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FINDING FEASIBLE SYSTEMS IN THE PRESENCE OF A PROBABILITY CONSTRAINT

Taehoon Kim[1], Sigrún Andradóttir[1], Seong-Hee Kim[1], and Yuwei Zhou[2]

[1]School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA, USA
[2]Kelley School of Business, Indiana University, Bloomington, IN, USA

ABSTRACT

We consider the problem of determining feasible systems among a finite set of simulated alternatives with respect to a probability constraint, where observations from stochastic simulations are Bernoulli distributed. Most statistically valid procedures for feasibility determination consider constraints on the means of normally distributed observations. When observations are Bernoulli distributed, one can still use the existing procedures by treating batch means of Bernoulli observations as basic observations. However, achieving approximate normality may require a large batch size, which can lead to unnecessary waste of observations in reaching a decision. This paper proposes a procedure that utilizes Bernoulli-distributed observations to perform feasibility checks. We demonstrate that when the observations are Bernoulli distributed, our procedure outperforms an existing feasibility determination procedure that was developed for a constraint on normally distributed observations.

1 INTRODUCTION

We consider the problem of identifying feasible systems among a finite number of simulated alternatives when observations are Bernoulli distributed. This problem occurs when a decision-maker considers a constraint on probabilities. For example, in a military operation, a commander may set the criterion of operation success as eliminating at least 70% of the enemy forces. The commander would aim to minimize the probability of operation failure. In this case, the commander can consider a constraint that the probability of eliminating at least 70% of the enemy forces by the end of the operation is less than or equal to $h = 1\%$. Such scenarios are not limited to military operations but frequently arise in manufacturing and service systems. For example, in an (s, S) inventory policy, a decision maker may want to identify inventory policies whose probability of yearly total cost exceeding 1.4 million dollars is no more than $h = 1\%$. As one can see from these two examples, we have a stochastic constraint in the sense that the probability needs to be estimated based on stochastic observations. Furthermore, basic observations are Bernoulli distributed with 1 (an event of interest occurs) or 0 (an event of interest does not occur).

Feasibility determination is a branch in the field of ranking and selection (R&S). R&S procedures have primarily been used to find a system with the best performance measure among a finite number of simulated systems where the definition of the best depends on the problem at hand. Kim and Nelson (2006b) and Hong et al. (2015) discuss four selection problems in simulation studies: selection of the best, comparison with a standard, multinomial selection, and Bernoulli selection. Among the four problems, the selection of the best is studied the most. In the selection of the best, observations are usually assumed to be normally distributed, and several approaches have been developed. For example, Nelson et al. (2001) and Kim and Nelson (2001) consider the fully-sequential indifference zone (IZ) approach with an IZ parameter $\delta > 0$, which is the smallest difference worth detecting between two systems. Chen et al. (2000) and Lee et al. (2010) propose optimal computing budget allocation (OCBA) procedures, while Frazier and Powell (2008) and Xie and Frazier (2013) employ the Bayesian approach.

If the decision maker considers finding a system with the largest or smallest probability of an event, Bernoulli selection can be used. Sobel and Huyett (1957) propose a procedure for selecting the system with the highest probability of success using an IZ approach that utilizes the difference between probabilities. Bechhofer et al. (1968) present a method for solving the Bernoulli selection problem sequentially using a random walk model and an IZ approach that utilizes the odds-ratio between probabilities. Research on solving the Bernoulli selection problem using the random walk model is also conducted by Tamhane (1985), and Paulson (1994) improves performance by proposing a method that eliminates inferior systems during the procedure in Koopman-Darmois populations including Bernoulli observations. Wieland and Nelson (2004) discuss three types of indifference-zone formulations in Bernoulli selection experiments (i.e., absolute difference, odds-ratio, and relative risk) and solve the Bernoulli selection problem with a gambler's ruin problem (Ross 2014) using the odds-ratio IZ parameter, aiming to create a more efficient procedure by narrowing down the decision-making region for selection as the procedure progresses.

