisfied by the model output. To do so, queue simulations were conducted to assess the probability of the queue waiting time not exceeding pre-fixed time within a 95% confidence interval, the same used in the integer programming model. These simulations were conducted for each facility identified by the model as capable of efficiently serving the population in an infectious disease testing program. Results and discussions will be presented in Section 4.3.4.

4 COMPUTATIONAL EXPERIMENTS AND RESULTS

This section details the computational experiments executed using the methodology described in the previous section. First, in Section 4.1, the instances created and the creation process are explained. Next, in Section 4.2, the study case parameters are introduced, followed by the computational results of the integer programming model, in Section 4.3, and the simulation, in Section 4.3.4.

4.1 Instances

The case study evaluated in this work is the state of Florida, in the southeast of the United States. Leader in the ranking of the most visited states of the United States by overseas visitors (National Travel and Tourism Office 2023), it is of special attention when dealing with a contagious disease threat, due to the high number of people entering and leaving the state. The three most populated counties, Miami-Dade, Broward and Palm Beach, were chosen as the evaluated regions, and their data are introduced in Table 1, sorted by total population from highest to lowest.

The population and geographic data were acquired from the official 2020 census website (US Census Bureau 2020a), while the healthcare units came from the Health Resources and Services Administration website (US Health Resources and Services Administration 2024). A brief introduction to the geographic hierarchy used in the census is presented in Section 4.1.1, while its processing procedures are explained in Section 4.1.2.

Table 1: General data from the three evaluated instances.

Instance	Total Population (2020 Census)	Census Tracts	Candidate Locations
Miami-Dade	2,701,767	707	244
Broward	1,944,375	417	27
Palm Beach	1,492,191	373	34

4.1.1 Census Geographic Division

The United States Census Bureau uses a hierarchical structure to define the geographic entities and their relationship. Starting from the National level, the subdivisions go up to the minimal geographic entity known as a Census Block, composed of an area bounded by visible features, such as roads and rivers, and invisible ones, like administrative limits (US Census Bureau 2020b). Figure 1 shows each hierarchy level.

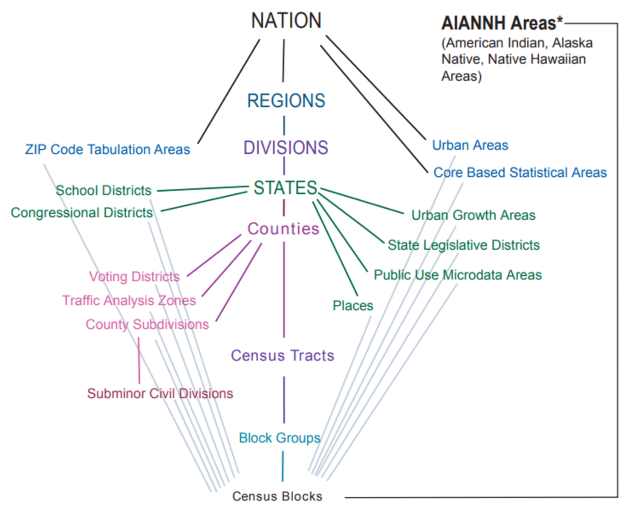

Figure 1: Census hierarchy, as defined by the United States Census Bureau. Source: census.gov

In this work, we considered as the smallest geographic entity the one known as a Census Tract, an area limited by visible or invisible features, composed of a set of Census Blocks. They usually have a population between 1,200 and 8,000, with an optimum size of 4,000 people. As it is highly dependent on the population size, their spatial size can vary, based on the populational density of the area. The Census Tracts boundaries are designed to last over the years, to allow the comparison between censuses. However, it can be split or merged into other census tracts, in case of population growth or declination (US Census Bureau 2020b). This relatively permanent structure justifies the choice of it being the smallest entity in this work, when talking about a population area, as the same procedure could be used over time, with only minor changes, if applicable. Figure 2 shows the maps of the three counties considered in this study, with their census tract subdivisions.

