SIMULATION OF EFFECTS OF UNCERTAINTY IN LARGE LINEAR MODELS

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ABSTRACT

Leontief Input Output (I-O) models contain large numbers of parameters which must be estimated. These models are widely used in energy and economic analyses without mention of the fact that relatively large estimation errors can cause large errors in model outputs. This paper discusses a simulation performed on a 90 sector I-O energy model in an attempt to statistically quantify the effects of these estimation errors on the entire Leontief inverse $(I-A)^{-1}$, the vector of total output X, and the vector of total primary energy intensities ε (i.e. the total out of ground energy embodied in a unit of output of each sector). Bias relative to published values, variance measures and their relation to error bounds and the sensitivity of the results to assumptions on estimation errors are discussed.

In particular, the sample means $\hat{\mu}$ are virtually all within 2% of their respective published values. It is further shown that $3\hat{\sigma}/\hat{\mu}$ (where $\hat{\sigma}^2$ is the unbiased estimate of the variance of an element of the result set) is less than 20% in most cases. Confidence intervals on $\hat{\mu}$ and $\hat{\sigma}$ are small enough that the above results would not change substantially if the sample statistics were replaced by the population means and standard deviations.

These results are further shown to be quite stable with respect to assumptions on estimation error statistics since doubling the estimation uncertainty only increased the output uncertainty by a factor of two. Finally, the stochastic error bounds on model outputs derived here are shown to be significantly lower than previously available worst case bounds which assumed no error cancellation.

1.0 INTRODUCTION

This paper describes a stochastic parametric sensitivity analysis of a detailed structural model of the U.S. economic system, in which model parameters were derived from physical observations of the system. Use of such models is becoming increasingly prevalent for mid-to long-range studies and policy analyses in government planning at all levels. Resource scarcity, foreign policy contingencies and other factors have made rapid structural change the object of analysis, not something one can assume away. Effective use of such models requires an understanding of the effects of parametric change and uncertainty.

We are concerned here with a linear static Input-Output model of the U.S. economy. Its parameters are derived from data on interindustry transactions compiled by the U.S. Department of Commerce (1974a and b). Due to funding limitations, measurement lags and the size and complexity of the economic system, these parameters are seven years out of date when published. Parametric uncertainty therefore can arise from two sources: observation of the system during the base year and structural changes during the seven year lag period. Estimates of uncertainty in the base year parameters were compiled by Bullard (1976).

The effect of parametric uncertainty on model outputs has been discussed by Sebald (1974) and Bullard and Sebald (1975). These papers quantified the maximum error tolerances that would result from the worst-case distribution of parametric errors. For this model, it was found that the process of matrix inversion could magnify input errors by more than 600%. Since worst-case distributions are not likely to occur, a methodology that could quantify the extent to which parametric errors cancel one another would provide much more realistic results.

The Monte Carlo simulation analysis described here was designed to answer that question. Base year interindustry transactions were characterized as random variables and the model parameters were derived from them. The results from each simulation were used to update a set of sufficient statistics to yield unbiased estimates of means, variances and some covariances. The simulations were performed to evaluate both the effect of doubling error tolerances on inputs and the effect of changing the structure of the model to enhance its usefulness for predictive work.

Section 2 describes the preparation of the data base and estimation of uncertainty on base year transactions. Section 3 details the simulation methodology, and the criteria for determining "acceptability" of simulated parameters. Section 4 presents the results of all simulations, and discusses the effects of aggregation, magnitude of input uncertainty and other variables.

2.0 DATA BASE PREPARATION

2.1 THE MODEL

The linear static input-output model of the U.S. economic system is described in detail by Bullard and Herendeen (1975). It is based on the theory developed by Leontief (1941), and relies largely on data assembled by the U.S. Department of Commerce, Bureau of Economic Analysis (BEA). Data are expressed in constant dollars, which act as a surrogate for physical units. In this particular model however, the inputs of energy to all sectors are expressed in physical units, to account for the fact that energy is sold to different users at different prices.

