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GLOBAL MULTI-OBJECTIVE SIMULATION OPTIMIZATION WITH LOW-DISPERSION POINT SETS

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ABSTRACT

Consider the context of a Multi-Objective Simulation Optimization (MOSO) problem. The goal of solving a MOSO problem is to identify decision points that map to the global Pareto set. For such problems featuring Lipschitz continuous objectives on a compact feasible set, we determine how one should tradeoff the number of decision points to sample using a low dispersion point set with the number of simulation replications per sampled point to ensure the estimated Pareto set converges to the true Pareto set under a novel performance measure called the modified coverage error. This performance measure enables us to quantify upper bounds on the deterministic and stochastic errors as a function of the dispersion of the decision points sampled and the number of simulation replications per point. The upper bounds and tradeoff analysis lead to an efficient method for solving global MOSO problems with probabilistic guarantees.

1 INTRODUCTION

MOSO is an emerging field that operates at the intersection of single-objective simulation optimization and multi-objective optimization, and many fundamental questions remain unanswered, especially in the problems featuring continuous objectives. We consider global MOSO problems,

$$\text{minimize} \quad \{f(x) = (f_1(x),\ldots,f_d(x)) := (\mathrm{E}[F_1(x,\xi)],\ldots,\mathrm{E}[F_d(x,\xi)])\} \quad \text{s.t.} \quad x \in \mathcal{X}, \tag{1}$$

where $f\colon \mathcal{D} \to \mathbb{R}^d$ is a vector-valued function composed of $d \geq 2$ unknown, real-valued, Lipschitz continuous and conflicting objective functions defined over the compact feasible set $\mathcal{X} \subseteq \mathcal{D} \subseteq \mathbb{R}^q$, $q \geq 2$ that can only be observed with a stochastic error, e.g., as the output from a Monte Carlo simulation oracle (Hunter et al. 2019). The solution to problem (1) is the *efficient set* and its image is the *Pareto set*, defined as

$$\mathcal{E} := \{x^* \in \mathcal{X}\colon \nexists x \in \mathcal{X} \text{ such that } f(x) \leq f(x^*), f(x) \neq f(x^*)\} \quad \text{and} \quad \mathcal{P} := f(\mathcal{E}).$$

Consider a straightforward procedure to solve problem (1), which is inspired by Yakowitz et al. (2000). First, select a set of m points at which to observe the objective function values called the *discretized feasible set*, $\mathcal{X}_m := \{\tilde{x}_1,\ldots,\tilde{x}_m\} \subset \mathcal{X}$, where $t > 0$ is the *dispersion*,

$$t := \mathrm{dist}(\mathcal{X},\mathcal{X}_m) = \sup_{x\in\mathcal{X}} \inf_{\tilde{x}\in\mathcal{X}_m} \|x-\tilde{x}\| = \sup_{x\in\mathcal{X}} \min_{1\leq i\leq m} \|x-\tilde{x}_i\|.$$

Let $\mathcal{E}_m$ and $\mathcal{P}_m$ represent the efficient and Pareto sets, respectively, for problem (1) when solved over $\mathcal{X}_m$. Then, observe n simulation replications for each $\tilde{x} \in \mathcal{X}_m$ and construct the sample-path problem,

$$\text{minimize} \quad \left\{\bar{F}(\tilde{x},n) = (\bar{F}_1(\tilde{x},n),\ldots,\bar{F}_d(\tilde{x},n)) := \left(\frac{1}{n}\sum_{i=1}^{n} F_1(\tilde{x},\xi_i),\ldots,\frac{1}{n}\sum_{i=1}^{n} F_d(\tilde{x},\xi_i)\right)\right\} \quad \text{s.t.} \quad \tilde{x} \in \mathcal{X}_m,$$

where $b = m \times n$. Now, estimate the solution to problem (1) as the *discretized estimated efficient set*,

$$\hat{\mathcal{E}}_m(b) := \{\tilde{x}^* \in \mathcal{X}_m\colon \nexists \tilde{x} \in \mathcal{X}_m \text{ such that } \bar{F}(\tilde{x},n) \leq \bar{F}(\tilde{x}^*,n)\}.$$

Given a set of points $\mathcal{X}_m$ and a total simulation budget b, we wish to measure the quality of the true image of the estimated discretized solution, $f(\hat{\mathcal{E}}_m(b))$, as a function of m and n. To facilitate the tradeoff analysis, we decompose the *optimality gap* into two distinct components: *discretization error* and *selection error*. While the Hausdorff distance enables this decomposition through the triangle inequality and is a natural choice for the performance indicator, it is challenging to analyze (Ondes and Hunter 2023). Therefore, we consider the coverage error (Sayin 2000) of the relevant upper images, which we call the *modified coverage error* (Hunter and Ondes 2023), enhancing the analytical tractability and yielding the bound

$$\underbrace{\mathrm{dist}_p(\mathcal{P}+\mathbb{R}^d_{\geqq}, f(\hat{\mathcal{E}}_m(b))+\mathbb{R}^d_{\geqq})}_{\text{“optimality gap”}} \leq \underbrace{\mathrm{dist}_p(\mathcal{P}+\mathbb{R}^d_{\geqq}, \mathcal{P}_m+\mathbb{R}^d_{\geqq})}_{\text{discretization error}} + \underbrace{\mathrm{dist}_p(\mathcal{P}_m+\mathbb{R}^d_{\geqq}, f(\hat{\mathcal{E}}_m(b))+\mathbb{R}^d_{\geqq})}_{\text{selection error}}.$$

2 MAIN RESULTS

The tradeoff analysis of m and n uses concentration inequalities for sub-exponential random variables from Vershynin (2018) to provide guidance on optimal allocation of the total simulation budget, b. Under the assumptions of Lipschitz continuity and certain regularity conditions, we establish that the probability of the optimality gap exceeding a certain bound diminishes as the number of decision points sampled, m, and the number of simulation replications per sampled point, n increase when selected in a appropriate balance. Let ℓ be a common Lipschitz constant, $c_q > 0$, $c > 0$, $\eta^* > 0$ and $\kappa > 0$ be constants.

$$\text{If } m = \left\lfloor \left(\frac{\kappa b}{\ln b}\right)^{q/(q+2)} \right\rfloor \text{ and } n = \left\lfloor \frac{b}{m} \right\rfloor, \text{ then } \mathbb{P}\left\{ \mathrm{dist}(\mathcal{P}+\mathbb{R}^d_{\geqq}, f(\hat{\mathcal{E}}_m(b))+\mathbb{R}^d_{\geqq}) > \frac{\ell c_q + 2c\eta^*}{\lfloor m^{1/q} \rfloor} \right\} \leq \frac{2d(m+1)}{mn},$$

where m is sufficiently large such that $\lfloor m^{1/q} \rfloor \geq 1$. This result is a simplified version of our main theorem.

3 CONCLUDING REMARKS

In the context of MOSO, we introduce a way to measure the optimality gap that is both tractable and allows us to establish upper bounds on the discretization and selection errors using modified coverage error. Additionally, we address the critical question of how to optimally balance the number of decision points sampled, m, with the number of simulation replications per sampled point, n, to ensure convergence of the estimated solution set to the true Pareto set at a fast rate.

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