CONSISTENCY OF RISK ATTITUDE IN THE INVESTMENT DECISION PROCESS

Moon K. Kim
Syracuse University

ABSTRACT

Risky investments are analyzed through a computer simulation process or with the use of the risk-adjusted discount rate and certainty-equivalent coefficient. A constant risk-adjusted discount rate or certainty-equivalent coefficient is frequently used in the evaluation of risky projects, while a constant risk-aversion coefficient is used in the simulation approach. The question raised in this paper is that one's consistent risk attitude is properly reflected in the risk analysis. Each approach was compared to each other to see if the risk-adjusted discount rate and certainty-equivalent coefficient become constant when the risk-aversion coefficient is held constant. Namely, this paper shows that the risk-adjusted discount rate and certainty-equivalent coefficient should be different from project to project to have a consistent risk attitude. Indeed, if a constant risk-adjusted discount rate or certainty-equivalent coefficient is used, variant risk-aversion coefficents or risk attitudes would be the result.

INTRODUCTION

In the capital allocation process, future uncertain cash flows of an investment are transformed into a certainty-equivalent cash value through the discounting scheme either with a risk-adjusted discount (RAD) rate or with a certainty-equivalent (CE) coefficient. Moreover, another widely used approach is the direct evaluation of projects through simulation. The simulation process is generally composed of three steps. First, a number of cash flow patterns through the life of an investment are generated by picking up a specific number out of each period's probability distribution of cash flows. Second, each cash flow pattern is discounted to obtain its net present value. Third, the probability distribution of net present values thus obtained in the second step is evaluated based on a mean-variance utility function (or quadratic loss function) where the variance is penalized against the expected net present value.

The RAD rate and CE coefficient have been well known to us but it is not easy to specify their values. Furthermore, a constant RAD rate or CE coefficient is repeatedly used through a set of projects. For example, if an investor claims that the above three approaches yield the same results, he should be able to reflect his consistent risk-aversion attitude in the RAD rate and CE coefficient. The purpose, then, of this paper is to compare the simulation approach to the other two approaches and to show, in fact, that the use of a constant RAD rate and CE coefficient may lead to variant risk attitudes.

THREE CERTAINTY-EQUIVALENT CASH VALUES

Assuming that an investment project has random cash flows, C_t, t=0,1, ... , n, with mean $E(C_t)$ and variance $\sigma^2(C_t)$, where n is the expected life of the investment, let us briefly examine the three approaches with which uncertain cash flows are transformed into a certainty-equivalent cash value.

(i) Risk-Adjusted Discount Rate Approach

The net present value (P_k) of cash flows discounted at the RAD rate is defined as:

$$P_k = \sum_{t=0}^{n} \frac{E(C_t)}{(1+k)^t}, \qquad (1)$$

where k is the average RAD rate over the life of the investment. An RAD rate is composed of two parts: the risk-free rate, i, and the risk premium rate (ϕ) associated with uncertainty about expected cash flows. As uncertainty about expected cash flows increases, a higher risk premium is required.

(ii) Certainty-Equivalent Coefficient Approach

Robichek and Myers [5] have proposed the CE coefficient approach:

$$P_\alpha = \sum_{t=0}^{n} \frac{\alpha_t E(C_t)}{(1+i)^t}, \qquad (2)$$

where P_α is the CE cash value and α_t is the CE coefficient. In this framework random cash flows are transformed into a CE cash value in each period and discounted at the risk-free rate to obtain a single CE cash value.

(iii) Simulation Approach

Instead of adjusting uncertain cash flows with α_t or k, we simply discount all possible patterns of cash flows with the risk-free rate and evaluate the probability distribution of such net present values. This approach is suggested by many authors including Hillier [3], Hertz [2] and Van Horne [6]. Cash flows are discounted at the risk-free rate since the variance of net present values thus obtained will be used as a risk measure. Accordingly, if the RAD rate is used, this would be a double adjustment for uncertainty [5]. Allowing P_i to be the net present value of a cash flow pattern discounted at the risk-free rate, i, results in

$$P_i = \sum_{t=0}^{n} \frac{C_t}{(1+i)^t} . \quad (3)$$

Since C_t is a random variable, P_i is also a random variable. Now, the expected value of the P_i distribution becomes:

$$E(P_i) = \sum_{t=0}^{n} \frac{E(C_t)}{(1+i)^t} . \quad (4)$$

Cash flows of different periods may or may not be independently distributed. Let ρ_{ts} be the correlation coefficient of cash flows between periods t and s. The variance of the P_i distribution becomes [1]:

$$\sigma^2(P_i) = \sum_{t=0}^{n} \frac{\sigma^2(C_t)}{(1+i)^{2t}} +$$

$$2 \sum_{t=0}^{n-1} \sum_{s=t+1}^{n} \frac{\rho_{ts}\,\sigma(C_t)\,\sigma(C_s)}{(1+i)^{t+s}} , \quad (5)$$

