THE CURRENT STATE OF RESEARCH IN JOB SHOP SCHEDULING

Carter L. Franklin, II
Assistant Professor of Industry
Wharton School of Finance and Commerce
University of Pennsylvania
Philadelphia, Pennsylvania

Abstract

The state of the art of job shop research is investigated via the development of a framework within which problems and results may be described. The bulk of the paper is devoted to the development of the framework and the research which arises from it. Few explicit references are made to the large body of published research on the problem.

As one sits down to draw up a review of the nature suggested by the title above, the first task to be completed is the creation of a context or framework within which the subject matter may be delineated and placed in perspective. Such a framework is helpful to those who would fully understand the work of others, very useful to the empiricist or theorist who would extend knowledge in the problem area and absolutely essential to the reviewer or compiler. If the literature on the job shop is explored the paucity of appropriate frameworks, implicit or explicit, is readily apparent. The need for a framework within which experiments and results may be couched is obvious. The reasons for the lack of one are not so obvious until the task of creating one is undertaken. The job shop problem _is_ difficult and complex, and the value of a framework rises in direct proportion to the magnitude of the difficulty in creating it. The old adage, "a problem well defined is a problem half solved" has never had a more appropriate application. Research leading to an understanding of the job shop process is largely unavailable and the years since the computer first allowed rapid and efficient application of computational resources to the problem have yielded little knowledge which may be generalized across the wide variety of job shops extant or conceivable. A "state of the art" review must primarily assess the distance travelled so far and the distance and direction of travel remaining. It is my own opinion that a very large part of the problem remains unsolved. We have only "brute force" computational procedures for dealing with complexity. We have no means of setting bounds on solution values and, in fact, no means of determining appropriate or inappropriate approaches to any particular problem. We are left in a position of having to rely on our intuition and the power of our machines. I am left with a feeling of discomfort and a motivation to suggest some remedies.

What follows is primarily the precursor of a "state of the art" analysis. A largely conceptual framework for viewing the job shop problem is presented, an extension of one I have developed during several years of research on the problem. Explicit references to the large literature on the problem are largely absent although the knowledgeable and perceptive reader will recognize much that has been done or suggested. A direction for future efforts is developed which arises as a natural and expected corollary to the development of a framework within which a problem is defined.

The abstraction which forms the basis of the proposed structure for the job shop problem is the following function.

$$\text{Value of Output} = f(\text{Scheduling Technique}, \text{Product Mix}). \quad (1)$$

The function explicitly states that value of the shop's output depends upon both the technique or rule employed for scheduling and the product mix or aggregation of jobs upon which the technique operates. The value of output is measured by a suitably defined objective function which evaluates schedule characteristics to determine schedule quality. The viability of this relation has been subjected to some very simple tests. The results of these tests [8] suggest that the relation is potentially of significant power in explaining the job shop process. Among the results of interest are (1) that value of output or schedule quality is linearly related to the work content of the product mix, (2) that schedule quality is related to the "job shoppiness" of the process by a cubic function, and (3) that "bottlenecks" early in processing are less deleterious to schedule quality than late "bottlenecks" which, in turn, are less deleterious than "bottlenecks" in the middle of the processing of the average job. A special case of the general relation was used for experimental purposes. It is

$$\text{Value of Output} = f(\text{Product Mix} \mid \text{Scheduling Technique}). \quad (2)$$

The objective function measured the completion time of the last job completed in the schedule. The results of the experimentation reported suggest that the relation does, in fact, represent the job shop process. It appears that the construct represented by the relation will enable us to formally structure any job shop problem in such a way that it is completely identified and such that we can accurately predict the performance of any scheduling technique. The construct is, however, not without limitations. While a theoretical relation is implied, the function does not suggest its form. That is left to our intuition or our experimental investigations. Due to the difficult and complex nature of the job shop problem, it is likely that the exact nature of the relation will have to be investigated through simulation.

One may identify four basic components of every experimental or theoretical approach to the job shop problem. They are: (1) the model of the process, (2) the scheduling technique, (3) the product mix, the particular problem under analysis, and (4) the objective function, the measure of solution quality. The kind of model employed to describe the job shop process defines the way in which the independent variables in the above function are described. It also constrains the nature of the schedules which may be created for any product mix. Models may be characterized by the presence or absence of the following restrictions on processing (see [4] and [20]): (a) whether or not jobs are required to be strictly ordered sequences of operations, without assembly or partition, (b) whether or not machines are continuously available for assignment, without planned or random interruptions of processing, (c) whether or not an operation can be performed by more than one machine in the shop, (d) whether or not multiple machines of a single type exist in the shop, (e) whether or not preemption is allowed, (f) whether or not "lap-phasing" is allowed, (g) whether or not each machine can perform more than one operation at a single point in time, (h) whether or not setups are considered explicitly, and (i) whether or not jobs are instantaneously transferred from one machine to the next. In addition, a broader characterization of models may be made. Models may be "static" or "dynamic." In the former case, all jobs are available for scheduling simultaneously while in the latter, jobs arrive continually during the scheduling interval. In the static case, all information concerning the product mix is available at the outset and "global" decision may be made. Here, Gantt charts may be created or integer linear programs may be employed. The dynamic model requires that a queueing or queueing-like simulation must be employed to obtain schedules. These decisions are "local" and events must be allowed to run their course before the schedule can be evaluated. The characteristics of the model used in any investigation of the job shop scheduling problem is the result of the experimenter's perception of what is appropriate for the investigation and his biases toward the problem.