While the selection of the best deals with a single performance measure, constrained R&S considers optimizing a primary performance measure subject to constraints on secondary performance measures. Thus, in constrained R&S, both feasibility determination and comparison are required. Several approaches have been developed for solving constrained R&S. For example, Lee et al. (2012), Hunter and Pasupathy (2013), Pasupathy et al. (2014), and Gao and Chen (2017) propose sampling frameworks that approximate the optimal computing budget allocation while considering stochastic constraints. Among the procedures that use the IZ approach, Andradĺőttir and Kim (2010), Healey et al. (2013), and Healey et al. (2014) propose constrained R&S procedures that find the best feasible system, while Batur and Kim (2010) identify a set of feasible solutions in the presence of multiple constraints. Hong et al. (2015) propose statistically-valid procedures that solve constrained selection of the best problem with secondary performance measures satisfy probabilistic constraints. For the Bayesian approach, Xie and Frazier (2013) discuss a Bayes-optimal policy for determining a set of simulated solutions with mean performances exceeding a fixed threshold value.

In R&S for feasibility checks, constraints are usually imposed on the expectation of normally distributed data. To the best of our knowledge, although there exist methods that formulate the Bernoulli feasibility problem as a hypothesis test on a probability (e.g., Fleiss et al. (2003)), there does not exist a statistically-valid fully-sequential procedure that solves the Bernoulli feasibility problem (where the observations are Bernoulli random variables). Theoretically, when observations are Bernoulli distributed, one can still apply existing procedures for feasibility determination by treating batch means of Bernoulli distributed data as basic observations. However, it is well known that a large batch size could cause inefficiency with unnecessary waste of observations in reaching a decision, especially in fully sequential type procedures, as pointed out in Kim and Nelson (2006a). Our proposed procedure collects only one basic observation from the systems in contention at each stage, which is expected to reduce the overall simulation effort required to find the set of feasible systems since we are able to identify apparently infeasible systems early in the experimentation. In this paper, we develop a fully-sequential IZ procedure for checking the feasibility of systems when a constraint is placed on a probability, which is the expectation of Bernoulli-distributed data with outputs of 1 ("success") or 0 ("failure"). The contributions of this paper are as follows: we (i) discuss how the IZ parameter based on an odds-ratio is formulated in feasibility determination on a probability constraint, (ii) solve such feasibility determination problems, (iii) prove the statistical validity of the proposed procedure, and (iv) demonstrate that our procedure outperforms an existing feasibility determination procedure for a constraint with normal observations due to Andradĺőttir and Kim (2010).

The rest of this paper is organized as follows: Section 2 provides our problem and notation. Our procedure is given in Section 3. Experimental results are shown in Section 4, followed by concluding remarks in Section 5. A more detailed version of this paper is provided by Kim et al. (2024).

2 PROBLEM, NOTATION, AND CORRECT DECISION

In this section, we describe our problem and notation in Section 2.1 and then define the correct decision event in Section 2.2.

2.1 Problem

We consider stochastic and terminating simulations of k systems. Let $\Omega = \{1, \ldots, k\}$ denote the index set of all possible systems. Observations Y_{in} represent whether an event of interest occurs ($Y_{in} = 1$) or not ($Y_{in} = 0$) from the nth replication of the ith system where $i \in \Omega$ and $n = 1, 2, \ldots$. We denote the probability of system i as $p_i = \mathrm{E}[Y_{in}]$. Observations are assumed to satisfy the following assumption:

Assumption 1 For each $i \in \Omega$, Y_{in} are independent and identically distributed Bernoulli distributed random variables with probability p_i.

If we do not use common random numbers (CRN), then observations from different systems, i.e., Y_{in} and $Y_{i'n}$ for $i \neq i'$, are independent. Note that CRN is not recommended in Bernoulli selection (Kim and Nelson 2006b) or feasibility determination (Zhou et al. 2022). Therefore, we do not consider the case when CRN is applied in this paper.

2.2 Correct Decision

In Bernoulli selection, whose goal is to find a system with the largest success probability of an event among k systems, three types of IZ settings are considered (Wieland and Nelson 2004). Let p_i denote the success probability for system i for $i = 1, 2, \ldots, k$ and $p_k > p_{k-1} \geq \cdots \geq p_1$. Then, a statistically valid selection procedure guarantees the selection of system k with at least $1 - \alpha$ probability under one of the following three types of IZ settings:

- Difference: $p_k - p_{k-1} \geq \delta > 0$.
- Odds-ratio: $\frac{p_k}{(1-p_k)} / \frac{p_{k-1}}{(1-p_{k-1})} \geq \theta > 1$.
- Relative risk: $p_k/p_{k-1} \geq \theta > 1$.

