PRODUCTION CONTROL SYSTEM FOR A PRODUCT LINE
WITH DIVERSE DEMAND PATTERNS

Aharon Gavriel Beged-Dov
Associate Professor
Department of Operations Analysis
College of Business Administration
The University of Toledo
Toledo, Ohio

Abstract

This paper describes a production control plan which may be used in a shop whose product line consists of numerous items which vary extensively in their demand pattern.

The plan is capable of (a) Stabilizing production by allowing inventories to fluctuate with moderate demand fluctuations; (b) Minimizing inventories of finished goods while providing protection again serious stockout; (c) Detecting significant changes in demand patterns and revising production schedules accordingly; (d) Scheduling production economically.

Employing several well-known concepts, in particular the control limits principle used in quality control, the plan considers simultaneously forecasted demand, forecast error, inventory position, and shop capacity to arrive at production decisions. For this purpose, control limits are applied to both demand and inventory, and production is stabilized with a simple production smoothing procedure. Simulation results, based on data taken from a medium size production shop, demonstrate the plan features and effectiveness.

Overall, the paper emphasizes operational principles rather than theoretical concepts. However, the development clearly shows that the plan is amenable to generalized treatment.

INTRODUCTION

Scope of Paper

The problem faced by the manager of a typical production facility may be stated as follows: given the future requirements of the facility, what production rates and workforce levels will minimize the total cost of regular payroll and overtime, hiring and lay-off, inventory and shortages, incurred to make the requirements? Of course, this problem is not new. People have sought ways to improve work efficiency since the early days of organized life. But systematic study of the management of production is a development of the 20th Century and especially of the past few decades. Attempts to solve production problems through the strict use of time, technical perfection of manufacturing methods, and carefully worked out incentive-wage payments were dominant up to the Second World War. However, only in the last twenty years are production problems being studied by operations researchers. Thus, the pioneering work of Holt, Modigliani, Murth, and Simon [5,6] in which they justified quadratic cost functions for the above variables and proceeded to develop and solve an intellectually appealing model to obtain an optimal (minimum cost) production program originated only in the last decade.

The purpose of this paper is to describe a production control plan which was developed for a medium-size shop (annual output approximately $4 million) owned by the world's largest manufacturer of communication equipment. Briefly, the system can be viewed as consisting of three interdependent plans. The first plan is concerned with the ordering and stock control of piece parts and raw materials, and is built upon the well-known "optimal lot size" method of analysis. The second plan is concerned with the problem of determining how many units of a given product (code) to make during the coming period. This is accomplished by considering simultaneously forecasted demand, forecast error, and the difference between actual and planned inventory levels. Implementation requires the manual solving of a few simple relationships (management indicated that computers will not become available for at least two years). The effectiveness of the plan hinges on detecting significant shifts in demand rate of individual codes. The tracking mechanism is based on the control limits concept of statistical quality control. The third plan is concerned with reconciling the output of the second plan with the shop labor-utilization policies.

For lack of space, the paper will dwell in detail on the second plan. Description of the first plan will be omitted altogether, and the third plan will be merely outlined.

The Shop and the Product Line

The shop studied in this paper operates within a much larger plant, one of many owned by the world's largest communication equipment manufacturers. The shop consists of four smaller units (subshops). It manufactures a product known as terminal strips which is used for wire terminations in switching equipment. The product is essentially row and column arrangements of very thin metal terminals. A typical unit consists of a block of laminated mounting strips, or cast resin or face plates and cast resin, which hold the terminals in place. The diversity of the product line arises from the many configurations, either in terms of the number of terminals and/or from the ways in which the terminals are held in position. There are approximately 440 different codes. They may be classified into about 150 types by grouping together codes which differ only in minor detail. The product line may be further divided into broad categories of laminated and cast resin. In turn, cast resin can be subdivided into conventional and module types.

The Production Problem

The terminal strips shop has been experiencing serious production problems. These manifested themselves in fluctuating output rate, overtime production, substantial inventory accumulation, poor customer service, and handling of numerous orders on emergency basis. All but the last symptom are easily discerned in Figure 1 which summarizes the shop's performance during the 72-week period preceding the project start. For example, it is obvious that notwithstanding the existence of substantial inventory, averaging about 50,000 terminal strips, the shop continuously experienced a large backlog of unfilled orders.

Very soon the team (headed by the writer) assigned to study the problem realized that the difficulties experienced by the shop could be traced to the cumulative impact of five major causes: continuously rising aggregate demand, widely fluctuating demand rate for many codes in the product line, long lead time to procure some raw materials, frequent arrival of orders requiring immediate attention, and the inability of the shop to schedule production effectively not knowing all of the orders it will be required to fill from the output specified by the schedule.

