# **ADAPTIVE SIMULATION OF EV CHARGING PROCESSES: EMPLOYING BAYESIAN INFERENCE WITH MARKOV CHAIN MONTE CARLO FOR DYNAMIC INPUT UPDATING**

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# **ABSTRACT**

Simulating the charging process of electric vehicles (EVs) at public stations is crucial for effective decisionmaking in the planning and management of EV infrastructure. Traditional models face challenges in reflecting the dynamic and uncertain nature of real-life EV charging. This study introduces a hybrid simulation framework that incorporates geospatial demand and utilizes Bayesian Inference with the Markov Chain Monte Carlo (MCMC) method to generate dynamic, probabilistic inputs. The proposed approach could (1) dynamically respond to changing observation data, (2) reflect the uncertainty and randomness of charging progress, and (3) integrate users' demand and geospatial factors in the charging station selection. A case study was conducted involving three charging stations in Fairfax City. The results explained the evolving charging patterns and evaluated the impact of unforeseen events on station utilization. This method offers a robust tool for planning, developing, and optimizing public EV charging infrastructure, adapting to changing behaviors and demands.

# **1 INTRODUCTION**

The shift towards electric vehicles (EVs) plays a pivotal role in reducing greenhouse gas emissions and diminishing the reliance on fossil fuels. As electric vehicles become more popular, the need for efficient and widely accessible public charging infrastructure intensifies significantly (Xiang et al. 2018). Simulating the charging behavior, such as when, where, and how the EV users charge their vehicles, is crucial for the charging infrastructure planning (Chen et al. 2020). However, the charging behaviors are dynamic and unpredictable, which includes uncertain factors such as arrival and departure times at charging stations, the duration of charging sessions, and the selection of charging locations, posing significant challenges to infrastructure planning (Uimonen & Lehtonen 2020). To mirror the dynamic real-life scenarios, simulation models could be used to design, analyze, communicate, and test the complex charging behaviors, which helps support critical decision-making in public EV charging infrastructure planning.

Accurate input data is critical for successful simulation models (Wu et al. 2019), particularly in the context of electric vehicle (EV) charging, where activities are marked by randomness and uncertainty. In most cases, the input variables are not fixed values but follow probability distributions. However, finding accurate parameters for model input is limited by the difficulty in obtaining sufficient population data and updating parameters according to changing conditions (Ji & AbouRizk 2017). Moreover, the accuracy of these models is further influenced by the decision-making mechanisms that reflect users' choices. Previous research efforts have focused on developing simulation models to mirror the complexities of real-world EV charging behaviors (Uimonen & Lehtonen 2020; Lee et al. 2019). These models, while innovative, often do not integrate new data dynamically nor ignore the uncertainty in fixed inputs, limiting their applicability in fluctuating real-world conditions. Therefore, being able to simulate the EV charging behaviors reliably and dynamically is critical for understanding the utilization situation of existing infrastructures and forecasting future uncertain scenarios, which could further assist decision-making in effectively planning, developing, and optimizing public EV charging infrastructure.

This research aims to introduce a hybrid framework to reliably simulate the charging process of electric vehicles (EVs) at public stations. Specifically, the objective is achieved by (1) incorporating Bayesian Inference with the Markov Chain Monte Carlo (MCMC) method to dynamically update model inputs (arrival intervals and charging durations); (2) developing a geospatial simulation model with a charging station selection mechanism considering users' demand and spatial factors; (3) demonstrating the feasibility of proposed approach using a case study in Fairfax City. By addressing these areas, the proposed hybrid simulation framework provides stakeholders with a sophisticated tool to facilitate the planning, development, and optimization of public EV charging stations in a way that adapts to changing charging behaviors and demands.