The odds-ratio represents how the number of successes per failure in one system compares to the number of successes per failure in another system. According to Wieland and Nelson (2004), there are two advantages of using the odds-ratio. First, as the probability approaches 0 and 1, it amplifies the difference between the two probabilities. For instance, when $p_{k-1} = 0.9$ and $\theta = 1.2$, p_k should be more than 0.915 for system k to satisfy the odds-ratio IZ setting. On the other hand, with the same θ, if $p_{k-1} = 0.5$, the corresponding value of p_k should be more than 0.545. That is, a 1% difference between two probabilities close to 1 is considered as a larger difference than a 1% difference in two probabilities close to 50% in terms of the odds-ratio. The same argument applies to two probabilities close to 0. This is desirable as it tends to take more effort to further increase a probability close to 1 or decrease a probability close to 0. Therefore, when the performance measure is a probability, using the odds-ratio is more appropriate. Second, the odds ratio makes it possible to use the gambler's ruin approach, which does not require either an initial sample size for variance estimation or the normality of observations (thus no batching). For these reasons, we employ the odds-ratio IZ setting in this paper.

In this paper, an odds-ratio IZ parameter is denoted by $\theta > 1$ and its value is specified by the decision maker. For a constraint that the probability of system i is less than or equal to a threshold h, similar as in AndradÍőttir and Kim (2010), we introduce the sets of desirable, unacceptable, and acceptable systems due to the fact that it is nearly impossible to guarantee a correct feasibility decision in terms of a stochastic constraint. Depending on the user-specified odds-ratio IZ parameter, the three sets are defined as follows.

- Any system i that satisfies $\frac{h/(1-h)}{p_i/(1-p_i)} = \frac{(1-p_i)h}{p_i(1-h)} \geq \theta$ is considered desirable with respect to threshold h. The set of all desirable systems with respect to threshold h is denoted as D:

$$D = \left\{ i \in \Omega \;\middle|\; \frac{(1-p_i)\,h}{p_i\,(1-h)} \geq \theta \right\}.$$

For those systems, we are expected to declare them feasible to threshold h.

- Any system i with $\frac{p_i/(1-p_i)}{h/(1-h)} = \frac{p_i(1-h)}{(1-p_i)h} \geq \theta$ is considered unacceptable with respect to threshold h, placing them in set U:

$$U = \left\{ i \in \Omega \;\middle|\; \frac{p_i\,(1-h)}{(1-p_i)\,h} \geq \theta \right\}.$$

For those systems, we are expected to declare them infeasible to threshold h.

- The remaining systems are considered acceptable and are placed in set A:

$$A = \Omega \setminus (D \cup U)\,.$$

When performing feasibility check for system i, we use CD_i to denote a correct decision event, and it is an event such that system i is declared feasible if $i \in D$ and infeasible if $i \in U$. For systems in A, any decision is considered as a correct decision. Then, a statistically-valid procedure that determines the feasibility of the k systems should satisfy the following statement:

$$\mathrm{PCD} = \Pr\left(\cap_{i=1}^{k} \mathrm{CD}_i\right) \geq 1-\alpha, \tag{1}$$

where $1-\alpha$ is the nominal confidence level.

3 PROCEDURE

In this section, we present our proposed procedure, namely BerF. The BerF procedure employs a random-walk model to process Bernoulli data without the need to employ batch means and estimate system variance. We need some additional notation before presenting BerF. First, for the overall confidence level $1-\alpha$, the parameter β, which corresponds to the nominal probability of error for each system, is defined as follows:

$$\beta = 1-(1-\alpha)^{1/k}. \tag{2}$$

Then, H is defined as the smallest integer such that

$$\beta \geq \frac{1}{1+\theta^{H}}. \tag{3}$$

Finally, for $i \in \Omega$, we let $I_{i1}, I_{i2}, \ldots$ represent independent and identically distributed dummy Bernoulli data with probability h independent of $Y_{i1}, Y_{i2}, \ldots$.