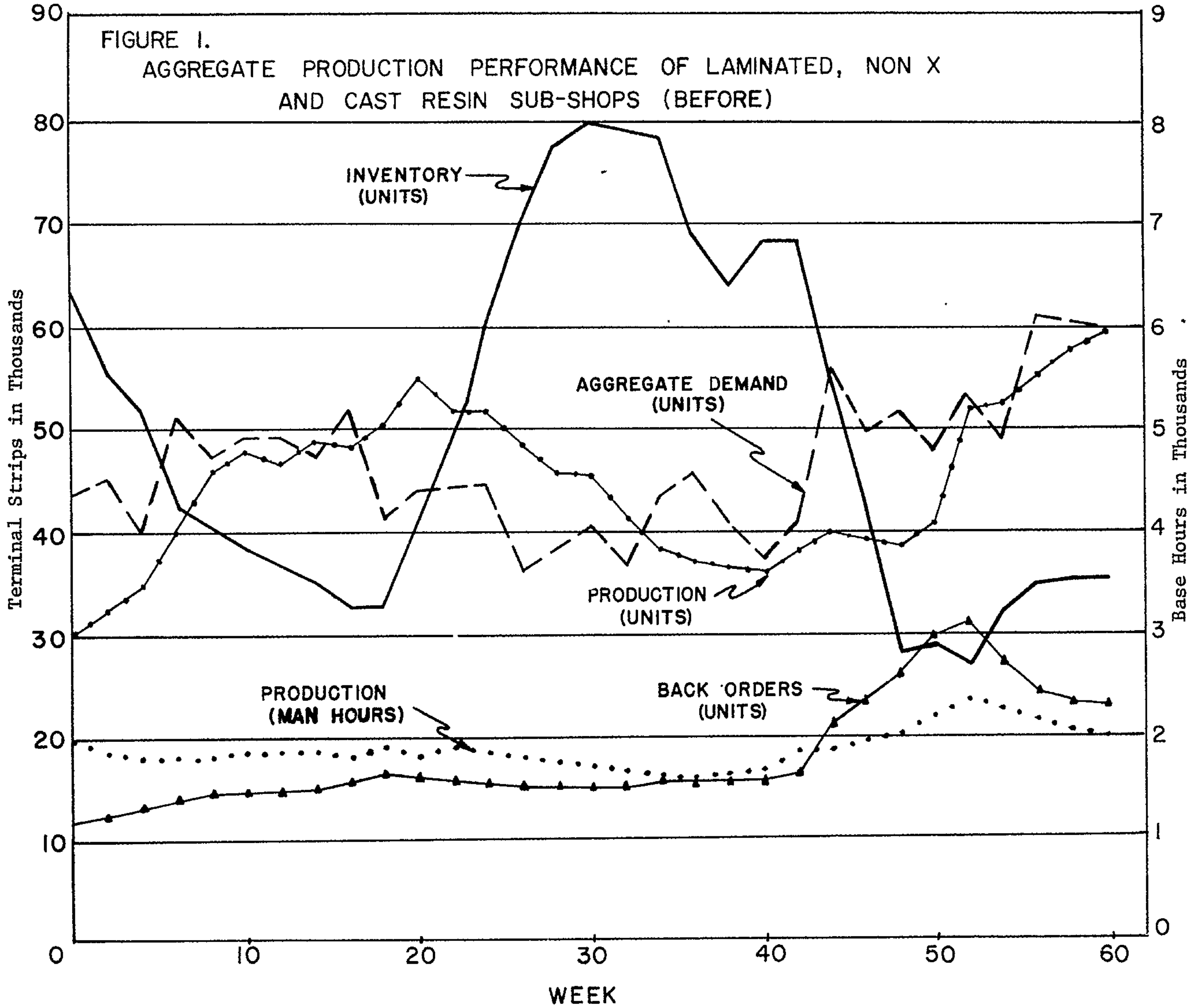
Preliminary Analysis

An analysis of the shop records revealed that

(1) A small number of codes make up the bulk of production.
(2) Annual demand for individual codes ranges from less than 100 and up to 500,000 units.
(3) Many codes have a highly variable demand rate.
(4) Aggregated demand rate tends to be more stable than that of most individual codes in the line.
(5) Considering both demand and production rates, the product line can be divided into three categories:
   (a) which will include the codes whose annual demand exceeds 20,000 terminal strips and/or which are manufactured on a continuous (daily) basis. Thus, such codes can be classified as continuous;
   (b) which will include codes with annual demand rate of 20,000 or less units and which are manufactured periodically (say every second or third week). Thus, such codes can be classified as periodic; and
   (c) which will include codes for which there is very small demand and which are manufactured only a very few times each year. Thus, such codes can be classified as occasional.

Discussions with shop management led to the identification of four manufacturing policies that could be employed. They are:

(1) Manufacture strictly to order. This method tends to cause large fluctuations in production rate as demand rate varies. But it eliminates the need to store inventories.
(2) Manufacture to order and some inventory. This method, useful where demand rate is relatively stable, tends to smooth production level at the expense of building small inventory.



(3) Manufacture both for order and inventory. This method tends to stabilize production and lessens the possibility of excessive stockout at the expense of building a relatively high inventory.

(4) Manufacture mainly for inventory. This method may provide a very large degree of production stability. However, it tends to build excessive inventories whenever demand rate decreases sharply (without warning).

Further discussion as to which policy should be employed in order to manufacture a given code type led to a consensus which is summarized in the matrix following. Here the entries 1st, 2nd, 3rd, and 4th designate the order of preference for each policy relative to code classification, 1st being most preferable, followed by 2nd, and so on. For example, for "continuous" codes Method (c) (i.e., manufacturing for demand and a buffer inventory capable of meeting reasonable fluctuations in demand rate) for "periodic" codes Method (b); and for "occasional" codes, Method (a) (i.e., manufacturing strictly for inventory) are deemed best. It should be noted that the remaining entries need not be discarded once the preference selection has been made. One can hardly expect the chosen alternatives to always provide the best option, and it is conceivable that in some cases a compromise between the first and second choices will prove superior to employing the first pure choice alone. (Table 1)

THE SOLUTION

Plan II: Preliminary Production Schedule Plan

The plan is based on five concepts. They are: Schedule Period (SP) which defines the length of the basic planning time interval; Revision Period (RP) which stipulates when periodic reviews of the current production schedule are made; Inventory and Demand(control) Limits, (IL) and (DL)respectively,

Table 1. Production Preference Matrix

MANUFACTURING METHOD	CODE CLASSIFICATION		
	continuous	periodic	occasional
(a) Manufacture Strictly To Order	4th	2nd	4th
(b) Manufacture Both To Order and Some Inventory	2nd	1st	3rd
(c) Manufacture To Order and Inventory	1st	3rd	2nd
(d) Manufacture Mainly For Inventory	3rd	4th	1st

which determine the operation characteristics of the plan, and Correction Parameters (CP), which regulate operation levels.

In theory, there is no restriction as to the length of SP. However, for several reasons the calendar month was chosen. First, and foremost, the entire company uses monthly production schedules. Second, the shop production control personnel and the operating foremen are well familiar and accustomed with planning production on a monthly basis. Third, this period in no way precludes weekly, or even daily reappraisal of current plans. Fourth, the period is long enough to both justify and allow realistic manpower planning, forecast adjustments, and, if necessary, changes in the current schedule.