# **2 LITERATURE REVIEW**

Previously, multiple studies have been conducted to simulate the EV charging behaviors to investigate the vehicle-transportation-grid trajectory (Chen et al. 2023), analyze the demand for the power grid (Ni & Lo 2020), and evaluate service capacity (Zhang et al. 2018). However, most of the research employed fixed variables as model input, which ignored the uncertainties and stochastic factors. These uncertainties arise from several sources, including variability of arrival and departure times due to mobility behavior, differences in charging duration due to state-of-charge on arrival, battery capacity, market price, etc. (Sun et al. 2020). To account for these variabilities, inputs are modeled as probability distributions because of their ability to incorporate randomness and uncertainties (Wu et al. 2019). Bayesian Inference with the Markov Chain Monte Carlo (MCMC) method has been demonstrated efficient in generating a reliable simulation model input in construction models (Ji & AbouRizk 2017; Wu et al. 2019). The main benefit of MCMC is that it doesn't need the entire likelihood function; instead, it just needs the posterior density's shape (Choi 2023). Rather than solving the complete simultaneous balance equation, the MCMC algorithm resolves local or detailed balance equations. This guarantees that the time average converges to a Bayes estimator by enabling it to manage the stability of the problems that come with Markov chains (Zhao 2021). One popular MCMC technique that creates a Markov chain is the Metropolis-Hastings algorithm. It works by recommending new samples from a proposed distribution and deciding whether to accept or reject them based on the ratio of next densities (Rocca 2019). This procedure provides a series of samples that converge to the actual posterior distribution after numerous iterations. Given the circumstances, Bayesian inference can be carried out using MCMC techniques even in cases where the posterior distribution is complex and multidimensional (Hofmeister 2021). The key steps are: (1) Set up a stationary distribution for the posterior in a Markov chain; (2) Take a sample from the chain; and (3) Use the sample data to conclude the unknown parameters.

# **3 METHODOLOGIES**

This research develops a hybrid simulation model to accurately reflect real-world electric vehicle (EV) charging dynamics by incorporating two fundamental elements: (1) Bayesian Inference coupled with the Markov Chain Monte Carlo (MCMC) method, which is utilized to generate dynamic, probabilistic inputs for the model; (2) a detailed simulation model that integrates users' demand and spatial distances into the decision-making process for selecting charging stations.

# **3.1 Bayesian Inference with Markov Chain Monte Carlo (MCMC)**

In the simulation model of EV charging progress, Bayesian Inference, coupled with the Markov Chain Monte Carlo (MCMC) method is used to generate dynamic, probabilistic inputs, including the arrival intervals of electric vehicles and charging durations of single vehicles. This approach could not only capture the distribution patterns inherent in historical data but also update the parameters at the time of observing new data, which reflects the uncertainty and variability inherent in real-life charging scenarios.

## **3.1.1 Bayesian Inference**

Bayesian inference is a method based on Bayes' theorem to update the probability for a hypothesis as more observations are available. The main components included in the Bayesian inference include the prior distribution likelihood function and posterior distribution. Based on Bayes' Theorem, the posterior distribution could be denoted as follows:

$$
P(\theta|X) = \frac{L(X|\theta) \times P(\theta)}{P(X)} \propto L(X|\theta) \times P(\theta)
$$
\n(1)

where:

- $P(\theta)$  is the prior distribution parameters  $\theta = {\theta_1, \theta_2, \theta_3, ..., \theta_n}$ .
- $L(X|\theta)$  is the likelihood function of data  $X = \{x_1, x_2, x_3, ..., x_n\}$  given the parameters  $\theta$ .
- $P(X)$  is the marginal probability of the data X.
- $P(\theta|X)$  is the posterior probability of the parameters  $\theta$  given the data X.