Under the constraints imposed by the model of the job shop process, the scheduling technique examines the characteristics of a subset of the product mix and makes decisions regarding processing sequence and time intervals to be assigned to the processing of jobs. A "rational" scheduling technique is one which attends to those characteristics of jobs for which cardinal measures exist. A "non-rational" technique makes sequence decisions without reference to the characteristics of the alternatives. A cross classification scheme for scheduling techniques involves whether or not the job to be processed "next" is actually available for processing. If the "next" job must be immediately available for processing, the technique may be called "non-delay." If the machine may stand idle while awaiting the next job, the technique is a "delay" technique. (In [4], this descriptor is applied to schedules, but it more appropriately characterizes a scheduling technique.)

The object of the action of the scheduling technique is the product mix to be scheduled. The result of the scheduling process is a schedule which is evaluated by the objective function. The product mix determines the range of values which the objective function may achieve, and for a particular scheduling technique, determines objective function value. If either the product mix or the scheduling technique exhibits stochastic properties, then only the distribution of objective function values may be predicted. Objective functions may be characterized in a variety of ways. A scheme is suggested in [4] wherein objective functions are either "regular measures of performance" or not. A regular measure is a function to be minimized, defined upon the completion times of jobs which increases only if one or more completion time increases. A cross classification scheme for objective functions would describe objective functions in terms of the measures upon schedules they constitute. Here, an objective function might measure mean flow time, maximum completion time, mean waiting time, etc. When a non-regular objective function is appropriate (e.g., minimize the maximum completion time), values of the objective function may be predicted as a function of product mix and scheduling technique. Objective functions of this type are generally measures of the extrema of the quantities measured by regular objective functions. Since distributions of schedule parameters may be predicted as a function of product mix and scheduling technique, then extrema of these distributions may also be predicted.

Thus far, we have briefly touched upon three of the four components necessary to an investigation of the job shop process. The fourth, product mix, has been deferred until now since we have wished to develop the argument that the product mix is instrumental to every analysis. It is upon product mix characteristics that the scheduling technique focuses for its decisions; it is the product mix which constrains the range of values which the objective function may achieve; and, as we shall see, much of the description of the model of processing may be stated in terms of the characteristics of the product mix. The strategem here is to burden the characterization of product mix with the major part of the problem definition. This seems to be the logical choice for the reasons cited immediately above. A generally viable solution to the job shop scheduling problem must, we believe, be stated in terms of the general function stated at the beginning of this paper, or upon some similar construct. The solution to the problem may then be obtained through a theoretical or empirical investigation of the nature of the function. It is unfortunate that a list of product mix characteristics significant in the function relating value of output to product mix and scheduling technique cannot be stated now. This problem is, however, part of the problem of finding a solution to the job shop scheduling problem. It seems appropriate at this point to create a list of product mix characteristics which may prove significant in our functional relation. The list will most probably be incomplete in some respects and redundant in others, since it is the first to be formulated. Following the presentation of the list of product mix characteristics, a revised description of models of the job shop process will be suggested and a way of looking at the problem represented by equation (1) will be developed.

The description of a product mix by the variables which are instrumental in the function cited at the outset of this paper must be available to the analyst a priori of any attempt at scheduling and independent of the scheduling technique to be employed. Most of the variables or descriptors suggested here will have some intuitive appeal to the reader. No arguments in support of the variables will be suggested due to the apparent complexity of the functional relation. There are three broad classes of variables which are suggested: (1) those which aggregate characteristics across jobs, (2) those which aggregate characteristics across machines, and (3) those which are not specifically related to either individual jobs or individual machines. Integral with the description of the variables will be a parenthetical reference to its class. The variables are as follows:

(1) Total work content of the product mix to be processed (3). This is simply the sum of all setup and processing times required to produce the product mix.

(2) The distribution of work content among jobs (1) or among machines (2). First and higher moments of these distributions may be explicitly considered.

(3) The total number of operations or precedence relations to be scheduled (1).

(4) The distribution of operations or precedence relations among machines (2).