The governing equation of the model is

$$(I-A)\ X = Y \qquad (2.1\text{-}1)$$

where X is an N-order vector of gross domestic outputs for each sector, Y is the vector of domestic final demands for the output of each sector, and A is the matrix of parameters describing the technology of producing goods and services during the base year. A typical element A_{ij} represents the amount of input from sector i required directly by sector j to produce one unit of its output. These parameters are derived from base year observations of interindustry transactions, T_{ij}, (amount of output from sector i sold directly to sector j):

$$A_{ij} \equiv \frac{T_{ij}}{X_j} \qquad (2.1\text{-}2)$$

In turn, these interindustry transactions are defined as the sum

$$T = DA + MDT + TF \qquad (2.1\text{-}3)$$

where DA_{ij} is the amount of product i sold directly to sector j, MDT_{ij} represents the transportation or trade margin i on all inputs to sector j, and TF_{ij} represents the amount of product j produced as a secondary output by sector i.

2.2 THE DATA

Estimates of all elements of the above matrices are collected and assembled by BEA at the 484 sector level of detail. Before publication however, they are aggregated to about 360 sectors. BEA personnel responsible for this compilation were interviewed; their subjective estimates of uncertainty on all base year transactions were obtained by Bullard (1976).

Before proceeding with the Monte Carlo simulation, these data were aggregated to 90 sectors.* This degree of aggregation was chosen to reduce computing costs while adhering closely to the most widely distributed and used version of the BEA input-output tables.*

* Names of sectors are given in Table 1.

Aggregating an input-output data base in a nontrivial operation since it must be done prior to the operation in (2.1-3). After aggregating the three matrices independently, zeroing the diagonal of TF, and summing to obtain T, X and A are computed using eqs. (2.1-1) and (2.1-2).

3.0 METHODOLOGY

3.1 POINT OF VIEW FOR STOCHASTIC ERROR ANALYSIS

There are several ways to interpret this problem, and the point of view affects both the methodology and the interpretation of results. One way is to act as a simulator of BEA's activities from data collection through matrix inversion. In an alternative viewpoint, the analyst attempts an a priori determination of the effect of mathematical transformation on uncertain observations. In either case, this information enables the analyst to assess the usefulness of the data for modeling purposes. We have adopted the latter point of view.

Within this framework, the analyst receives measurements** of the economic system associated with each interindustry transaction as well as total output and value added. Actually, each of these signals from an industry is the sum of many signals from individual establishments. The signals appear to be uncorrelated; that is to say, the analyst's only information on their correlations comes from accounting identities requiring income to equal outgo.

Each set of measurements is processed by BEA, resulting in upper and lower bounds and a "published value" representing their best estimate of the most likely true value of the parameter. We then characterize BEA's knowledge of the transactions as random variables. The resulting distributions drive a Monte Carlo analysis,*** the important outputs of

* The BEA tables are published at the 83 sector level of detail, while the 90 sector model used here retains more detail in the transportation and energy sectors of the economy.

**All measurements correspond to the model base year. Due to the size and complexity of the economic system, frequent measurement is economically prohibitive so virtually no time series data are available.

***Although we have adopted the point of view of attempting to determine the effect of mathematical transformations on uncertain observations, we have attempted to carefully model the parametric correlations induced by control total constraints. In particular, each parameter is first sampled independently and steps are taken to assure that external balance conditions are satisfied. Although it is unrealistic to completely simulate BEA's activities, many of which are judgemental, undocumented and not reproducible, in this case we have attempted to follow their procedures where possible. Specific instances of non compliance are detailed in section 3.3.

which comprise a solution set S. For each element of S, second order statistics are generated and compared with the deterministic results obtained by BEA and by Bullard and Sebald (1975).

3.2 SAMPLING RANDOM VARIABLES

All of the basic data (transactions, industry output, final demands) are characterized as random variables having either normal or lognormal distributions.* Small magnitude entries truncated to zero by BEA are modeled with a "folded normal" random variable, which is simply the absolute value of a normal random variable with mean 0. Non-zero cells are modeled using either normal or lognormal random variables with the former used in those cases where the published value is relatively accurate. In situations where the data is less well known, BEA personnel tended to use a multiplicative factor to bound their estimates rather than an additive error bound. A lognormal distribution is appropriate in such a case because of its property of multiplicative symmetry about the median. That is, if X_0 is the median of a lognormal random variable X, then Prob. $(X \geq X_0D)$ = Prob. $(X \leq X_0/D)$ for any factor D. For example, if an analyst states that his estimate has probability α of being correct within a factor of D, then a lognormal random variable with α = Prob. $(X_0/D \leq X \leq X_0D)$ will be used to model the situation.