The purpose of periodic revision is to compare planned performance with current status. This makes it possible to detect unanticipated changes in demand, or production, early enough.

In brief, the plan stabilizes production by manipulating inventory level within a specified range. The plan stipulates corrective action when it is discovered that the current inventory of a code is below a number defined as the (code's) lower inventory limit, or exceeds another (much larger) number defined as the (code's) upper inventory limit. In the first case, the response tends to increase scheduled production to meet current demand and also to build up the lagging inventory. In the second case, the response curtails scheduled production, and thus tends to bring inventory within the specified range. The degree of adjustment depends upon the discrepancy between the appropriate inventory limit and current inventory on one hand, and the values of the response constants K and $\overline{K}$ (which will be defined shortly), on the other.

The purpose of demand limits is to detect significant changes in (assumed) demand rate. This is extremely important because manpower planning and inventory levels, to mention two key activities, are mainly influenced by demand rate forecast. The tracking is accomplished using a procedure of Statistical Quality Control. First, actual demand in the period just ended is compared to two quantities defined as a lower demand limit and upper demand limit. Then a decision is made if it is reasonable to assume that demand has changed significantly from the assumed rate. If the answer is yes, all the decision and control parameters that are functions demand rate are revised, and all further computations are carried out, using the new figure, until the next revision.

The interrelationships of these parameters can best be described by discussing the underlying ordering formula. The formula is as follows:

(A) $\hat{p}_{i,t} = d_{i,t} - \beta_i(I_{i,t-1} - TI\ )$

where

$P_{i,t}$ = production amount of code i in period t (units)

$\hat{d}_{i,t}$ = forecasted demand for code i in period t (units)

β_i = constant associate with code i, such that $0\le\beta_i\le 1$

$I_{i,t-1}$= net inventory position of code i at beginning of period t (units)

TI_i = target inventory for code i (units)

The formula states that the amount of scheduled production (of a code in any one period) is computed by subtracting from forecasted demand a percentage of the difficulty between actual and target inventory. The formula simply formalizes the decision process used by the shop to schedule production. It recognizes that production quotas directly affect both production rate and service levels. To use the formula, it is necessary to set the values of the decision variables β and TI. Probably the most reliable way of determining the value of β is by simulation. To do this, it is necessary to formulate some relevant measure of efficiency such as production cost, or production stability, or inventory level, or service level and so on, and then chose the one which optimizes the measure of efficiency. This was the method used by the team. More will be said on this subject later. In determining the value of target inventory TI, the team concluded that it should consist primarily of codes which are in continuous demand of significant dimensions, i.e., "continuous" codes. Otherwise, production smoothing would be accomplished by controlling the inventory levels of slow-moving codes. Clearly, this is undesirable since if the process is allowed to proceed for a length of time, large inventory investment with practically no leverage will be built eventually. One may argue that by stocking slow-moving codes, some savings in setup charges will accrue. However, setup charges are negligible in terminal strip manufacturing. Furthermore, the determination of economic lot size quantities by considering all relevant cost elements, including setup cost, is quite straight-forward. Specifically, the level of target inventory was determined in conjunction with the employment rules followed by the shop. The necessity for this is best realized by reflecting that employment level and inventory level are interdependent. For example, flexible transfer policies will tend to lower inventories. On the other hand, reluctance to hire or lay off will produce either excessive use of overtime or excessive inventory buildup in reaction to persistent demand rate oscillations. (Magee [11] shows that the expected magnitude of production fluctuations is roughly proportional to $\sqrt{\beta L/(2-\beta)}$. Here β is defined as in equation (A), and L is the time between two consecutive production decisions (review period). Magee also obtains the combination of β and L which will tend to minimize incremental production cost by simulation.)

The above line of reasoning and the realization that the plant management will be reluctant to invest too large a sum in finished goods inventory led the team to adopt an approach which can be summarized as follows: "Let management specify the average investment it is willing to allow, and then determine how to allocate this amount among the individual terminal strips codes selected to make up target inventory." This was done. Management specified 40,000 terminal strips, an amount representing about 80 per cent of the average inventory maintained by the shop at the time. The makeup of target inventory, hereafter designated by T terminal strips units was made with two main considerations in mind. First, as already noted, it should contain strips which should consist primarily of "continuous" type codes, and to a lesser degree of "periodic" type codes (recall from Table 1 that "occasional" codes will not be stocked at all but will be made on demand). Second, code target inventory TI, should reflect (1) present and anticipated demand for the code, and (2) the investment it represents relative to gross target inventory investment. In other words, the allocation rule used was developed by answering two related questions: first, how to divide the (relevant) product line into a small number of groups, and second, how to allocate T first among the groups and then within the codes in each group. Let:

T_g = target inventory for group g. $g = 1, \ldots, G$.

S_g = average annual sales of all the codes in the group. $g = 1, \ldots, G$.

TI_{ig} = target inventory for code i in group g. $i = 1, \ldots I_g$. $g = 1, \ldots, G$.

V_{ig} = demand variance of code i in group g. $i = 1, \ldots, I_g$. $g = 1, \ldots, G$.

Then the answers to these questions may be stated as follows:

$$(B)\quad T_g = T \frac{S_g}{S_1 + S_2 + \ldots + S_G}$$

$$(C)\quad T_{ig} = T_g \frac{V_{ig}}{\sum_{ig} V_{ig}}$$

Equation (B), though arbitrary, reflects the assumption (which was substantiated later) that changes in demand rate, expressed in absolute units, are somewhat proportional to average demand rate. Intuitively, this assumption is quite reasonable. For example, demand rate can easily rise from say 1,000 to 1,300 units per week. But it is most unlikely that weekly demand rate will jump from 50 to 350 units. Equation (C) can be justified in a similar fashion.

Using the Plan

The operation of the plan can best be described by referring to Tables 2 and 3. Table 2 summarizes the values of the operational parameters recommended by the team for use by the shop. As already noted, the values of IL and CP were determined by simulation. The other values appearing in the table were determined by rational analysis. Table 3 illustrates the preliminary determination of production quota for a code designated as 140D, as shown.