4.1.2 Data Processing

After determining the evaluated regions, the set of candidate places to deploy testing facilities must be defined. In this work, the set of candidate places was obtained by a tool made available by the Health Resources and Services Administration, a government agency aimed to provide equitable access to healthcare services (https://findahealthcenter.hrsa.gov). Every facility inside the county was considered as being a candidate place. Regarding the costs, as the considered facilities were already listed as healthcare units, no opening costs were considered, and the tester's wage will be the only cost of each facility.

The next step in the creation of the instances is to calculate the distance between candidate places and the census tracts. Concerning the candidate locations, their geographic position can be considered with no additional processing. However, the census tracts must have a reference point because, right now, only their area is available. In this work, the following procedure defines the reference point of each census tract: the most populous census block is identified, and its geometric centroid is calculated. Then, the shortest distance between each census tract reference and candidate facility pair is calculated, using the real road network of the region. This is possible by the use of an open-source project called Project OSRM, which

Figure 2: Each county considered in this work, Miami-Dade (1), Broward (2) and Palm Beach (3), with their census tract subdivision.

stands for Open Source Routing Machine (https://project-osrm.org/). This project uses data from the open and collaborative mapping project called OpenStreetMap (https://www.openstreetmap.org/). This approach allows for a more precise calculation of the distances, instead of approximation procedures, like using the Euclidean distance. Figure 3 shows the Broward County map as an example, with all paths from one of the census tracts reference points to every candidate location to receive a testing facility.

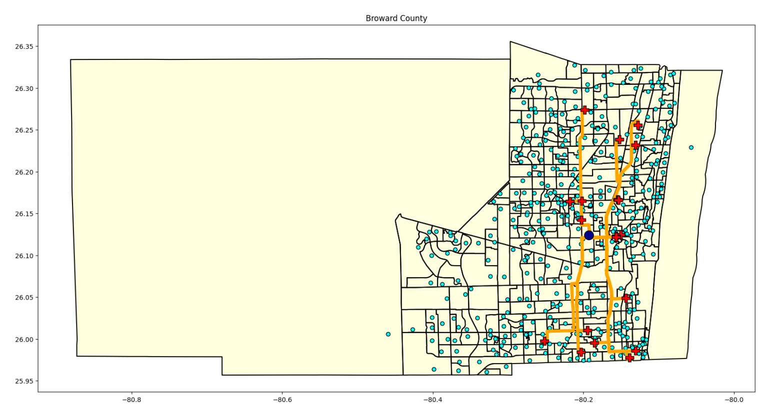

Figure 3: Paths from a census tract reference point to each candidate location in Broward County. The cyan dots represent the reference point of each census tract. One of them is highlighted, in blue. The red crosses represent the candidate places and the orange lines represent the paths to them, departing from the highlighted reference point, using the real road network. The black lines represent the limit of each census tract.

4.2 Experiments Premises

In this study, the value of the wage of each tester is set to \$6,659.17. This is the mean monthly wage of a nurse in Florida, according to the Bureau of Labor Statistics of the United States (US Bureau of Labor Statistics 2022). Therefore, in contracts with one tester, the cost of the contract will be \$6,659.17 and, with 10 testers, \$66,591.70. Regarding the number of testers per contract, the maximum value is set to 10, as physical constraints could make it harder to have more than 10 testers serving at the same time in a testing facility.

Another main point in the experiments is the definition of the demand of each census tract. In this study, the instances considered, for each census tract, an hourly demand value of 0.2% of the total population of each census tract. This value is justified by being an average value to complete a testing campaign in about one month, reaching 100% of the population, in 8-hour shifts.

After defining the contracts and the demands, the quality of service and the queue parameters also must be determined. In this research, the quality of service, denoted by α, was set to 95%. This parameter enforces the minimum probability of an individual not waiting in the queue for more than a predefined number of minutes. This maximum time in the queue, denoted by τ, in turn, was set to 15 minutes in the base scenario, with alternative evaluations with τ = 10 and τ = 20. For the service parameters, this study considered that a tester can serve an individual in a mean of 2 minutes. Therefore, the hourly service rate, μ, was set to a mean of 30 individuals per hour.