The following conditions are satisfied by all parametric random variables in the situation:

1) The sample is drawn from a folded normal, normal or lognormal population.

2) The distributions are truncated to prevent samples that are absurd (e.g., negative transactions). Truncation eliminates samples in the upper and lower 0.15% tails in the normal and lognormal cases and in the upper 0.3% tail in the folded normal case. This corresponds to the percentage of probability outside 3 standard deviations from the mean in a normal population.

3) The expected value of the sampled result is equal to the published value, M, of the entry in question (except in the folded normal case where the published value is zero).

4) Before truncation, the random variable X from which we sample has a confidence interval defined by a parameter b, δ or D.

   a. Folded Normal Case

      Prob $(X \leq b) = .997$
      (i.e., b amounts to 3 standard deviations of the underlying normal random variable.

   b. Normal Case

      Prob $(\mu_X - \delta\mu_X \leq \mu_X + \delta\mu_X) = .997$
      (i.e., δ amounts to 3 standard deviations of X expressed as a fraction of the mean, μ_X=M)

   c. Lognormal Case

      Prob $(X_0/D \leq X \leq X_0D) = .997$

In all three cases the sampling procedure is based on a standard normal random variable* (i.e., mean = 0 and variance = 1). Details are given in Bullard, et. al. (1976).

3.3 AGGREGATING RANDOM VARIABLES

Based on subjective uncertainty estimates made by BEA Personnel, probability distributions were defined at the 368 sector level of detail. For these simulations, data were aggregated to the 90 sector levels of detail. The means of the aggregated variables are easily obtained but specification of the distributions of the aggregate variables is a non-trivial task which was undertaken in the following way.

Since all transactions, margins, etc. at the 368 order are in fact aggregates of data obtained initially from individual establishments grouped by 5 or 6 digit Standard Industrial Classification codes, the original BEA specification of a distribution for these aggregates was a crude assumption in itself. The basis for specifying the distribution at the 90 sector level is equally subjective. For computational convenience and reproduciblity, we assumed that the variance, V, of each aggregated element is the sum of the variance of all its constituents. If $3\sqrt{V}$ is less than 40% of the aggregated mean, μ, assign a normal $N(\mu,V)$ distribution to the variable. If $3\sqrt{V}$ is greater than 40% of μ, a lognormal distribution is assumed. If μ equals zero, a folded normal distribution is used. It is felt that the subjective nature of the disaggregated uncertainty estimates did not warrant a more rigorous approach.

3.4 CONSTRUCTING THE TRANSACTIONS MATRIX

The relationship between the matrices of transactions, (T), final demand (FD), imports (M) and gross domestic outputs (GDO) is given by

$$\sum_{j=1}^{N} T_{ij} + \sum_{k=1}^{10} FD_{ik} - M_i = GDO_i \qquad (3.4\text{-}1)$$

These random variables are sampled from normal or lognormal distributions as described above. Each element in the first row (i=1) is sampled first independently, just as BEA analysts receive these values from apparently independent sources. Since eq. (3.4-1) is an external balance condition that is not satisfied in general, we force this condition to be satisfied in much the same manner as BEA does. The lognormally distributed variables in the row are generally those obtained from unreliable sources or computed using surrogate variables. Therefore these values are scaled proportionately to satisfy

* In a few cases a negative entry in the data is modeled by the negative of a lognormal random variable (which necessarily takes only positive values). This set of circumstances is handled so much like the usual lognormal case that it is not discussed separarately in what follows.