The BerF procedure declares the feasibility of system i with respect to threshold h as

$$\begin{cases} \text{feasible} & \text{if } \sum_{n=1}^{r} (Y_{in} - I_{in}) \leq -H, \\ \text{infeasible} & \text{if } \sum_{n=1}^{r} (Y_{in} - I_{in}) \geq H, \end{cases} \tag{4}$$

where r denotes the number of observations collected so far. In other words, the feasibility of system i with respect to threshold h is determined by assessing whether $\sum_{n=1}^{r}(Y_{in} - I_{in})$ first reaches either $-H$ or H. Figure 1 shows a sample path where system i is declared feasible with respect to threshold h. A detailed description of the procedure is provided in Algorithm 1.

Theorem 1 proves the statistical validity of our proposed procedure. The proof is provided in Kim et al. (2024).

Theorem 1 For k systems with a probability constraint and threshold h, BerF guarantees $\mathrm{PCD} = \Pr(\cap_{i=1}^{k}\mathrm{CD}_i) \geq 1-\alpha$.

The main idea in the proof of Theorem 1 is to compare systems individually, with statistical guarantee of $1-\beta$, with a dummy system whose success probability is h and determine which system has a larger probability. Notice that whenever an observation Y_{ir} is collected, a dummy random variable I_{ir} is also collected, and $Y_{ir} - I_{ir}$ takes values in $\{-1, 0, 1\}$. Therefore, we are able to use the monitoring statistics $\sum_{n=1}^{r}(Y_{in} - I_{in})$ for $r = 1, 2, \ldots$ as states in a discrete-time Markov chain and model a state-change process as a random-walk process. The detailed proof is included in Kim et al. (2024).

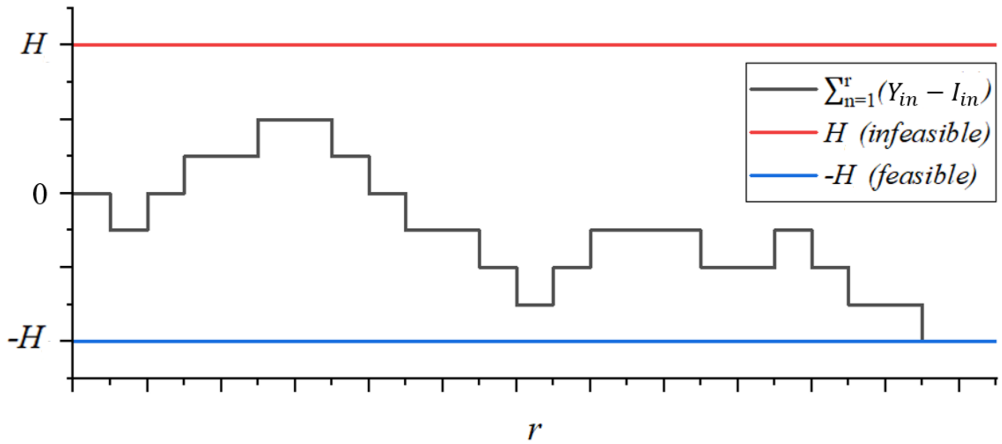

Figure 1: An example of a feasible decision with respect to the threshold h for BerF.

Algorithm 1 Procedure Bernoulli Feasibility, BerF

[**Setup**:] Choose confidence level $1-\alpha$, threshold h, and odds-ratio IZ parameter $\theta > 1$. Set $\Omega = \{1, 2, \ldots, k\}$ and H as the smallest integer such that Equation (3) holds, where β is determined as in Equation (2).

for each system $i \in \Omega$ **do**

[**Initialization**:]

Set $r = 1$ and $Z_i = -1$.

Generate Y_{ir} and $U_{ir} \sim U(0,1)$ independent of Y_{ir}.

[**Feasibility Check**:]

Set $I_{ir} = 1$ if $U_{ir} \leq h$ and 0 otherwise.