4.3 Computational Results

In this section, the experiments using the integer programming model introduced in Section 2 were executed. It is important to highlight that, given that its objective function evaluates values from different magnitudes (costs and distances), a normalization step is done in both values before the data is sent to the model. The experiments were run on a computer with an AMD Ryzen^{TM} 5 5600G 3.9 GHz processor and 32GB RAM, running Windows 11 and a Python 3.10 environment. The solver used was the IBM CPLEX 22.1.1, with the DOCplex Python library.

First, Table 2 and Table 3 present the outcomes of the experiments on the three regions in the three scenarios, and the general results are briefly discussed. Next, the results of each region are detailed, in Sections 4.3.1 to 4.3.3.

Table 2: Results from all the three evaluated instances (Used Facilities and Testers Hired).

Instance	Used Facilities (% from Total)			Testers Hired		
	$\tau = 10$	$\tau = 15$	$\tau = 20$	$\tau = 10$	$\tau = 15$	$\tau = 20$
Miami-Dade	33 (14.35%)	33 (14.35%)	33 (14.35%)	216	212	208
Broward	15 (55.56%)	15 (55.56%)	15 (55.56%)	147	144	142
Palm Beach	14 (41.18%)	13 (38.23%)	13 (38.23%)	116	112	112

Table 3: Results from all the three evaluated instances (Used Capacity and Costs).

Instance	Used Capacity			Costs		
	$\tau = 10$	$\tau = 15$	$\tau = 20$	$\tau = 10$	$\tau = 15$	$\tau = 20$
Miami-Dade	96.53%	96.10%	96.38%	$1,438,380.72	$1,411,744.04	$1,385,107.36
Broward	98.40%	98.69%	99.07%	$978,897.99	$958,920.48	$945,602.14
Palm Beach	96.91%	98.38%	96.98%	$772,463.72	$745,827.04	$745,827.04

Being the county with the biggest population, Miami-Dade needed more facilities than the other two counties combined, in all scenarios. However, it had the lowest used/total facilities rate, because the number of candidates was almost ten times bigger than Broward and Palm Beach. In terms of hired people, as expected, Miami-Dade also was the first in the rank. At the same time, it was in the last place if we consider the percentage of used capacity. This can be explained by the fact that it was less costly to open a new facility and assign a tester to it than to make an individual travel long distances. The map in Section 4.3.1 shows this situation, where there are clusters of census tracts around an opened facility, without long-distance trips. On the other hand, Broward had the highest usage rate in all scenarios, due to the number of candidates being the lowest of the three counties. Palm Beach, being the smallest of the three, had compatible results, with the smallest costs and opened facilities values, only with a slightly higher usage rate than Miami-Dade in 10 and 20-minute scenarios, but achieving a higher result in the base scenario, of almost 2%.

Additionally, the cost analysis for the time variant from 15 minutes to 10 minutes allows us to conclude that it is possible to achieve a 33% improvement in expected waiting time, with a cost increase ranging between $19,977.51 and $26,636.68, depending on the location.

In addition to the data related to the facilities, another analysis can be done on the travel distances data. The total traveled distance of Miami-Dade, the biggest county in total area, was the smallest of the three instances, in all scenarios. On the other hand, the smallest county in total area, Broward, had the biggest travel distance. This can be explained by the number of candidate places in the county, the smallest of the three instances. This forces more lengthy trips to accommodate some demands, as can be seen in the map in Section 4.3.2. In the next sections, all evaluated data is detailed, for each region under investigation.

4.3.1 Miami-Dade County

The largest of the three instances, Miami-Dade County would need only 33 out of 240 possible facilities to handle all the demand, regardless of the maximum waiting time. This represents only 14.35% of them. In terms of testers hired, it would need 212 professionals in the base scenario, resulting in a mean of 6.42 testers per facility and a total cost of $1,411,744.04. When reducing the waiting time to 10 minutes, the number of testers needed increased by only four units, or 1.89%, with a mean of 6.54 testers per facility. On the other hand, when relaxing the time to 20 minutes, the number of testers decreased in the same proportion, with a mean of 6.3 testers.