#### **3.1.2 Analytical Derivation**

In the EV charging simulation model, both the arrival intervals of electric vehicles and the charging durations for individual vehicles are not predetermined with fixed values. Instead, these variables are characterized by specific distributions to incorporate uncertainty and randomness. Based on Bayes' Theorem, the analytical solution for EV arrival intervals will be proved mathematically as an example. In the simulation, exponential distribution  $exp(\lambda)$  is typically used to model times between arrivals (arrival intervals)  $x$ . The probability density function is given by:

$$
f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x}, & x < 0 \\ 0, & x \ge 0 \end{cases}
$$
 (2)

When the variable follows the exponential distribution, the usual conjugate prior is the Gamma distribution with parameter  $(\alpha, \beta)$ :

$$
P(\lambda) = \frac{\lambda^{\alpha - 1} e^{-\beta \lambda}}{\Gamma(\alpha)} \propto \lambda^{\alpha - 1} e^{-\beta \lambda},
$$
  
where  $\Gamma(\alpha) = (\alpha - 1)!$  (3)

According to the Equation (2), the likelihood function has the form:

$$
L(X|\lambda) \propto \prod_{i=1}^{n} \lambda e^{-\lambda x_i} \propto \lambda^n e^{-\lambda x}
$$
  
where  $X = \sum_{i=1}^{n} x_i$  (4)

Therefore, the posterior distribution has the form:

$$
P(\lambda|X) \propto L(X|\lambda) \times P(\lambda) \propto \lambda^{a-n-1} e^{-(\beta + X)\lambda}
$$
 (5)

From Equation (5), the posterior distribution is also a gamma distribution with parameters  $(a - n, \beta + \beta)$  $X$ ), which is denoted as:

$$
P(\lambda|X) = Gamma(a - n, \beta + X)
$$
\n(6)

For the distribution of arrival intervals, the mean of the posterior Gamma distribution is used to represent the analytical parameter  $\lambda$ , which is denoted as:

$$
\lambda = \frac{a - n}{\beta + X} \tag{7}
$$

Similarly, this research uses  $beta(\alpha, \beta)$  to model charging durations. The derivation process will not be discussed in detail here.

#### **3.1.3 Markov Chain Monte Carlo**

In real-life practice, the analytical solution may not exist or will be difficult to derive (Ji & AbouRizk 2017). In such cases, the Markov Chain Monte Carlo (MCMC) is used to find the target posterior distributions  $P(\lambda|X)$ . As a widely used method in MCMC, the Metropolis-Hastings algorithm constructs a Markov chain  $(\lambda^1, \lambda^2, \lambda^3, \dots, \lambda^n)$  that moves through the parameter space, with the transition from one state (or sample) to the next based on a probabilistic decision rule. The steps are as follows:

- 1. Starting from the current state  $\lambda^i$ , a new proposed candidate is generated:  $\lambda^{i+1} = \lambda^i + \Delta \lambda$ , where  $\Delta\lambda \sim normal(u, \sigma)$ .
- 2. Decide whether to accept  $\lambda^{i+1}$  based on the posterior probability ratio to its previous step. The ratio is given by the Equation (8). This ratio determines whether the new candidate state is more likely (or less likely but still potentially acceptable) compared to the current state. The new move is accepted if the new sample is more probable than the existing sample. Otherwise, the move is accepted with the acceptance probability  $\rho$  or the move is rejected.

$$
\rho = \min\left(1, \frac{P(\lambda^{i+1}|X)}{P(\lambda^{i}|X)}\right)
$$
\n(8)

- 3. To achieve the logic in step 2, compare the ratio  $\rho$  to a random value  $u$  sampled from a uniform distribution  $U(0,1)$ . Accept the proposed candidate  $\lambda^{i+1}$  if  $\rho$  is larger than u. Otherwise, reject the proposed candidate and redo step 2.
- 4. After iterating step 1 to step 3 for certain times, a list of samples will be saved, and a histogram could be generated to show the distribution of parameter  $\lambda$ .

By following this procedure, the Metropolis-Hastings algorithm ensures that the generated Markov chain will converge to the desired stationary distribution.

#### **3.2 EV Charging Simulation Framework**

The simulation framework is intricately designed to simulate several key aspects: the generation of demand points, the selection of public charging stations, and the subsequent charging process. A generic simulation model is developed in the Simphony.NET environment.