Computation Procedure

The computations required to implement the plan can best be discussed with reference to Table 3.

Table 2. Summary of Plan Two Parameter Values

Code Group (g)	Average Demand (units/week) d	Revision Period (RP)	Forecast Constant α	Inventory Limits (IL) Low	Inventory Limits (IL) Target	Inventory Limits (IL) High	Demand Limits (DL) Low	Demand Limits (DL) High	Correction Parameters (CP) $\underline{K}$	Correction Parameters (CP) $\bar{K}$
One	20-50	Month	.6	0	2d	4d	.3d	3d	1	1
Two	51-200	Month	.5	.3d	d	4d	.3d	3d	1	1
Three	201-500	Two Weeks	.3	.3d	d	3d	.3d	2d	.6	.6
Four	501-2000	Week	.1	.3d	d	2d	.4d	2d	.5	.5
Five	2001-4000	Week	.05	.2d	.5d	1d	.7d	1.4d	.2	.2
Six	4001 and over	Week	.03	.2d	.5d	.8d	.8d	1.2d	.1	.1

Table 3. Schedule Form

Code Serial Number	Code Group	Revision Period	Forecast Constant	Correction Parameters $\underline{K}$	Correction Parameters $\bar{K}$
140D	4	1 week	.1	.5	.5

(1) Revision Period	(2) Forecast Demand	(3) Actual Demand	(4) Actual Ship-ments	(5) Inventory Corr.	(6) Inventory On Hand (6)+(8)-(4)	(7) Scheduled Production (2)+(5)	(8) Actual Production	(9) Uncom-mitted Inventory (6)+(8)	(10) Inventory Limits Low	(10) Inventory Limits Target	(10) Inventory Limits High	(11) Demand Limits Low	(11) Demand Limits High
1/I	7200	7506	7506	0	1200	7200	7200	8400	360	900	1800	900	2700
2/I	7135	6315	6315	0	894	7135	7200	8029	360	890	1780	890	2670
3/I	8270	8226	8226	-40	1779	8230	8300	10000	330	820	1650	820	2470
4/I	6408	6466	6466	-126	1853	6282	6400	8135	320	800	1600	800	2400
5/I	6548				1787								

The form, designed to aid the shop to carry out the computations in a systematic manner, consists of repetitive sets of eleven entries. Of these, four are read from Table 2 (i.e., code group, (RP), α and the correction parameters (CP))*, one is the code serial number, and the others are computed by the analyst. To be specific:

Column (1) specifies the appropriate schedule period. The value of SP is either the calendar month, for most "periodic" codes, or a week, for some "periodic" and all the "continuous" codes. The apparent rule, "the higher the demand rate, the shorter the revision period" tends to minimize unwarranted rise in inventories as a result of sudden drop in demand. For example, should the demand of some code, currently averaging 10,000 units per week say, drop substantially, several thousand terminal strips will be added to inventory in a matter of a few weeks, unless the change is detected, and appropriate action is taken early enough. The notation, in case of weekly reviews, is w/m where w, in Arabic numerals, states the week, and m, in Roman numerals, states the month in question. In case of monthly reviews, only Roman numerals are used.

Column (2) specifies the current demand forecast (of a code) for the entire Scheduling Period (SP). The forecast is obtained by the formula

(a) $$\hat{D}_m = nd + n\,(n + 1)\,\frac{\Theta^*}{2}$$

where $\hat{D}_m$ = forecasted demand for the m-th schedule period units

d = current average weekly demand rate (of the code)

n = number of weeks in the current schedule period

Θ^* = demand trend estimate

Column (3) states the actual demand D_m incurred during the scheduling period. In general, D_m is unlikely to match forecasted demand $\hat{D}_m$.

Column (4) states the actual number of terminal strips shipped during the scheduling period. This amount may or may not correspond to actual demand, depending on whether partial and/or late shipments, to cover orders for preceding months, took place.

*Actually, α, $\underline{K}$ and $\overline{K}$ being a function of demand rate should be assigned their own column. However, inasmuch as the likelihood of a code changing from one demand rate group to another is small, and in order to simplify the form, they are treated as functional constants. Of course, should a change be required, it can be carried out regardless of where the affected parameters are listed.

Column (5) states the inventory correction, the amount by which tentatively scheduled production will differ from $\hat{D}_m$. It equals

(b) $\underline{K}$ [(Lower Inventory Limit) -(Inventory on Hand)],when inventory on hand is less than the Low inventory limit.

(c) $\overline{K}$ [(Upper Inventory Limit) - (Inventory on Han)], when inventory on hand exceeds the High inventory limit.

(d) Zero, when inventory on hand lies between the Low and High inventory limits. That is, inventory correction may be a negative zero, or a positive quantity.

Column (6) specifies the amount of finished goods inventories available for shipments at the beginning of the Scheduling Period m. The formula is:

(e) $$(\text{Inventory on Hand})_m = (\text{Inventory on Hand})_{m-1} + (\text{Actual Production})_{m-1} - (\text{Actual Shipments})_{m-1}$$

That is, inventory on hand at the beginning of the current scheduling period equals inventory on hand in the beginning of the previous scheduling period, plus actual production during the previous scheduling period. Note that equation (e) can yield a negative answer.

Column (7) states the shop load for the current scheduling period based on the latest revised figures. The formula is:

(f) Scheduled Production = (Forecasted Demand) + (Inventory Correction).

When the sum is either zero or negative, there is no production at all, since negative production has no meaning.

Column (8) states the number of terminal strips actually produced during the current scheduling period. This figure is necessary for a number of reasons (such as excessive spoilage or input shortage) actual production can, and does, differ from the scheduled production quantity.

Column (9) states the expected quantity which will be available for shipment during the current scheduling period, i.e.,

(g) $$(\text{Expected Quantity Available for Shipment})_m = (\text{Inventory on Hand})_m + (\text{Scheduled Production})_m$$

Columns (10) and (11) are completed from Table 2.

Sample problem

To illustrate the plan, consider an example covering a three-month history of a hypothetical "continuous" code - 140D say. The following initial conditions are assumed:

average demand rate = 1,800 terminal strips;
beginning inventory = 1,200 terminal strips.