* The standard normal random number generator used was the International Mathematical Statistical Library routine GGNRF. Tests of randomness and normality were performed for verification purposes and are described by Bullard et al. (1976).

eq. (3.4-1).*

Proceeding in this manner through N rows, a complete data set is constructed satisfying row constraints. The rows are not independent, however, because the value of all outputs (GDO) of a sector must equal the value of all commodity inputs (from the other N sectors) plus "value added" (a term, VA, accounting for wages, taxes, and profit). VA is measured independently by federal agencies and provides BEA analysts with another external condition to satisfy. Their method for satisfying this was too complex to model, so a simpler check had to be devised for this Monte Carlo study.**

The method employed is based on BEA's response to the following question: "If the criterion for terminating the iterative process of balancing the I-O table were based on uncertainty of the VA values, how much could be tolerated?" The answer indicated that out of 90 sectors, at least 88 must be within ±20% of the "true" value.*** If the condition was not met, the matrix was rejected. This condition was never violated in the actual simulation.

Next, the terms in eq. (3.4-1) are used to compute the coefficients

$$A_{ij} = \frac{T_{ij}}{GDO_j}$$

and the Leontief inverse matrix $(I-A)^{-1}$ is finally calculated. Aside from checking the eigenvalues of A, there is no a priori check that can be performed to guarantee positivity of the inverse matrix.**** Therefore, each matrix was checked after inversion. Again, the simulation was completed without this condition being violated.

*In fact, BEA analysts actually estimate many of these uncertain values by computing the difference between GDO and the sum of the well known (normally distributed) variables and allocating proportional to some surrogate variables (e.g. employment).

**In the 1967 input-output study, consistency between row and column sums was assured by assigning responsibility for individual sectors to different analysts and after each independently estimated initial row values, the resulting columns were presented to each analyst for independent verification. After many iterations and some undocumented judgement decisions, the "published" values were agreed upon.

***Philip M. Ritz (1976) Interindustry Economics Branch, Bureau of Economic Analysis, U.S. Department of Commerce, personal communication.

****If all variables were expressed in current-year dollars, some a priori test are available. In the general case such as this one, where the energy sector outputs are expressed in physical units, no such tests exist.

3.5 RESULTS SAVED FOR ANALYSIS

Our attention was focused on the means, variances and confidence intervals for the elements of $(I-A)^{-1}$ and selected subsets and linear combinations thereof. To calculate these, it was necessary to save a set of sufficient statistics on disk after each iteration. The running sum and the sum of the squares for each element of the following set of results, s, was saved.

1. The entire $(I-A)^{-1}$ matrix;
2. The total primary energy intensity vector, ε; and
3. The sector output vector, X.

The total primary energy intensity vector is a linear combination of the energy rows of $(I-A)^{-1}$, and a typical element ε_j represents the amount of basic energy resources required directly and indirectly to produce one unit of output from sector j for final consumption.* The sector outputs X are computed from the simulated $(I-A)^{-1}$ matrix using the base year domestic final demands as weighting factors:

$$X_i = \sum_j (I-A)^{-1}_{ij} \left(\sum_{k=1}^{10} FD_{kj} - M_j \right) \quad (3.5\text{-}1)$$

This is done because I-O models are frequently employed to estimate total sector outputs corresponding to a specified final bill of goods, and a significant amount of additional error cancellation may be achieved.

In order to ascertain the nature of the distribution of typical random variables, each simulated value was saved for source results. The variables saved were X, ε, and the electricity sector row of $(I-A)^{-1}$. Goodness of fit tests performed on these variables are described in Section 4.

Finally, since most applications of the particular models examined are in the area of energy policy analysis, it was decided to save sufficient statistics for recovering covariances of the energy sector rows of $(I-A)^{-1}$. Since all possible linear combinations were not of interest - only row and column combinations - storage requirements were considerably reduced. It was sufficient to save the running sums of products of all pairs of entires appearing together in such linear combinations. If other combinations are ever needed, they will be recoverable from $(I-A)^{-1}$ matrices saved on an archive tape.

With this set of results it is possible to estimate the total energy requirements to meet arbitrarily

*The energy rows utilized are those corresponding to coal, crude oil and gas, and the fossil fuel equivalent of hydro and nuclear electricity:

$$\varepsilon_j = (I-A)^{-1}_{1j} + (I-A)^{-1}_{2j} + 0.6\,(I-A)^{-1}_{4j}$$

specified final demands, and to compute linear combinations of energy intensities similar to the "total primary" one described earlier.