**Demand Points Generation**: Demand points are randomly generated across the geographical landscape, representing EVs that need charging. The arrival intervals between these EVs follow a distribution derived from the MCMC method.

**Public Charging Station Selection Mechanism**: A selection index is formulated to prioritize public charging stations based on user demand and spatial proximity. The index is defined by the formula:

$$
I_i = \frac{t_i}{t_{total}} + \ln\left(\frac{d_i}{d_{total}}\right)
$$
 (8)

Where  $t_i$  is the yearly historical charging time for public EV charging station  $i$ ,  $t_{total}$  denotes the total charging time of all public charging stations within the geographic scope.  $d_i$  is the travel distance from the

demand point to the public EV charging station  $i$ ,  $d_{total}$  is the cumulative distance from the demand point to all stations within the geographic scope. After each demand point is generated, an index is calculated for each public charging station assuming that the charging price of all public charging stations is constant. The selection order for the charging stations is then determined in descending order based on the index values. For example, if  $I_1 > I_2 > I_3$ , the selection order would be station 1, followed by station 2, then station 3. In cases where a preferred station is at full capacity, the user will proceed to the next available option. All selection logic will be implemented by writing code in the Execute element in Simphony. Net.

**Charging Process Simulation**: The charging duration for each session is modeled using a probability distribution derived via the MCMC method, which captures the inherent variability and uncertainty in actual charging times.

#### **4 CASE STUDY**

In this research, three public EV charging stations in Fairfax City were selected to demonstrate the feasibility and applicability of the proposed approach. Firstly, the probabilistic model inputs (arrival intervals and charging duration) were modeled using the MCMC method to incorporate uncertainty. Then, a simulation model was designed using the generated inputs to simulate real charging behaviors and possible special scenarios.

**Data Source.** The dataset used in this research contains detailed information on EV users' charging behaviors in 2023. It stores EV charging records for three public charging stations in Fairfax city. The information includes location, connection time, disconnection time, total charging time, number of chargers, etc. The arrival interval is obtained by subtracting two adjacent connection times and charging time. The charging time has already been calculated by subtracting the disconnection time from the connection time.

**Bayesian Updating of Input Models.** In real-world scenarios, the challenge of obtaining sufficient data and the need to adaptively update model parameters in response to changing conditions present significant limitations for establishing and refreshing dependable simulation inputs. Therefore, in this research, the MCMC method is used to generate distribution for arrival intervals and charging duration. Based on historical data and expert insights, the arrival interval is assumed to follow an exponential distribution, with prior distribution *Gamma* (2,1) for  $\lambda$ ; charging duration is assumed to follow a Beta distribution with prior distribution *Normal*(5,2) for  $\alpha$ , and  $\beta$ .

Given that the prior distribution for arrival intervals is conjugate to the likelihood function, the analytical solution is calculated to validate the effectiveness of the Bayesian updating process. Figure 1 shows the 5000 iteration samples of  $\lambda$  generated by the MCMC method. The initial value is set to 2 and it reaches stable after around 50 iterations (as shown in Figure 2). The analytical value is 0.0082 and the estimated value generated by the proposed method is 0.0079. The difference is 3.7% of the analytical value.



Figure 1: Trace plot for parameter  $\lambda$ .



Figure 2: Trace plot for parameter  $\lambda$  (200 Iteration).

Unlike the conjugate prior for the arrival intervals, it is hard to find an analytical solution for the distribution of charging duration for three stations with a non-conjugate prior. Therefore, it requires numerical methods, Markov Chain Monte Carlo (MCMC), for estimation. The initial values for  $\alpha$ ,  $\beta$  are set at 0.5, 5. The results will be compared with parameters fitted from cumulative observations. The results and differences are shown in Table 1. Figure 3 shows trace plots of the parameter  $\alpha$ ,  $\beta$  got from MCMC for three public charging stations.

	raone i. Shape parameters.							
	EV Charging Station	Estimated $(\alpha, \beta)$	Fitted $(\alpha, \beta)$	Difference				
		using MCMC		$\frac{1}{2}$ of fitted value)				
		(0.69, 11.26)	(0.70, 11.61)	(1.4, 3.0)				
		(0.74, 3.46)	(0.73, 3.38)	(1.3, 2.3)				

Table 1: Shape parameters.