The first figure enables one to read directly from Table 1 that: the code category is Group Four: $\alpha = .1$; the inventory limits are .3d, .5d, and 1d, respectively; the demand limits are .5d and 1.5d, respectively; and finally, that both correction parameters have a value of .5.

Now it is possible to proceed to compute the various figures, beginning with the first week of January (1/I) as shown in Table 3. The interested reader can easily verify the entries.

Plan III: Production Scheduling

Assume that R hours are required to produce the output specified by Plan Two. Whether the associated schedule need, or need not, be modified can be determined by consulting the rules summarized in Table 4. The table was developed by consulting existing company policies with regard to employment. To explain the table, it is best to describe the meaning of a few entries. For example, the decision shown for the "30%, one week" combination means that when 30% more production hours are required to meet increased demand, but production will return to present level after one week, this should be accomplished through overtime production. The reason is that it is uneconomical to hire workers as a response to conditions which will persist for only one week. On the other hand, if only 10% increase in manpower is required, the recommended action is to maintain production at current level, but use inventories to meet excess demand. Decrease in production rate is interpreted in an analogous manner. Thus, a 10% production drop persisting up to four weeks will be counteracted by maintaining current production level and using slack capacity to build inventories. For periods exceeding four weeks, transfer is stipulated.

Assume then, that R hours are required to produce the output specified by Plan Two. Whether the associated schedule need, or need not, be modified will be determined by consulting Table 4. If transfer, or hiring, or layoff are recommended, the problem of modifying the schedule will not, normally, arise because the changes in manpower can be planned for a new steady production level. Suppose, however, that a condition such as the "plus 10%, one week" arises and it is necessary to meet the additional demand with current manpower. Clearly, this can only be accomplished by using some inventory. But how? To begin with, the excess production can be absorbed by considering first codes with large scheduled production quantities. Second, the current inventory position of these codes must be taken into account to prevent serious inventory inbalance. Then why not define a status index which incorporates both consideration, i.e.,

$$ISI_i = Q_i \frac{IH_i - TI_i}{TI_i} \quad i = 1, \ldots, I$$

where

ISI = Inventory Status Index of code i

IH_i = Inventory on Hand of code i

TI_i = Target Inventory of code i

I = Number of codes scheduled for production (in period m)

Q_i = Number of terminal strips of code i scheduled for production (in period m).

Note that unlike Q, IH, and TI, which by definition, must be positive, IH can assume negative value. But when ISI_i is negative, production of code $_i$ cannot be decreased. This difficulty is easy to overcome by eliminating from consideration every code with negative, and even low positive, inventory status index. Therefore, rank the indexes in order of decreasing value, i.e.,

$$ISI_1 \geq ISI_2 \geq \ldots ISI_k > 0$$

Let r_i+ be the number of production hours required to make

$$Q_i - \beta_i\,(IH - TI) \text{ units of code}$$

$$i, \text{ where } \beta_i \leq Q_i/(TH_i - TI_i).$$

Then, to reduce production by R hours, one only needs to curtail Q_i by the amount $\beta_i\ (TH_i - TI_i)$

such that

$$R \simeq r_1^+ + r_2^+ + \ldots + r_k^+ \quad k \leq I$$

It is not difficult to see that the procedure is equally applicable if it is desired to fill a slack, rather than excess, of R hours. Now the ISI's will be ranked in order of decreasing value, and scheduled production will be increased by r_i-. This is shown in Figure 2.

Some Results

To test the production control system described above, the second and third plans of the system were combined into a single model and programmed for computer simulation. But to take advantage of the capabilities of the computer, some minor refinements were introduced. (This, however, did not change in any significant way the features of the manually oriented system.)

In brief, the computer model operates as follows: initial production is computed using a formula

Table 4. Employment Guide Matrix

Production Level	Duration 1 Week	2 Weeks	4 Weeks	8 Weeks	12 Weeks
+30%	Overtime Production	Overtime Production	Hire	Hire	Hire
+20%	Overtime Production	Overtime Production	Hire	Hire	Hire
+10%	Use Inventory	Increase Production	Overtime Production	Overtime Production	Hire
Current	---	---	---	---	---
-10%	Increase Inventory	Curtail Production	Transfer Workers	Transfer Workers	Transfer Workers
-20%	Curtail Production	Transfer Workers	Transfer Workers	Transfer Workers	Lay Off Workers
-30%	Transfer Workers & Decrease Production	Transfer Workers	Lay Off Workers	Lay Off Workers	Lay Off Workers

such as (A): initial production equals forecasted remains plus some portion of the difference between present inventory and some predetermined target inventory. Demand is forecasted with a second order exponential smoothing formula and production schedules are modified according to rules intended to protect against excessive inventory or stock-outs: i.e.,

(A) A forecast four weeks (periods) ahead is computed using second order exponential smoothing.
(B) Inventory position is checked.
 (a) If inventory is negative:
  (1) The fourth period forecast is increased by some proportion of the negative inventory.
  (2) The present (first period) production schedule is increased by some proportion of the difference between present inventory and twice the target inventory.
 (b) If inventory is positive:
  (1) Inventory is checked against twice the target inventory. If greater than twice target:
   (i) The fourth period forecast is decreased by some proportion of the excess over two times target.
   (ii) The present (first period) forecast is decreased by some proportion of the excess over two times target.
(C) The fourth period production schedule is computed by adding some proportion of the difference between present inventory and target inventory (note that if present inventory exceeds target inventory, the amount added to forecast is negative).
(D) Inventory position is checked against half the target inventory.
 (a) If current inventory is less than half the target inventory, the second period production schedule is increased by some proportion of the average production for that code.
 (b) Otherwise the second period production schedule is not modified.
(E) Inventories are updated by adding current production and subtracting current shipments.
(F) Shipments, current production, inventories, and back orders for the individual codes are cumulated and printed out.
(G) The cycle is repeated for the next week.