3.6 STOPPING RULE

One of the major difficulties associated with Monte Carlo simulation is knowing how many runs will be required to attain reasonable confidence intervals on the results of the simulation. There are two major problem areas. If one is considering whether or not to use Monte Carlo techniques, an estimate of the required number of runs is crucial to determination of simulation costs. It may be, for example, that reasonable confidence intervals may require a prohibitively expensive number of runs. The second problem arises after the decision has been made to use Monte Carlo methods. One needs to know when enough runs have been made.

In the first problem area, present practice dictates running several small scale simulations of a similar nature to the one of interest in order to be able to extrapolate the number of runs in the smaller cases to the probable runs needed in the larger. In the second area, good statistical practice dictates that before taking any samples, one must determine how to stop sampling in a way that doesn't bias results. Executing additional runs if the resulting confidence intervals are too large is considered unwise since one runs the risk of biasing the simulation results by stopping when the desired outcome occurs.

A method was developed for determining, based on a very small number of runs, the proper number of total runs the simulation should require. The method elucidates the cost/benefit tradoff between the cost of additional runs and the benefits of increased accuracy. Since this method is based on just the first few runs, biasing of the simulation will not occur. Since it is based on a very small number of runs, the method is a cost effective way to decide whether a Monte Carlo analysis is economically feasible. A detailed description is given by Bohrer and Sebald (1976).

4.0 ANALYSIS OF RESULTS

The basic results of the simulation will be given for the 90 order I-O matrix. This includes information on bias relative to published values, variance measures and their relation to error bounds, the sensitivity of the results to uncertainties on the variances of the underlying BEA data and the effects of aggregating to 30 sectors. As a prelude, we begin by discussing the goodness of fit tests which were required to verify some distributional assumptions inherent in the simulation.

4.1 GOODNESS OF FIT

The methodology for the goodness of fit tests was developed by Stephens (1976), who describes a test for normality based on the Cramer-von Mises statistic which may be employed when the population mean and standard deviation are not known. Stephens' test compares a given sample distribution function to a normal distribution with mean and standard deviations given by the sample mean and sample standard deviation. Included in Stephens' paper is a table of significance levels for the statistic given the hypothesis that the random variable being tested is normal. Thus, a test of normality may be made by calculating the value of Stephens' statistic for a given sample and comparing it to the tabulated values which characterize normal behavior.

The first series of tests using this method was made to test the normality of the Z random variables defined by averaging every ten consecutive sample points obtained for the entries in the simulation results. In all, 270 of these random variables were tested, one for each entry in the electric utility sector row of $(I-A)^{-1}$, the total primary energy vector, ε, and the total output vector, X. Table 4-1 shows the upper tail percentage points calculated by Stephens along with the observed percentages of the Z random variables which fell into the various categories.

OBSERVED PERCENTAGE	16.0	12.22	6.29	3.33	1.48
NORMAL PERCENTAGE	15.0	10.0	5.0	2.5	1.0
STEPHENS' STATISTIC	.091	.104	.126	.148	.178

Table 4-1. A Comparison of Observed and Theoretical Upper Tail Percentage Points for Goodness of Fit Tests on the Random Variables Z.

For example, Stephens predicts that 10% of all normal samples will achieve sample statistics larger than .104; we observed 12.2% above that mark. Even if the 270 random variable being tested are interdependent, the expected value of the observed percentages should equal the theoretical percentages if the normality hypothesis is satisfied. Thus the results are very reassuring and seem to justify treating the average variables as normal.

A second series of tests was undertaken to examine the distributional properties of the raw data for the same 270 entries. In the absence of averaging there is little reason to suspect that these random variables are normal. However, the results were suprising in that very many of the 270 sample statistics were small and therefore indicate good fit to a normal distribution curve. Those entries that displayed decidedly non-normal behavior were virtually all unimodal but slightly skewed to the right. It is interesting to conjecture why some entries seem to be roughly normal while others are not; perhaps in the process of inversion some elements of $(I-A)^{-1}$ get a better mix of elements of the A matrix. At any rate it is useful to know that the entries are all more or less unimodal and symmetric. If such is the case then 3σ may be conveniently employed as an error bound on the distance from the mean, μ. While Chebychev's inequality guarantees that $\mu+3\sigma$ contains at least 89% of the total probability in an arbitrary distribution, this percentage rises to 99.7 in the normal case. Presumably the percentage is also high for any random variable whose density function is roughly unimodal and symmetric. For all but one of the entries examined here, at least 99% of the sample points fell within three sample standard deviations of the sample mean. Thus, $3\hat{\sigma}$ may be thought of as an approximate bound on deviation from the mean for the entries in the simulation results, even if many of those entries are not very close to being normal.