**Simulation Model.** The model is designed to simulate EV charging progress in three public charging stations within one day. The stations, designated as "S1," "S2," and "S3," are each modeled as a resource within the simulation framework, reflecting the available charging infrastructure. The number of charging units at each station is represented by the quantity of resources in the model, indicating the station's capacity. The geographic scope of the study is defined as a 2-mile radius from the centroid of these three charging stations (shown in Figure 4).

S3 (0.31, 4.07) (0.30, 3.90) (3.0, 4.0)

In this simulation model, 100 electric vehicles (EVs) are generated as input entities, each assigned geographic coordinates within the defined scope. These entities arrive at intervals following an exponential distribution with a rate parameter of 0.078. The number of entities exceeds the actual demand to prevent the simulation from halting due to input limitation. The simulation's temporal boundary is set at 540 minutes, equivalent to a 9-hour window, aligning with typical office hours as the three public charging stations are close to workplaces, anticipating higher charging demand during these periods. The selection index  $I_i$  is calculated based on Euclidean straight-line distance and yearly charging time (in minutes) accumulated from historical data. The travel time to reach a station is deduced from the distance and an assumed urban travel speed of 25 miles per hour. Charging durations at the three stations—labeled "S1," "S2," and "S3" are modeled to follow beta distributions with parameters  $beta(0.69, 11.20)$ ,  $beta(0.74, 3.42)$ , and  $beta(0.31, 4.07)$ . The simulation model is shown in Figure 5.





Figure 3: Trace plots for the parameter of 3 stations.



Figure 4: Geographic scope.



Figure 5: Simulation model.

**Simulation Model Results**. Data is segmented into four quarters for analysis, with each quarter's data used to update the parameters of the distribution. The distribution parameter  $(\lambda)$  for charging intervals is recorded as 0.0083, 0.007, 0.008, and 0.0095 for each consecutive quarter, respectively. A slightly growing rate indicating an increased likelihood of shorter charging intervals was observed from Q2 to Q4, suggesting a rising demand for charging services.

Distributions of utilization rates for three public charging stations are shown in Figure 6. Average utilization rates for three charging stations, derived from a simulation model, are presented in Table 2. All three stations have a high frequency of low utilization, indicating underutilization overall. There is a very gradual trend of increasing utilization over the four quarters. This is reflected by a small but steady decrease in the lowest utilization bracket and a corresponding increase in the middle brackets, which can be attributed to the rising demand. Charging Station 2 (S2), despite being geographically close to the highly frequented

Charging Station 1 (S1), exhibited lower total utilization compared to the other stations. This discrepancy is primarily due to S1, which possesses a greater number of charging units, capturing most of the demand from the surrounding area. In such a situation, implementing flexible pricing models that incentivize charging in some under-occupied charging stations can help balance the load across the system. Charging Station 3 (S3), located further from S1 and S2, had significantly fewer charging occurrences, constituting only 3% of the total. Nevertheless, its utilization was higher than that of S2. This higher rate of utilization underscores S3's essential role within its local neighborhood, highlighting its uniqueness and irreplaceable nature despite its lower overall usage. This phenomenon emphasized the importance of incorporating geospatial factors in decision-making progress. Additionally, the low but increasing utilization suggests there might be potential for growth in demand or a need to review the placement and promotion of the charging stations to increase their use.

