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USING FUNCTIONAL PROPERTIES TO SCREEN SOLUTIONS FOR MULTI-OBJECTIVE SIMULATION OPTIMIZATION

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ABSTRACT

We propose a class of screening procedures designed for multi-objective simulation-optimization problems. The procedures screen out solutions that are deemed strictly Pareto-dominated by leveraging known or assumed properties of the functions describing solution performance, such as convexity and Lipschitz continuity, while simulating only a small subset of all solutions. Whether to screen out a given solution is determined by solving a non-convex mixed-integer program, the size of which scales with the number of simulated solutions and the number of objectives. Screening decisions are accompanied by several statistical guarantees, including uniform confidence and consistency. We discuss the computational tractability of the underlying optimization problem and applications in large-scale problems.

1 INTRODUCTION

Multi-objective simulation optimization (MOSO) procedures generally provide a probabilistic guarantee that the recommended solution will converge to a locally efficient one as the simulation budget approaches infinity (Bonnel and Collonge 2014). In contrast, posteriori MOSO methods, which aim to identify or approximate the entire efficient set or approximate it, have popularized the use of metaheuristic algorithms that lack convergence guarantees (Blank and Deb 2020). For a subclass of problems called multi-objective ranking-and-selection (MORS), convergence to the efficient set can be achieved if the total simulation budget approaches infinity (Hunter et al. 2019). A common MORS guarantee is to ensure that the probability of returning the exact efficient set is above a specified threshold $1 - \alpha$.

Let the performance of a solution $x \in \mathbb{X} \subseteq \mathbb{R}^s$ be described by a vector $\boldsymbol{\mu}(x) \in \mathbb{R}^d$. A solution x is said to be strictly dominated by another solution x' if and only if $\boldsymbol{\mu}(x) > \boldsymbol{\mu}(x')$, where the inequality sign between the two vectors is componentwise. A solution that is not strictly dominated by any other solution is called weakly efficient. Our goal is to find the weakly efficient set $\mathcal{E} = \{x_0 | \boldsymbol{\mu}(x_0) \not> \boldsymbol{\mu}(x) \text{ for all } x \in \mathbb{X}\}$. We consider several guarantees for the returned subset $\mathcal{S}$:

Definition 1 **(Finite-sample confidence)** For any sample sizes, $\mathbb{P}(\mathcal{E} \subseteq \mathcal{S}) \geq 1 - \alpha$.

Definition 2 **(Asymptotic confidence)** For sufficiently large sample sizes, $\mathbb{P}(\mathcal{E} \subseteq \mathcal{S}) \gtrsim 1 - \alpha$.

Finite-sample confidence can be delivered when the underlying distribution of the simulation outputs is known; asymptotic confidence can be achieved by batching and appealing to the Central Limit Theorem.

Definition 3 **($\mathsf{S}(\mathsf{X})$ Consistency)** For sufficiently large sample sizes, $\mathbb{P}(x_0 \in \mathcal{S}) \to 0$ for all $x_0 \notin \mathsf{S}(\mathsf{X})$, where $\mathsf{S}(\mathsf{X})$ is the set of solutions that are possibly weakly efficient, if the performance function $\boldsymbol{\mu}$ were observed at a set $\mathsf{X} = \{x_1, x_2, \ldots, x_k\} \subseteq \mathbb{X}$.

2 SCREENING WITH FUNCTIONAL PROPERTIES

When the feasible region is too large to be simulated exhaustively, Plausible Screening methods can be used to screen out unsimulated solutions (Eckman et al. 2022). These methods assume some prior knowledge about properties of the performance function $\boldsymbol{\mu}$, such as convexity or Lipschitz continuity for each objective. As an illustration, we consider the case where $\boldsymbol{\mu}$ is known to be convex for each objective. The set of vector-valued performance functions that are convex for each output and for which a given solution x_0 is weakly efficient is defined as

$$\begin{aligned}\mathscr{M}(x_0) = \{\boldsymbol{m} = (m_1, m_2, \ldots, m_d) \in \mathscr{F}^d :& \\ &\text{for all } x \in \mathbb{X}, \text{ there exist } \boldsymbol{g} = (\boldsymbol{g}_1, \boldsymbol{g}_2, \ldots, \boldsymbol{g}_d) \in \mathscr{F}^{d\times s} \text{ such that} \\ &m_r(x) - m_r(x') \leq \boldsymbol{g}_r(x)(x - x') \text{ for all } x' \in \mathbb{X} \text{ and } r = 1, 2, \ldots, d, \\ &\boldsymbol{m}(x_0) \not> \boldsymbol{m}(x) \text{ for all } x \in \mathbb{X}\}, \qquad (1)\end{aligned}$$