As discussed before, this county had a grouping of the served census tracts around near facilities, resulting in the smallest usage rate in all scenarios. In absolute terms, each facility served an average of 21.42 census tracts. Figure 4 shows the outcomes of the model in the map, in the base scenario. Each facility is marked with a cross and receives a color, also given to each census tract centroid served by such facility. The same color is given as well to the paths between them.

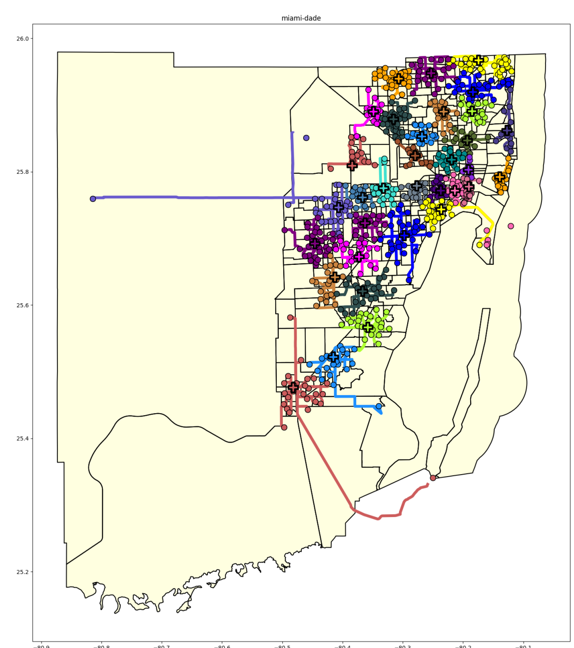

Figure 4: Miami-Dade County results in the base scenario, with 15 minutes of maximum waiting time, with the colors showing the census tract served by each opened facility.

4.3.2 Broward County

In the Broward County instance, 15 testing facilities would suffice to successfully serve the population in the case of a contagious disease testing program, in all scenarios. This number represents 55.56% of the total number of mapped candidates, 27. Regarding the number of testers hired, the county would need 144 testers in the base scenario, with a mean of 9.6 testers per facility and a total cost of $958,920.48. When reducing the waiting time to 10 minutes, the number of testers needed increased by three units, or 2.08%, with a mean of 9.8 testers per facility. Conversely, the number of testers had a small reduction, of 2 units, or 1.39%, when increasing the waiting time to 20 minutes, with a mean of 9.47 testers per facility.

Concerning the census tracts served by each facility, there is a mean of 27.8 per facility, with a mean usage rate of up to 99.07%. The county/facility association for the base scenario can be seen in Figure 5.

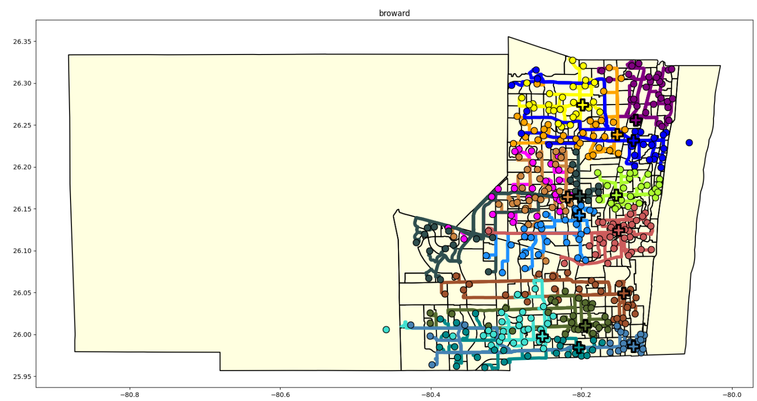

Figure 5: Broward County results in the base scenario, with the colors showing the census tract served by each opened facility.