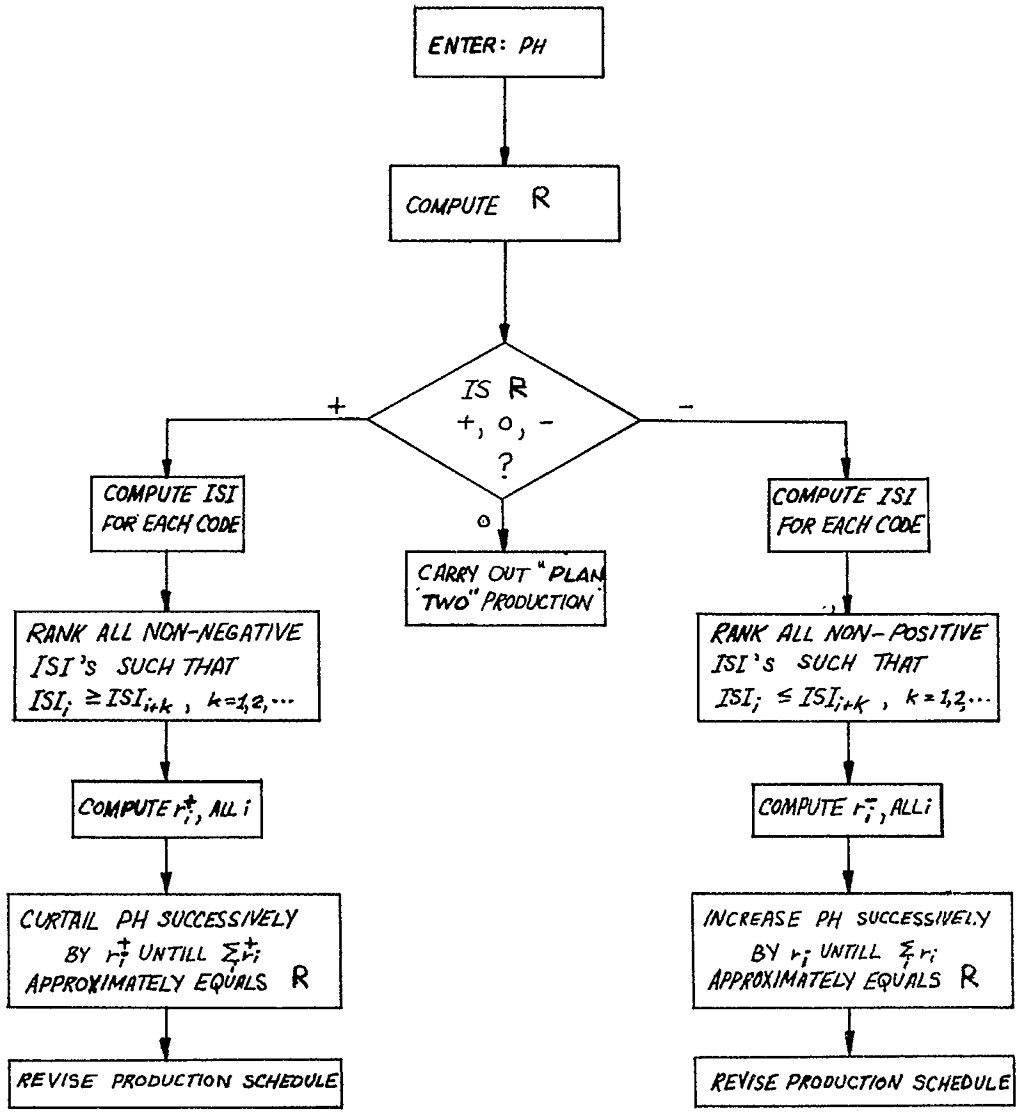


Figure 2. Logic of Schedule Adjustment Routine

Essentially, the model smooths production via exponential smoothing, and maintains inventory control by changing production schedules when inventory differs from target inventory (i.e., exceeds twice target inventory, or is less than half of target inventory). The model flow chart and computer program (written in Fortran) are given in Figures 3 and 4, respectively.

The model was tested by comparing the actual performance of the shop indicated in Figure 1 with that of the model. Several simulation runs were executed, each associated with different set of parameter values. In all the cases simulated, the model yielded vastly superior results. In fact, in one run demand rate of some codes was increased by fifty percent without serious consequences as far as production control is concerned.

The details summarizing this run are presented in Figure 5. The following parameter values were used to obtain the results shown:

- i. α, the second order exponential smoothing constant was 0.05.
- ii. Δ, the production smoothing constant which limits the amount by which fourth period production can vary from third period production was 0.05.
- iii. β, which specifies how much of the difference between actual and target inventory is to be added or subtracted from the