where $\mathscr{F}$ is the space of functions mapping from $\mathbb{X}$ to $\mathbb{R}$. The solution x_0 will be screened out (i.e., deemed implausibly weakly efficient) if $\widehat{\boldsymbol{\mu}}$, the observed sample means at the set X, strongly disagrees with the hypothesis that $\boldsymbol{\mu} \in \mathscr{M}(x_0)$. We measure this disagreement by the optimal value of the optimization problem $\min_{\mathbf{m}\in\mathbb{M}(x_0)} D(\mathbf{m}, \widehat{\boldsymbol{\mu}})$, in which

$$\begin{aligned}\mathbb{M}(x_0) = \{\mathbf{m} = (\mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_k)^\intercal \in \mathbb{R}^{k\times d} :& \\ &\text{there exist } \mathbf{m}_0 \in \mathbb{R}^d \text{ and } \mathbf{g} \in \mathbb{R}^{(k+1)\times d\times s} \text{ and } \boldsymbol{\alpha} \in \mathbb{R}^d \text{ such that} \\ &\mathsf{m}_{ir} - \mathsf{m}_{jr} \leq \mathbf{g}_{ir}(x_i - x_j) \text{ for all } i, j = 1, 2, \ldots, k \text{ and } r = 1, 2, \ldots, d, \\ &\mathsf{m}_{ir} - \mathsf{m}_{0r} \leq \mathbf{g}_{ir}(x_i - x_0) \text{ for all } i = 1, 2, \ldots, k \text{ and } r = 1, 2, \ldots, d, \\ &\mathsf{m}_{0r} - \mathsf{m}_{ir} \leq \mathbf{g}_{0r}(x_0 - x_i) \text{ for all } i = 1, 2, \ldots, k \text{ and } r = 1, 2, \ldots, d, \\ &\mathbf{g}_0^\intercal \boldsymbol{\alpha} = \mathbf{0}_s,\ \boldsymbol{\alpha} \geq \mathbf{0}_d,\ \mathbf{1}_d^\intercal \boldsymbol{\alpha} = 1, \qquad (2) \\ &\mathbf{m}_0 \not> \mathbf{m}_i \text{ for all } i = 1, 2, \ldots, k\} \qquad (3)\end{aligned}$$

and $D(\mathbf{m}, \widehat{\boldsymbol{\mu}})$ is the discrepancy between $\mathbf{m}$ and $\widehat{\boldsymbol{\mu}}$, for example,

$$D(\mathbf{m}, \widehat{\boldsymbol{\mu}}) = \max_{i=1,2,\ldots,k} \max_{r=1,2,\ldots,d} \frac{\sqrt{n_i}}{\widehat{\sigma}_{ir}} |\widehat{\mu}_{ir} - \mathsf{m}_{ir}|.$$

$\mathbb{M}(x_0)$ is the projection of $\mathscr{M}(x_0)$ onto $\mathbb{R}^{k\times d}$ corresponding to solutions in X. The confidence and $\mathsf{S}(\mathsf{X})$ consistency guarantees can be delivered by comparing the optimal value with a predetermined cutoff.

3 COMPUTATIONAL TRACTABILITY

Constraints (2) and (3) introduced by (1) make $\mathbb{M}(x_0)$ a non-convex set. Constraint (3) can be reformulated as non-convex mixed-integer constraints, while the constraint $\mathbf{g}_0^\intercal \boldsymbol{\alpha} = \mathbf{0}_s$ in (2) is bilinear. Therefore, the optimization problem is a non-convex mixed integer program with $kds + 2kd + ds + d + 1$ decision variables and $(k+1)^2 + kd + s + 1$ constraints.

In a synthetic experiment with two objectives, we simulated 20 replications at 50 randomly chosen solutions and were able to screen out 738 out of 961 alternative solutions. The total inference time, i.e., the time required to solve all 961 optimization problems, was less 2 seconds. However, the scale of the optimization problems increases quadratically with the number of simulated systems, posing a challenge for large-scale problems. Potential ways to address this challenge, such as exploiting parallel computing, are being explored.

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