If $\sum_{n=1}^{r}(Y_{in} - I_{in}) \leq -H$, set $Z_i = 1$;

Else if $\sum_{n=1}^{r}(Y_{in} - I_{in}) \geq H$, set $Z_i = 0$.

[**Stopping Condition**:]

If $Z_i \neq -1$, return Z_i. Otherwise, set $r = r+1$, obtain Y_{ir} and U_{ir}, and go to [**Feasibility Check**].

end for

4 EXPERIMENTS

In this section, we demonstrate the performance of the proposed procedure BerF when a single system is considered (i.e., $k = 1$). We compare the performance of BerF with $\mathcal{F}$ due to Andradĺőttir and Kim (2010). As discussed in Section 1, we use batch means of the data as basic observations for $\mathcal{F}$ since $\mathcal{F}$ is designed for feasibility checks with respect to normally distributed data.

We set the true probability of the single system p_1 to either 0.01 or 0.15. We consider two values for the odds-ratio, i.e., $\theta \in \{1.2, 1.5\}$. We choose the threshold under the so-called *slippage configuration* (SC) and non-SC. Note that the SC corresponds to the most difficult case to solve in the odds-ratio IZ setting (Bechhofer and Goldsman (1986)) where the mean of the system falls exactly on the boundary of the desirable or unacceptable set of the threshold. In particular, we consider threshold h such that $\frac{p_1(1-h)}{(1-p_1)h} = \theta$, which means that the threshold is set as $h = \frac{p_1}{p_1+(1-p_1)\theta}$. With this value of h, the system becomes unacceptable. For the two values of odds-ratio considered, $h = 0.008347$ when $\theta = 1.2$ and $h = 0.006688$ when $\theta = 1.5$. On the other hand, in the non-SC, we halve the value of h from the SC and set it as a new threshold value.

To ensure a fair comparison, we choose the tolerance level ϵ of $\mathcal{F}$ as the absolute difference between the threshold h of the SC and p_1. We set $n_0 = 10$ for $\mathcal{F}$ to perform the required variance estimation. Furthermore, since the basic observations of $\mathcal{F}$ are batch means, we consider the batch size $b \in \{1, 32, 100\}$ for both $\theta \in \{1.2, 1.5\}$ and further include $b \in \{300, 400, 500\}$ for $\theta = 1.2$ and $b \in \{200, 300\}$ for $\theta = 1.5$ when $p_1 = 0.01$. Those additional values of the batch size are used to demonstrate the required batch size to ensure the desired statistical guarantee.

Table 1 shows the experimental results when $p_1 = 0.01$, where we report the estimated PCD and the required number of observations to conclude the feasibility decision (OBS). Firstly, from the PCD perspective, $\mathcal{F}$ does not provide statistical guarantee of $1-\alpha$ when $b \in \{1, 32, 100, 300, 400\}$ when $\theta = 1.2$ and when $b \in \{1, 32, 100, 200\}$ when $\theta = 1.5$ under the SC. This is expected because Bernoulli data with $p_1 = 0.01$ are heavily skewed, requiring a large batch size to achieve approximate normality of batch means. We see that one needs roughly a batch size of 500 for $\theta = 1.2$ and 300 for $\theta = 1.5$ to ensure the statistical guarantee of $\mathcal{F}$. When $b = 500$, BerF requires slightly fewer OBS compared with $\mathcal{F}$ when $\theta = 1.2$, but 37% fewer OBS than $\mathcal{F}$ when $b = 300$ and $\theta = 1.5$ under the SC. On the other hand, under the non-SC, which is an easier case in terms of feasibility determination, the superiority of BerF is obvious, resulting in up to 67% reduction in OBS.