(5) The distribution of the ratio of setup time to processing time for each machine (2), each job (1), or the aggregate across all operations of all jobs (1).

(6) The distribution of job "slack" (1). Job slack is defined as due date less release date less the work content of the job.

(7) The distribution of tardiness penalty function parameters (1). It is assumed here that tardiness penalties are assessed by a function common to all jobs.

(8) The distribution of job value (1). The value of an individual job may be either the revenue or the profit it produces.

(9) The "job shoppiness" of the product mix (3). This variable, ρ, is a measure of how job shop like (as contrasted to flow shop like) the processing of a product mix is. Many such measures may be constructed. One such is described by the following steps:

   (a) Noting that the numbering of machines is arbitrary, define β_i for each machine i as the average percentage of completion of the jobs processed on i as they arrive at i.

   (b) Renumber machines in order of increasing β_i.

   (c) For each i, form the distribution of percentage completion of jobs processed on i.

   (d) Correlate the percentage complete distribution with machine number.

   (e) If the correlation is perfect, the processing is that of a pure flow shop, if zero, the processing is that of a pure job shop.

(f) The correlation coefficient, ρ, is a measure of the processing of a product mix, placing it on the continuum whose extremes are defined by pure job shop processing (ρ = 0.0) and pure flow shop processing (ρ = 1.0).

(10) The distribution of arrivals of jobs in the shop (1). If a "static" model is employed, the appropriate measure involves the distribution of times when jobs are available to the shop (1) and the distribution of times when machines become available for processing jobs in the product mix (2). The latter distribution may also be represented as a plot of machine availability versus β_i' (2).

(11) The fraction of work content to be executed by each machine as numbered in order of β_i (2). This serial measure will identify bottlenecks in the processing of jobs in the product mix.

(12) The number of operations or precedence relations to be scheduled on each machine as ordered by β_i' (2). Here a different construction may be given to the term "bottleneck."

(13) Technological Risk: The production of any product mix is not a completely deterministic process, although we have implicitly assumed determinism to this point. The notion of technological risk must be introduced at this point. Technological risk is defined as that natural variation in the production process which is beyond the control of management, and arises out of an inability to accurately define operations in the time dimension. Assuming all technological variation is normally distributed, we may define the technological risk of a product mix (3) as the sum of the coefficients of variation of all parameters which are stochastically defined. If technological risk occurs as the joint result of two or more stochastic processes at some point, the risk at that point is the product of the coefficients of variation.

(14) The technological risk of a job (1) is the sum of the coefficients of variation at all points of risk in the definition of the job.

(15) The technological risk associated with a machine (2) is the sum of the coefficients of variation at all points of risk for the operations of individual jobs processed on that machine.

(16) Technological risk may be defined across machines through a plot of risk versus machine number as defined by β_i' (2).

(17) Technological uncertainty: The production of a product mix may not be completely determined in that the shop may exert control over the production requirements of jobs through taking advantage of processing flexibility resulting from the characteristics of the resources of the shop, the characteristics of individual jobs, or characteristics of the jobs to be processed on a single machine. This flexibility is designated as technological uncertainty and occurs when the shop's management may choose from among several (usually discrete) alternative means of producing a job. Technological uncertainty usually occurs as the result of the existence of an alternate machine for processing an operation, an alternate sequence for the processing of a contiguous subset of the operations of a job, or sequence dependent setup times. Each of these is discussed.

(a) Where alternate machines exist, the processing of the operation may proceed at equal or greater cost than the processing on the nominal machine. The former occurs when two or more identical machines exist in the shop, the latter when two or more functionally similar but economically distinct machines exist. In the former case, some distortion in the computation of ρ may occur due to the arbitrary choice of a machine to process an operation. For this reason, the calculation of β_i' and ρ should be carried out as if only one of each machine type were extant. Since work may be apportioned among identical machines in any way, its distribution should follow the dictates of the relation between value of output and the distribution of work among machines or the distribution of operations among machines. The appropriate measure of this type of technological uncertainty (3) is obviously dependent upon explicit knowledge of the relation cited immediately above.

(b) The existence of alternate processing sequences represents the ability of the shop to choose from among several alternative jobs for inclusion in its product mix. The appropriate measure depends upon explicit knowledge of the relation between value of output and ρ, the distribution of work among machines, etc. (3).

(c) A measure of the variability of value of output as a function of the alternative setup times possible for an operation is difficult to obtain. The difficulty is due to the inability to predict what setup alternatives are available to the scheduler at any point in time. The setup alternatives available depend directly upon the jobs available for processing at a point in time which depends, in turn, upon the product mix and scheduling technique extant in the shop. This recursiveness has confounded many investigators and will not be resolved here. Its solution is a statement of the status of every job at all points during the schedule interval. The measure of this type of technological uncertainty which we suggest is the sum of the coefficients of variation of the rows of the matrix defining the setup alternatives (2). This measure may also be summed across machines (3) for a measure on the product mix as a whole.