4.3.3 Palm Beach County

The results for Palm Beach were compatible with the size of the instance. Being the smallest of the three evaluated counties, it used the lowest number of facilities, 13, in the base and the 20-minute scenarios. However, in the 10-minute scenario, the number of facilities increased by one unit. This indicates that, in the base scenario, the service rate was near the limit to admit more demand without making an individual take long trips. This number represents 38.23% of the candidates in scenarios with 15 and 20 minutes and 41.18% in the 10-minute scenario. Regarding the average of the census tracts served by a facility, this region had the highest of all instances: 28.69 in the 15 and 20-minute waiting time scenarios. When tightening this limit to 10, the average number of census tracts served by a facility becomes 26.64. This is explained by the fact that, with a smaller waiting time, the demand a facility can serve in a unit of time also decreases, and so does the number of census tracts that can be served, if the facility is already operating at full capacity.

Similar to the other regions, the number of testers hired was related to the defined waiting time, but with a slight difference. With 15 minutes, the county would need 112 testers to handle the demand, with a mean of 8.61 per facility, with a total cost of $745,827.04. When decreasing the time to 10 minutes, this number increased by four units or 3.57%. However, unlike the other counties, when increasing the time to 20 minutes, the number of testers stayed the same. This can be explained by the fact that the gap generated by the increase in the service capacity per hour was not big enough to reduce the number of servers and/or reallocate a tract to a more interesting facility, for example. This can be analyzed by its average usage rate, which decreased from the 15 to the 20-minute scenario, from 98.38% to 96.98%, keeping all the associations between opened facilities and census tracts present in the base scenario. Figure 6 displays the outcome of the base scenario in the map.

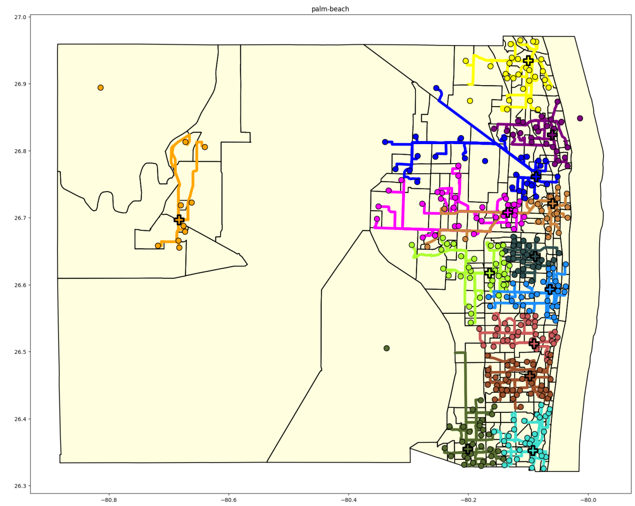

Figure 6: Palm Beach County results in the base scenario, with the colors showing the census tract served by each opened facility.

4.3.4 Simulation Results

In this subsection, our goal is to validate that the descriptive parameters of each facility can generate scenarios where queue wait times exceeding a predefined time limit τ occur in at most 5% of cases. To evaluate the simulations, we established a base case using a 15-minute time limit ($\tau = 15$ minutes) as the primary reference for the analysis. Additionally, to explore other types of behavior, we generated some alternative scenarios.

We assumed that the facilities would operate for 8 hours per day and the projected results considered the operation over 100,000 days. The simulation parameters are determined according to the experimental premises outlined in Section 4.2. Since the evaluation of queue time analysis is independent of the region, the tests were conducted considering facilities from all counties grouped together.

Consider a feasible solution $s = \langle \bar{y}, \bar{x} \rangle$ to the TFL-CQTP. With that, let $n = \sum_{j \in F} \bar{y}_j$ denote the number of facilities deployed, and define $\bar{F} = \{ j \in F \mid \bar{y}_j = 1 \}$ as the set of facilities that are deployed in the solution s. For each facility $j \in \bar{F}$, we stored the variable $\xi_j = P(w_j \geq \tau)$, which denotes the probability of an individual experiencing a queue time surpassing τ.

The M/M/m queue represents a model context where 'm' servers handle tasks arriving following a Poisson process (where the time intervals between arrivals are exponentially distributed), and service times are also exponentially distributed (the M denotes a Markov process). Therefore, at each simulation, we generated random values to represent both the arrival intervals between individuals and the service times, adhering to the respective distribution parameters ($\frac{1}{\lambda}$ and $\frac{1}{\mu}$ respectively).