4.2 CONFIDENCE INTERVALS

This section discusses the precision of the sample statistics obtained for various simulation results in light of the goodness of fit tests just discussed. Because the Z variables are approximately normal, standard techniques may be used to derive confidence intervals for the mean and variance of a Z variable and hence for the mean and standard deviation of the associated entry. After 1000 inversions, a 97.5% upper confidence bound σ_u on the standard deviation σ of an entry is given by $\sigma_u = \hat{\sigma} * 1.16$. Thus, $\hat{\sigma}$ is a fairly good estimate of σ for any given entry.

The confidence intervals on the sample means are even smaller. In more than 90% of the entries in the inverse the population mean is within 2% of the sample mean with 95% confidence. All the entries of ε and X are accurate to within 1% with 95% confidence.

4.3 VARIABILITY OF THE ELEMENT IN THE RESULT SET

Histograms of $3\hat{\sigma}/\hat{\mu}$ were prepared in order to show the relative amount of variability in the entries of the results set. Three such histograms, one for the whole inverse, one for ε and one for X, are displayed in Figure 4-1. For half of the entries of the inverse, $3\hat{\sigma}/\hat{\mu}$ is less than 20% while virtually all the entries of ε and X have $3\hat{\sigma}/\hat{\mu}$ less than 20%. The above discussion of confidence intervals suggests that these histograms would not change substantially if the sample statistics were replaced by the population means and standard deviations. Since these entries are roughly unimodal and symmetric, the histograms may then be taken as a good measure of the variablility in the entries of the various subsets of the results. The large decrease in variability from the elements of the inverse to the elements of X suggests that significant error cancellation occurs as linear combinations of many I-O coefficients are computed.

In addition to those discussed above, histograms for $\frac{(3\sigma_u + \hat{\mu} - p)}{p}$ and $\frac{(3\sigma_u + p - \hat{\mu})}{p}$, where p = published value, were also computed in order to relate p to the upper and lower bounds on the uncertainty in an entry. Because $\hat{\mu}$ is generally very close to p and because σ_u is only slightly larger than $\hat{\sigma}$, these histograms are very similar to the histograms for $3\hat{\sigma}/\hat{\mu}$ except that the values are all slightly larger.

4.4 BIAS ON ELEMENTS OF THE RESULT SET

In standard statistical language, bias is usually defined as the difference between the mean of an estimator and the true value of the quantity to be estimated. We use the term in a fundamentally different way to denote the difference between the mean of the simulation output variables and their corresponding published values. The mean values of the assumed distributions of each element of the transactions matrix are equal to their respective published values. One important result of the simulation is to determine the bias introduced by normalization and inversion in passing from the transactions matrix to $(I-A)^{-1}$.

Fig. 4-2 details histograms of the ratio of sample mean to published value, $\hat{\mu}/p$, for three important and disjoint subsets of the result set, viz the vectors of total output X and total primary energy intensity ε and the entire inverse $(I-A)^{-1}$. Three aspects are noteworthy:

1) Nearly all $\hat{\mu}$ cluster within 2% of their published values.
2) Within this cluster, $\hat{\mu}$ tends to have a positive bias more often than a negative one.
3) Essentially none of the $\hat{\mu}$ fall below 98% of their respective published values while, especially in the inverse, a small number of $\hat{\mu}$ range well above the published value.

The reason for this positive bias is unclear. The best explanation may be that transactions reported by BEA as zero were assigned a small positive value in the simulation to account for the fact that no transaction is known to be exactly zero. The large percentage excess over the published value may result for the same reason, since an inverse element may be affected (percentagewise) quite significantly if its corresponding direct coefficient A_{ij} changes from zero to some finite value.