Table 1: Estimated PCD and OBS for a single system with $p_1 = 0.01$.

		b	SC		non-SC	
			PCD	OBS	PCD	OBS
$\theta = 1.2$	BerF		0.959	9441.6	1	2917.6
	$\mathcal{F}$	1	0.095	8567.1	0.100	6622.6
		32	0.888	8542.9	0.961	6505.5
		100	0.919	8592.3	1	6453.8
		300	0.936	8932.7	1	6707.0
		400	0.946	9153.6	1	6844.2
		500	0.951	9514.8	1	7166.9
$\theta = 1.5$	BerF		0.960	2214.7	1	1199.9
	$\mathcal{F}$	1	0.097	2625.7	0.095	2986.9
		32	0.876	2579.3	0.958	3083.2
		100	0.918	2688.5	1	3165.5
		200	0.945	2971.7	1	3310.9
		300	0.965	3506.2	1	3689.2

Table 2 shows the experimental results when $p_1 = 0.15$. As the Bernoulli data in this case are less skewed, $\mathcal{F}$ achieves estimated PCD above the nominal level when $b = 32$ and $\theta \in \{1.2, 1.5\}$. With $b = 32$, BerF needs slightly more OBS than $\mathcal{F}$ when $\theta = 1.2$, while BerF spends 50% fewer OBS than $\mathcal{F}$ when $\theta = 1.5$ under the SC. BerF clearly outperforms $\mathcal{F}$ under the non-SC, bringing as large as 75% reduction in OBS when $b = 32$.

5 CONCLUSION

In this paper, we address the problem of identifying feasible systems among a finite number of simulated alternatives in the presence of a probability constraint. Due to the nature of the probability constraint, the observations follow a Bernoulli distribution. We develop a novel procedure that employs a random walk model with an odds-ratio IZ parameter. Our procedure is statistically valid and does not need either

Table 2: Estimated PCD and OBS for a single system with $p_1 = 0.15$.

		b	SC		non-SC	
			PCD	OBS	PCD	OBS
$\theta = 1.2$	BerF		0.957	707.6	1	197.8
	$\mathcal{F}$	1	0.785	600.9	0.806	388.8
		32	0.951	655.0	1	440.3
		100	0.980	1058.4	1	1000.7
$\theta = 1.5$	BerF		0.964	166.1	1	82.3
	$\mathcal{F}$	1	0.787	169.0	0.800	179.3
		32	0.983	328.2	1	325.7
		100	1	1000.0	1	1000.0

an initial sample size for variance estimation or batching to achieve the approximate normality of basic observations. Our experimental results show that the proposed procedure outperforms an existing procedure that is originally designed for normally distributed data.

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AUTHOR BIOGRAPHIES

TAEHOON KIM is an officer in the Republic of Korea Army. He received his M.S. in Operations Research from the H. Milton Stewart School of Industrial and Systems Engineering at the Georgia Institute of Technology. His research interests include simulation optimization and the application of simulation in the defense and healthcare fields. His email address is tkim638@gatech.edu.

SIGRÚN ANDRADÓTTIR is a Professor in the H. Milton Stewart School of Industrial and Systems Engineering at the Georgia Institute of Technology. Prior to joining Georgia Tech, she was an Assistant Professor and later an Associate Professor in the Departments of Industrial Engineering, Mathematics, and Computer Sciences at the University of Wisconsin – Madison. She received her Ph.D. in Operations Research from Stanford University in 1990. Her research interests include simulation, applied probability, and stochastic optimization. She is a member of INFORMS and served as Editor of the Proceedings of the 1997 Winter Simulation Conference. She was the Simulation Area Editor of *Operations Research Letters* from 2002 to 2008, and has served as Associate Editor for various journals. Her email address is sa@gatech.edu and her website is isye.gatech.edu/faculty/sa.

SEONG-HEE KIM is a Professor in the H. Milton Stewart School of Industrial and Systems Engineering at the Georgia Insitute of Technology. She received her Ph.D. in Industrial Engineering and Management Sciences from Northwestern University in 2001. Her email address is skim@isye.gatech.edu and her website is https://www2.isye.gatech.edu/∼skim/.

YUWEI ZHOU is an Assistant Professor in the Kelley School of Business at Indiana University. Prior to joining Indiana University, she was a Postdoctoral Principal Researcher in the Booth School of Business at the University of Chicago from 2021 to 2024. She received her Ph.D. in Operations Research from the H. Milton Stewart School of Industrial and Systems Engineering at Georgia Institute of Technology in 2021. Her research interest includes simulation optimization and data-driven decision making, with an emphasis on public sector applications. Her e-mail address is yuwzhou@iu.edu and her website is https://yuweizhou3.github.io.