For a given expected life of investment, the mean and variance of P_i's can be mathematically defined as above. If, however, n is also a random variable, it is not quite so easy to obtain the mean and variance (especially the variance) from mathematical formulations. In this case, it is preferred to run a number of simulations from which cash flow patterns are enumerated. Then, the mean and variance of net present values of those cash flow patterns are calculated. Once we obtain the expected value and variance of the P_i distribution, we can utilize the expected mean-variance utility function [4]. The mean-variance utility function takes the following form:

$$P_\lambda = E(P_i) - \lambda\,\sigma^2(P_i), \quad (6)$$

where λ is the risk-aversion coefficient and P_λ is the CE cash value obtained with the simulation approach.

RELATIONSHIPS AMONG THE THREE APPROACHES

Granted, an investor may evaluate CE cash value of future uncertain cash flows with one of the above three approaches. If, however, he has a certain attitude toward risk, what should be the relevant RAD rate? More specifically, if he has a certain risk-aversion coefficient, what should be the risk premium over the risk-free rate?

First, let us assume that the CE values derived with the RAD approach and the risk-aversion coefficient approach are the same, hence $P_k = P_\lambda$, or

$$\sum_{t=0}^{n} \frac{E(C_t)}{(1+k)^t} = E(P_i) - \lambda\,\sigma^2(P_i). \quad (7)$$

Since $E(P_i)$ is defined in equation (4) and $\sigma^2(P_i)$ in equation (5), if the λ value is supplied, the right-hand side of equation (7) becomes a constant. It follows that the procedure to solve for k is exactly the same as the internal rate of return is solved from a present value equation; and k is unique given the level of λ.

The $P_k = P_\lambda$ relationship indicates that each project should be discounted at a unique k for a certain attitude toward risk. Yet, if two projects have different $E(P_i)$ and $\sigma^2(P_i)$ values for a given λ, the k values may be different. The k value is a function of the risk-aversion coefficient as well as $E(P_i)$ and $\sigma^2(P_i)$ for each project. Conversely, if one uses a uniform k throughout investment projects, it implies that he employs different attitudes toward risk because $E(P_i)$ and $\sigma^2(P_i)$ vary from project to project.

FIGURE 1

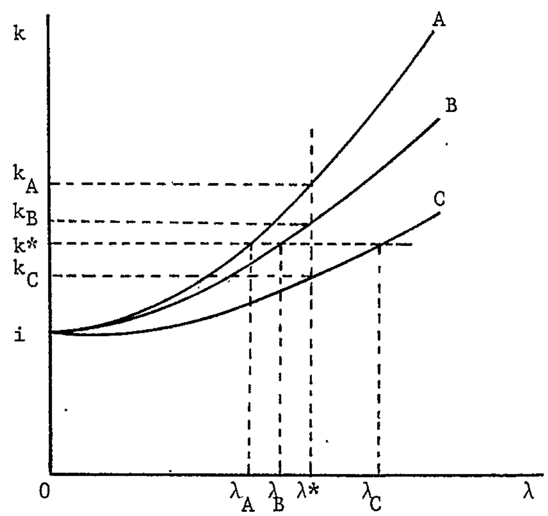

Figure 1 shows three different curves of k for projects A, B, and C as λ varies. Given a λ* value, the RAD rates of projects A, B, and C are k_A, k_B, and k_C, respectively. Instead, if uniform RAD rate, k*, is used through the three projects, the risk-aversion coefficient then implicitly used for each project varies. This is contrary to the assumption that an investor has a consistent risk-aversion attitude.

The relationship between the CE coefficient and the RAD rate already is shown in Robichek and Myers [5]. Suppose that CE cash values with both approaches are the same, then $P_k = P_\lambda$. From equations (1) and (2).

$$\sum_{t=0}^{n} \frac{E(C_t)}{(1+k)^t} = \sum_{t=0}^{n} \frac{\alpha_t\, E(C_t)}{(1+i)^t}. \tag{8}$$

From equation (8),

$$\alpha_t = \left(\frac{1+i}{1+k}\right)^t. \tag{9}$$

Thus, if k is known, α_t can be known. If k varies from project to project, α_t also varies.

As we found in the relationship between P_k and P_λ, k varies from project to project for a given λ value, which, in turn, provides variant α_t values. Indeed, one's consistent risk attitude requires different RAD rates or CE coefficients for different projects. Otherwise, one will not maintain his consistent risk attitude.