Figure 3

COMPUTER FLOW CHART - PLANS TWO AND THREE

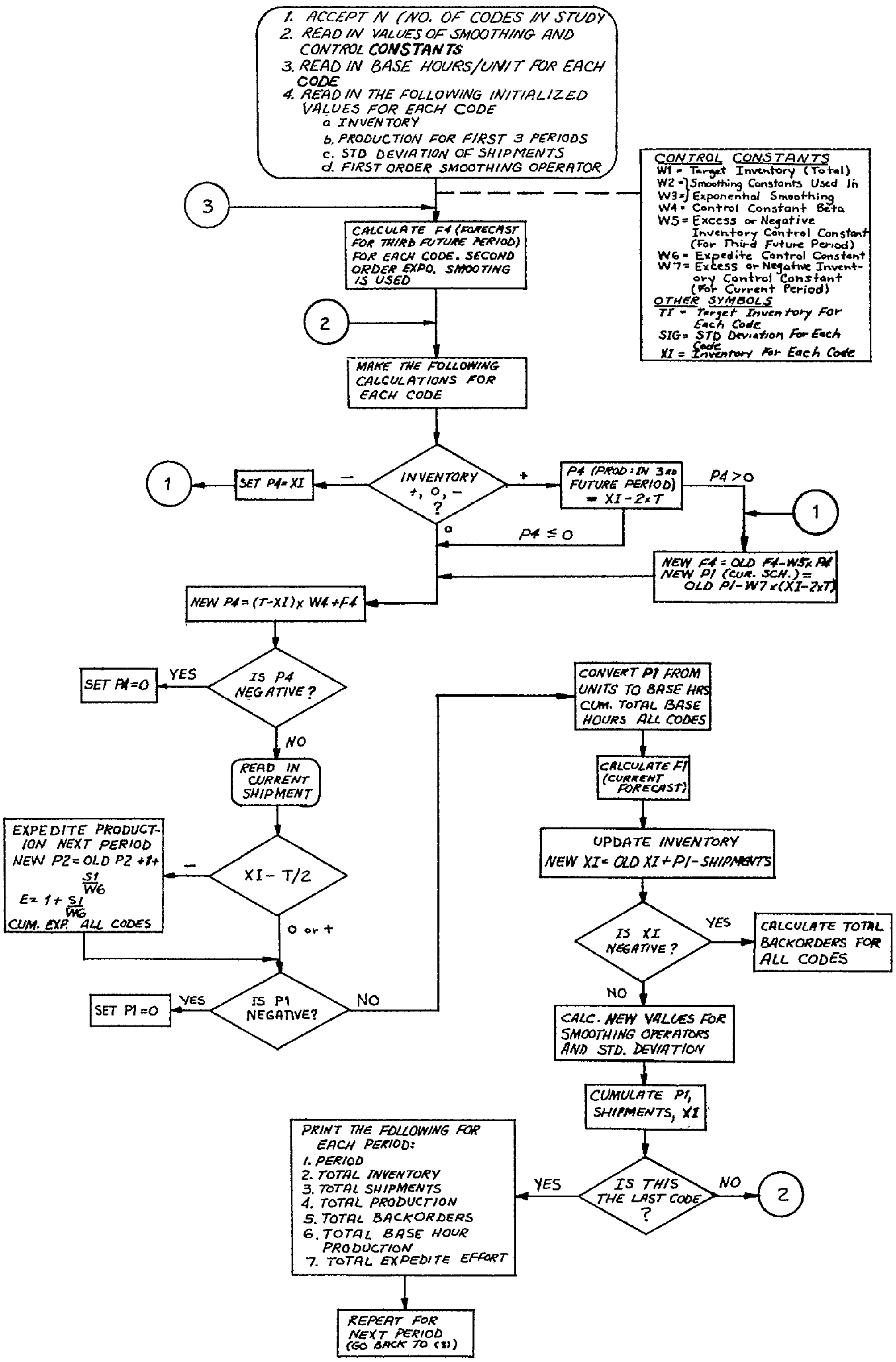


Figure 4

COMPUTER PROGRAM - PLANS TWO AND THREE

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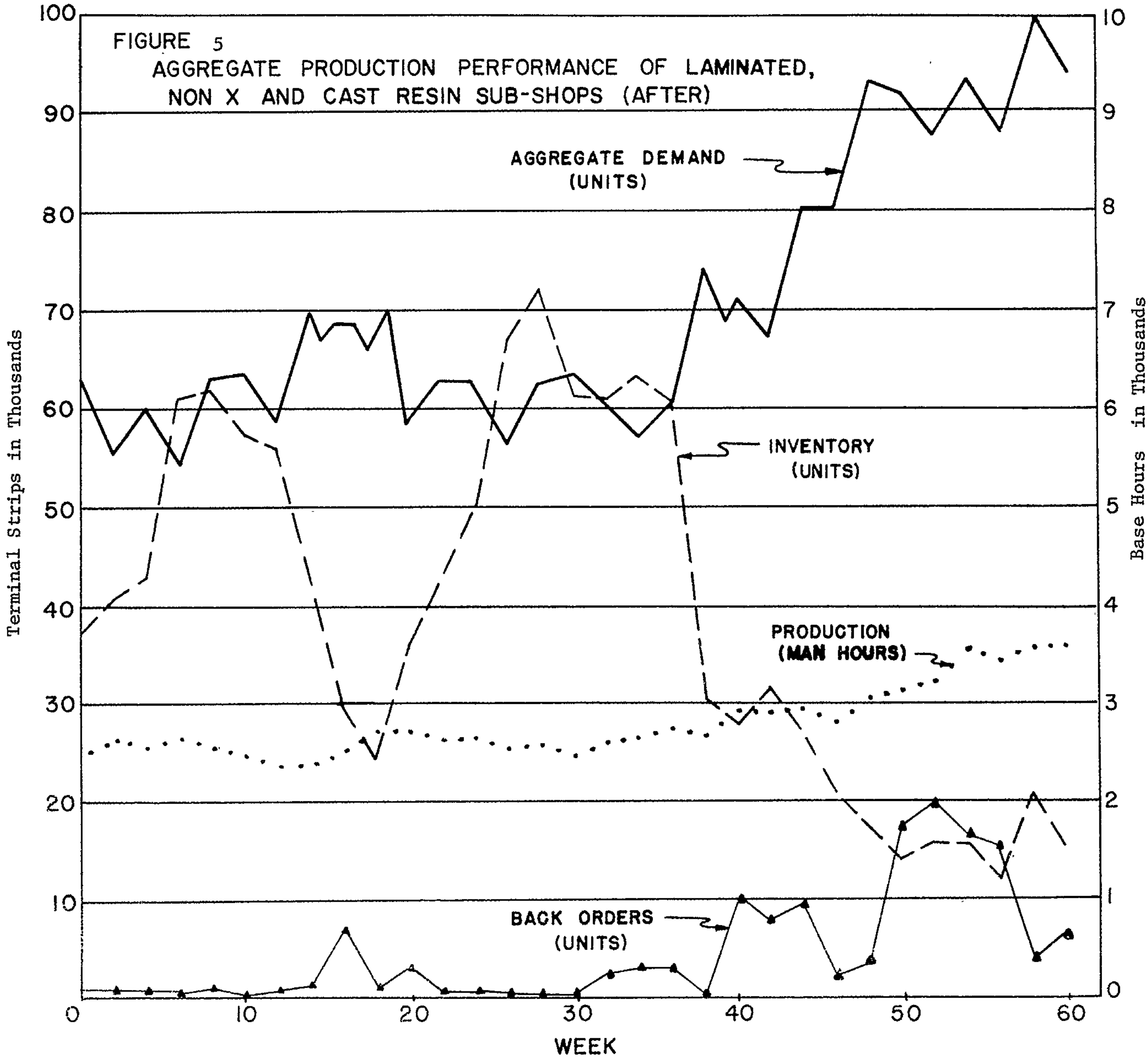
      DIMENSION S1(50), S2(50), P1(50), P2(50), P3(50), X(50), F4(50)
      DIMENSION SIG(50), W(8), BHR(50)
      ACCEPT, N
      DO 1 K=1,N
1     READ, W(K)
      W(8)=W(2)/(1.-W(2))
      DO 70 K=1, N
70    READ, BHR(K)
      TF1=0.
      DO 2 K=1,N
      READ, X(K), P1(K), P2(K), P3(K), SIG(K), S1(K)
      TF1=TF1+S1(K)
2     S2(K)=S1(K)
      PAUSE
3     K2=1
4     SSIG=0.
      XN=0.
      TBHR=0.
      DO 5 K=1,N
      SSIG=SSIG+SIG(K)
5     F4(K=(4.*(W(8*(S1(K)-S2(K)))+2.*S1(K)-S2(K))
8     TA=W(1)/SSIG
      TP1=0.
      TSH=0.
      TX=0.
      TF1=0.
      TP3=0.
      TP4=0.
      E0.
      DO 19 K=1,N
      M=K
      IF(X(K)) 81,84,82
81    P4=X(K)
      GO TO 83
82    P4=X(K)-TA*SIG(K)*2.
      IF(P4 84,84,83
83    F4(K)=F4(K)-W(5)*P4
      P1(K)=P1(K)-(X(K)-TA*SIG(K)*2.)*W(7)
      M=K+200
84    P4=(SIG(K)*TA-X(K))*W(4)+F4(K)
      IF (P4) 9,10,10
9     P4=0.
10    READ, SH
      IF (SH) 40,110,110
110   IF(X(K)-TA*SIG(K)/2.) 11,12,12
11    P2(K)=P2(K)+1.+S1(K)/W(6)
      E=E+1.+S1(K)/W(6)
      M=K+100
12    IF(P1(K)-10000.) 13,14,14
13    KP=P1(K)
      P1(K)=KP
      IF(P1(K)) 71,14,14
71    P1(K)=0.
14    BH=P1(K)*BHR(K)
      TBHR=TBHR+BH
      F1=2.*S1(K)-S2(K)
      X(K)=X(K)+P1(K)-SH
      IF(X(K)) 201,202,202
201   XN=XN+X(K)
202   SIG(K)=SQR(SIG(K)**2*(1.-W(3))+(F1-SH)**2*W(3))
      S1(K)=(1.-W(2))*S1(K)+W(2)*SH
      S2(K)=(1.-W(2))*S2(K)+W(2)*S1(K)
      TP1=TP1+P1(K)
      TSH=TSH+SH
      IF(X(K)) 16,15,15
15    TX=TX+X(K)
16    IF(SENSE SWITCH 2) 17,18
17    PUNCH, M, P1(K), SH, X(K), F1
18    P1(K)=P2(K)
      P2(K)=P3(K)
      P3(K)=P4
19    CONTINUE
      PRINT, K2, TX, TSH, TP1, XN
      PRINT, TBHR
      PRINT, E
      XN=-XN
      PUNCH, K2, TP1, TSH, TX, XN
      K2=K2+1
      GO TO 4
40    STOP
      END
```



demand forecast to arrive at fourth period production forecast, was .250.

iv. γ, the constant to smooth forecast deviation, was 1/8.

v. T, the target inventory, was 35,000 units.

Altogether, the simulation revealed that the size of target inventory is far less important than its composition. At the tactical level, the simulation revealed that small values of α and β should be assigned to determine either production or forecast demand of large runners, i.e., all "continuous" and some "periodic" type codes. The simulation shows that production level is maintained at a reasonably steady level while inventories and back orders fluctuate rather widely. These results come about principally from the choice of