Having presented our list of variables describing a product mix, we may return to a consideration of the model of the process, the scheduling technique, and the objective function. Reconsidering the characterization of models, the terms static or dynamic must still be used as well as the terms global and local. These are general descriptors and are necessary to the definition of how the model replicates the process. The list of descriptors of the necessary details of the replication may be sharply truncated. Many of these descriptors have been subsumed in the list of variables defining a product mix. This is appropriate since the presence or absence of variability in these characteristics of product mix in a set of jobs to be scheduled under the model does not impair the function of the model in any way. Four of the nine descriptors may be eliminated, leaving (b) the continuity of machine availability, (e) the possibility of preemption, (f) the possibility of "lap-phasing," (g) the multiple processing ability of machines, and (i) the transfer times of jobs between operations. The last of these is the obverse of "lap-phasing" and may, if desired, be viewed as a special case of lap-phasing.

The description of scheduling technique may be couched in terms of the variables describing product mix. The general descriptors, rational and non-delay, are appropriate for the general description and a description of precisely how the rule operates may be drawn in terms of the product mix upon which it operates. At any and every decision point in the scheduling process a rational scheduling technique looks at the characteristics of a subset of the product mix and assigns to each member of the subset a position on a cardinal scale. The job to be processed "next" is usually the one assigned maximum or minimum value. If the technique is a non-delay technique the subset is that available for immediate processing on the machine in question. If a delay technique is used, the subset of jobs also (usually) includes those jobs in process on other machines.

The classification of the objective function in terms of product mix is not possible. Even though product mix defines the range of objective function values (values of output) which may be obtained, the nature of the objective function reflects the goals or object of the shop with respect to processing. The objective function for the rational shop should reflect the value of the shop's capability to itself and its environment. For example, if the process is capital intensive in the sense that costly raw materials are employed, then the objective should reflect this through evaluating flow times or work in process inventory. If the process is capital intensive in the sense that the processing employs costly machines, then the objective function should attend to the utilization of machines. If the process is labor intensive, then the objective function should reflect this. If the shop gains a competitive advantage from rapid delivery or short lead times, the objective function should have a strong "due date" orientation.

The objective functions cited have a common characteristic. They measure schedule quality or value "unidimensionally." Objective functions of this type have commonly been employed in job shop research to date. This research may be characterized as an investigation of the characteristics of the function.

$$\text{Value of Output} = f(\text{Scheduling Technique} \mid \text{Product Mix}) \quad (3)$$

The research has been successful in the sense that a technique which is the best of those tested may be discovered empirically. The research has been unsuccessful in citing a dimensional space for describing scheduling techniques, in finding the optimal scheduling technique for any problem (i.e., product mix and objective function), and in providing insights into the nature of the relation. A larger problem exists, however. It is rarely the case that a "Unidimensional" or "pure" objective function is appropriate for describing the goals of a "real" job shop. If we may assume that an appropriate objective function is a linear combination of two or more unidimensional functions and we realize that each shop is unique (and perhaps unique at each point in time), then the need for insight into the behavior of the above function for "mixed" objective functions is apparent. Since it appears that (from past research) a unique, "best" scheduling technique may be found for each "unidimensional" objective function, it is possible that the "best" technique for a "mixed" objective may be a linear combination of the "best" techniques for the components of the "mix" in identical or similar combination. This proposition is certainly testable empirically, and, if true, allows the solution of any problem for which an objective function may be stated. If the proposition is false, the need for a more appropriate theory is clear or a search for a single scheduling technique, "best" for all objectives, may be initiated.

A parallel line of investigation has also been suggested, that which centers around determining those variables significant in (2) and the nature of the relations among them. These investigations are complementary in view of the fact that scheduling techniques may be described in terms of the product mix characteristics appropriate for the dominant ranking of potential "next" jobs. The ultimate problem is an explicit statement of (1), which may not be possible for the general case. The obvious infinity of functions (1), may, however, yield to insight from analysis of (2) and (3). It is counter-intuitive that the product mix descriptors significant in (2) will be much different from those in (3) for describing dominant scheduling techniques.

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Biographical Sketch

Carter L. Franklin, II, Ph.D. is Assistant Professor of Industry at the Wharton School of Finance and Commerce of the University of Pennsylvania. He holds a B.S. in Chemistry from Carnegie Institute of Technology (1961) and a Ph.D. in Industrial Administration from Purdue University (1968). Dr. Franklin teaches courses in Operations and Production Management at the MBA and Ph.D. level for the Wharton School. In addition, he serves in an advisory capacity to several companies and the United States Air Force on a variety of operations planning problems. His specific research interests include the theory and design of scheduling systems and related problems of resource allocation.