EV		Οl	O <sub>2</sub>	Q <sub>2</sub>	Q3	O3	O4	O4
Charging	Utilization	StdDev	Utilization	StdDev	<b>Utilization</b>	StdDev	Utilization	StdDev
Station	$\frac{1}{2}$		$\left(\%\right)$		$(\%)$		$(\%)$	
S1	21	0.14	18.5	0.135	20.8	0.145	23.3	0.147
S <sub>2</sub>	3.2	0.081	2.9	0.076	3.5	0.082	3.8	0.089
S3		0.209	14.7	0.196	15.9	0.200	19	0.223

Table 2: Average utilization of four EV charging stations.

By analyzing the standard deviation and distribution pattern, it becomes clear that despite a generally low utilization rate, there still exists uncertainty in EV charging behaviors, especially for S3. In Q4, the likelihood of the utilization rate exceeding 66.7% stands at 4.3%, coupled with a standard deviation of 0.223, suggesting a wider spread in the data and hence higher variability or uncertainty in utilization. This variation implies that the station experiences periods of both high and low use, with usage patterns that are less consistent than S1 and S2. To further explore the maximum capacity of this study area, the arrival duration was hypothetically reduced to zero, simulating a scenario where there is a continuous demand for charging stations. For addressing the unpredictable elements of charging, such as the occasional forgetfulness to disconnect, beta distributions were employed to model the variability in charging times. It was estimated that the maximum vehicle throughput for the area is approximately 24 to 26 vehicles (as shown in Figure 7).

**Special Scenario**. In 2023, charging stations S1 and S2 experienced a notable dip in usage over two months due to unforeseen events. Given the proximity of S1 and S2 to one another, it's plausible to conclude that both were simultaneously non-operational, likely because of roadwork or upgrades to the electric grid network. Consequently, station S3 would have absorbed the entire charging demand. To accurately simulate this real-world scenario, S1 and S2 were excluded from the model. The average arrival duration for S3 ( $1/\lambda$ ) in the exponential distribution) was adjusted to 120, 90, and 60. These adjustments represent the compensatory increase in demand due to the outage of S1 and S2. As demonstrated in Figure 8, there is a marked escalation in utilization correlating with the increased demand. Notably, when the average arrival interval was reduced to 60, the probability of utilization rates exceeding 66.7% spiked to 62.5%. This highlights periods of near or full capacity at Station S3, signaling a potential necessity for expanded resources or additional stations to manage the surge in demand during such special situations.





Figure 6: Utilization of S1, S2, and S3 across four quarters.



Figure 7: Distribution of maximum capacity.



Figure 8: Utilization of S3 with different λ.

# **5 CONCLUSIONS**

This study introduces a hybrid framework for simulating the EV charging process at public charging stations, using a Markov Chain Monte Carlo (MCMC) approach to create dynamic probabilistic model inputs. This approach can effectively capture the variability and uncertainty in EV charging scenarios. A case study of the City of Fairfax demonstrates the model's ability to generate reliable input parameters and explore unforeseen scenarios, thus more accurately reflecting real-world scenarios. In theory, the model can perform sophisticated simulations of complex charging behaviors and dynamically update inputs as new data is available. In practice, it provides detailed insights into user behavior and traffic patterns, aiding in operational management and strategic planning of public EV charging infrastructure. This makes the framework a valuable decision-support tool for addressing a variety of scenarios, such as peak loads, behavioral shifts, or policy changes.

However, this research study still has limitations. Firstly, while the model does consider user demand and spatial distances, other factors such as charging costs and station availability could also be integrated into future models. Secondly, considering the model's efficiency, this study used data from three charging stations in Fairfax City, which provides valuable insights but may not fully capture the variability in charging behaviors across different locations and user demographics. Thirdly, given the computational resources, this research divided observation data into four sections and updated the results a limited number of times. Although dynamically responding to changing observation data would more accurately reflect real-life situations and support decision-making, it would also increase the computational cost of the model. To address these limitations, more sophisticated and efficient methods need to be developed in future research that could incorporate a more diverse set of data sources, covering a broader geographic area and including various types of charging stations and user profiles.

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