rather low smoothing and control constants. Such a choice of constants ensures that production will be relatively steady. Since demand is much more volatile than production, the difference between demand and production must be reflected in inventory change and in back orders. Figure 5 demonstrates that it is possible to have concurrently high inventory and rising back orders, although back orders and inventory generally move in opposite directions. The inventory peaks represent periods of falling demand and steady production; while the falling inventories and rising back orders toward the end of the run represent rapidly increasing demand and steady production. The point to emphasize is that the choice of model parameters depends largely on the goals of the system. That is, steady production is achieved at the expense of inventory swings. Steady inventories are achieved at the expense of fluctuation in production levels.

Concluding Remarks

The writer believes that the production control system described in this paper can be adapted profitably in most shops faced by similar problems. One may argue that the central assumption underlying the system, the existence of "well-behaved" demand patterns is rarely met. But, to paraphrase a former teacher, some structural premises must be accepted as an article of faith; the admonition that anything can happen leads to the culmination of all study. Indeed, the use of almost any statistical concept is contingent on a presumption of general stability in the universe from which the data is drawn.

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Biography

A. G. Beged Dov's background spans several disciplines.Before joining the University of Toledo in 1967, he was a Senior Research Engineer with the Western Electric Company. There he was responsible for directing multi-million dollar projects. From 1947 to 1953 he served in the Israeli Army in combat and staff assignments. Since 1960 he has served as a consultant to firms here and abroad. Professor Beged Dov has authored three books and numerous papers and reports on subjects such as applied mathematics, economics, and operations research. His articles have appeared in the IEEE Transactions, IEEE Proceeding, the Journal of Industrial Engineering Management Science, Electronics and Power, The Western Electric Engineer, Simulation, The American Economic Revue, and other journals. Some of his papers were translated in Russia and other countries and also appeared in several books of readings. Professor Beged Dov holds a BS, MS, and MBA degrees from the University of Toledo, and a Ph.D. in Operations Research from the University of Pennsylvania. He is married and the father of two sons and a daughter.