For the base case, the integer programming model indicated the deployment of 61 facilities for the combined area of the 3 counties. As a result, the 'Riviera Middle School' facility in Miami-Dade County exhibited the highest probability of queue wait times exceeding τ, with $\xi_{max} = \max_{j \in \bar{F}} \xi_j = 2.49\%$, which is notably lower than the $\alpha = 5\%$ tolerance threshold.

Additionally, upon averaging the individual probabilities ξ_j obtained in each configuration, we derived $\xi_{avg} = \dfrac{\sum_{j \in \bar{F}} \xi_j}{n} = 0.99\%$. It's noteworthy that the low average of the probabilities aligns with the entry rate considered in most facilities being lower than the maximum supported service capacity of the system.

Furthermore, we conducted an 'entry rate stress test', where for each facility j, we adjusted the average entry rate to match the facility capacity $\lambda^{\alpha j}_{m(c)}$, as referenced in Section 2. As expected, we observed a general increase in the values of ξ_j obtained for each facility, and the highest probability was $\xi_{max} = 3.67\%$, which remained within the 5% tolerance threshold. Meanwhile, considering the average of all probabilities, ξ_{avg}, increased to 1.99%. This implies that even when assessing the extreme scenario of server saturation predicted by the integer programming model, the simulation results indicate that the desired 95% confidence interval was maintained for all facilities.

Finally, we conducted supplementary simulations across scenarios characterized by varying time limits (τ) by 5 minutes, similar to the previous section. The aim was to assess their minimal impact on the results. Regarding the hourly service rate, variations in the waiting time did not alter the values obtained in the base scenario. Table 4 provides a comparison involving both increasing and decreasing the time limit.

Table 4: Comparative results for alternative scenarios.

	$\tau = 10$	$\tau = 15$	$\tau = 20$
ξ_{avg}	1.85%	0.99%	0.84%
ξ_{max}	3.29%	2.49%	2.46%

The tabulated values (ξ_{avg} and ξ_{max}) respectively represent the average and maximum probabilities of queue wait times surpassing the respective time limits as determined in the simulations. Despite the reduction of the time limit from 15 to 10 minutes leading to increases in both average and maximum probabilities, these statistics remained notably below the maximum tolerated threshold of 5%. Conversely, extending the time limit to 20 minutes led to marginal changes in these probabilities. In summary, as expected, Table 4 demonstrates that queue wait time probabilities were affected by variations in time limit constraints, but these variations in the probabilities were consistently below 5%, thus validating the model.

5 CONCLUSIONS AND FUTURE WORKS

This paper first applied the Testing Facility Location with Constrained Queue Time Problem to the three biggest counties, in terms of population, of Florida, USA. This problem, presented as an integer programming model, aims to define the best places to deploy testing facilities in the case of an infectious disease outbreak while constraining the maximum time in the queue. Further, a Monte Carlo simulation was employed, using the opened facilities output by the model, to validate the maximum waiting time in queue constraint.

The testing plans delineated by the integer programming model, especially when comparing the different values of τ, made it possible to achieve interesting insights, like the relatively small investment needed to improve 33% the expected waiting time, taking as a starting point the base scenario.

Additional scenarios were also assessed, including maximum allowable wait times of 10 and 20 minutes at the 95% confidence level. The results indicated small deviations from the base scenario, which were successfully validated by the simulation. The 'entry rate stress test', obtained by adjusting the entry rate to match each facility capacity ($\lambda = \lambda_\alpha$), was also simulated. Even under this extreme scenario, the simulation results indicate that the desired 95% confidence interval was maintained for all facilities.

For future works, an interesting path is the addition of other queue types, like priority queues, and evaluate their impact on the results (Wang, Baron, and Scheller-Wolf 2015).

ACKNOWLEDGMENTS

The authors thank CNPq, FAPERJ, and CAPES for partially funding this research.

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