AN EXAMPLE

Suppose an investment proposal has the following cash flows which are mutually independent of other periods' cash flows:

Period	Expected Cash Flow	Standard Deviation of Cash Flows	Coefficient of Variation
0	$-800	$8	0.01
1	400	4	0.01
2	400	4	0.01
3	400	4	0.01

In this example the coefficient of variation $\sigma(C_t)/E(C_t)$ for each period is maintained constant: that is, the degree of uncertainty is the same. First, the mean and variance of the probabilistic distribution of net present values are derived from equations (4) and (5) at i = 5%:

$$E(P_i) = \sum_{t=0}^{3} \frac{E(C_t)}{(1+i)^t} = 289.30$$

$$\sigma^2(P_i) = \sum_{t=0}^{3} \frac{\sigma^2(C_t)}{(1+i)^{2t}} = 103.62$$

Then, from equation (7) with λ = 0.5, we obtain:

$$-800 + \frac{400}{(1+k)} + \frac{400}{(1+k)^2} + \frac{400}{(1+k)^3} =$$

$$289.30 - (0.5)\ 103.62$$

If we solve for k, k=0.0764 and ϕ=0.0264. The risk-aversion coefficient of 0.5 is equivalent to the RAD rate of 0.0764 for this project. The CE coefficients can be obtained from equation (9) given the k value: $\alpha_1 = 0.9755$, $\alpha_2 = 0.9516$, and $\alpha_3 = 0.9055$.

Table 1 shows various ϕ and α values as λ varies from 0.0 through 0.50 with the above example. ϕ and α values as the coefficient of variation changes are shown in Table 2. As the risk-aversion coefficient and coefficient of variation increases, the risk premium becomes larger and the certainty-equivalent coefficient decreases. For an extremely large λ or coefficient variation, ϕ will have positive infinity, then negative infinity. But this case is not practical and will not be considered.

TABLE 1

The risk premium and certainty-equivalent coefficient values as the risk-aversion coefficient varies when $i = 5\%$ and $\sigma(C_t)\ /\ E\ (C_t) = 0.02$

Risk-Aversion Coefficient (λ)	Risk Premium (ϕ)	Certainty-Equivalent Coefficient (α_1)
0.0	0.0	1.0
0.05	0.0103	0.9903
0.10	0.0210	0.9804
0.15	0.0320	0.9704
0.20	0.0434	0.9603
0.25	0.0552	0.9500
0.30	0.0674	0.9396
0.35	0.0801	0.9291
0.40	0.0933	0.9184
0.45	0.1070	0.9075
0.50	0.1212	0.8965

TABLE 2

The risk premium and certainty-equivalent coefficient values as the coefficient of variation varies when $i = 5\%$ and $\lambda = 0.10$

Coefficient of Variation	Risk Premium (ϕ)	Certainty-Equivalent Coefficient (α_1)
0.0	0.0	1.0
0.01	0.0051	0.9952
0.02	0.0210	0.9804
0.03	0.0493	0.9552
0.04	0.0933	0.9184
0.05	0.1595	0.8682
0.06	0.2601	0.8015
0.07	0.4222	0.7132
0.08	0.7191	0.5935
0.09	1.4471	0.4205
0.10	7.4635	0.1233

SUMMARY

The certainty-equivalent cash values can be achieved with three approaches: (1) the conventional net present value method with the risk-adjusted discount rate, (2) the certainty-equivalent coefficient method, and (3) the simulation approach with the probabilistic distribution of net present values. The relationships among the three approaches are derived under the assumption that they generate the same certainty-equivalent cash values when an investor evaluates a project.

The risk premium included in the RAD rate was expressed as the function of one's risk-aversion coefficient and the mean-variance of cash flows through time periods. The result shows that any two projects which have different mean and variance values will not have the same risk premium and certainty-equivalent coefficient. This implies that an investor cannot use the constant RAD rate or CE coefficient for different projects. If he does, his utility function would vary. Lastly, the use of a constant RAD rate may result in wrong capital investment decisions.

BIBLIOGRAPHY

[1] Bussey, Lynn E. and Stevens, G.T. Jr., "Formulating Correlated Cash Flow Streams," Engineering Economist, (Fall, 1972), pp. 1-30.

[2] Hertz, David B., "Risk Analysis in Capital Investment," Harvard Business Review, (January-February, 1964), pp. 95-106.

[3] Hillier, Frederick S., "The Derivation of Probabilistic Information for the Evaluation of Risky Investment," Management Science, (April, 1963), pp. 443-57.

[4] Markowitz, Harry M., Portfolio Selection: Efficient Diversification of Investment, John Wiley & Sons, Inc., New York, 1959.

[5] Robichek, Alexander A. and Myers, Stewart C., Optimal Financing Decisions, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1965.

[6] Van Horne, James C., "Capital-Budgeting Decisions Involving Combinations of Risky Investments," Management Science, (October, 1966), pp. 84-92.