4.5 SENSITIVITY OF SIMULATION RESULTS TO ASSUMPTION ON INPUT UNCERTAINTIES

Since the variances assigned to input quantities such as the transactions matrix, FD and GDO are only estimates of the true variances, simulation results have meaning only if small changes in these assumed variances, do not cause very large changes in simulation outputs. This sensitivity to changes in input variances was investigated by repeating the simulation with the standard deviations of all normal quantities doubled and dispersion factors on lognormal inputs doubled. Three major effects were noted:

1) The ratio of CI to $\hat{\mu}$, where CI is the length of the 95% confidence interval for the mean, was doubled by the factor of two increase in standard deviation.
2) The ratio of $3\hat{\sigma}$ to $\hat{\mu}$ doubled on the average by doubling the input standard deviations.
3) Increasing the input variability made the biases slightly more negative. This is thought to be the result of increasing simulation sensitivity to the larger elements of the transactions matrix and decreasing relative sensitivity to the smaller elements discussed in section 4.4.

Since output uncertainties only doubled with a factor of two increase in input uncertainties, the simulation is probably very stable with regard to assumptions on input variances.

The absolute magnitudes of these results with doubled input uncertainties may be useful in assessing the general viability of I-O results applied far beyond the base year (the uncertainty of base year parameters increases over time). Moreover, if institutional factors make it unlikely (as some claim) that government can fairly estimate uncertainty of its own data, then these results show the effect of a 50% underestimate of the actual uncertainty.

4.6 THE EFFECT OF AGGREGATION

The effect of aggregating the 90 order model to 30 order was analyzed for two reasons:

1) It was felt that although variances at the 30 order were smaller than those of the 90 order due to the aggregation, more error cancellation should exist at the 90 order where more input elements combined to form elements of X, ε and $(I-A)^{-1}$.

2) Since much I-O work is done at the 360 order, it is of interest to determine whether the expansion of the simulation to 360 order would likely require more than 1000 runs used in the 90 order case.

Aggregation produced effectively no change in the simulation output uncertainties:

1) The ratio of $3\hat{\sigma}$ to $\hat{\mu}$ remained virtually unchanged by the aggregation. This implies that the two effects mentioned above virtually cancel one another.

2) The already very small biases of Fig. 4-2 were made slightly more negative by aggregating to the 30 order.

3) Since the ratio $\sigma_u/\hat{\sigma}$ is a function only of the number of simulation runs, it is unaffected by aggregation.

These results give no indication that more than 1000 runs would be needed in the 360 sector case.

REFERENCES

1. R. Bohrer and A. V. Sebald "On Quantifying the Tradeoffs Between Monte Carlo Iterations and Smaller Confidence Intervals" (forthcoming)

2. C. Bullard and A. Sebald, "Effects of Parametric Uncertainty and Technological Change in Input-Output Models," Review of Economics and Statistics, (forthcoming). Also available as CAC Document No. 156, Center for Advanced Computation, University of Illinois, Urbana, IL 61801 March 1975.

3. C. Bullard, A. Sebald, D. Putnam, D. Amado, "Stochastic Analysis of Uncertainty in a U.S. Input-Output Model," CAC Document No. 208, Center for Advanced Computation, University of Illinois, Urbana, IL 61801, September 1976

4. C. Bullard and R. Herendeen, "Energy Cost of Consumption Decisions," Proc. IEEE, March 1975. Also available as CAC Document No. 135, Center for Advanced Computation, University of Illinois, Urbana, IL 61801.

5. C. Bullard, "Uncertainty in the 1967 U.S. Input-Output Data," CAC Document No. 191, Center for Advanced Computation, University of Illinois, Urbana, IL 61801, April 1976.

6. W. Leontief, The Structure of the American Economy 1919-1939, Oxford University Press, New York, 1941.

7. A. Sebald, "An Analysis of the Sensitivity of Large Scale Input-Output Models to Parametric Uncertainties," CAC Document No. 122, Center for Advanced Computation, University of Illinois, Urbana, IL 61801, November 1974.

8. M. A. Stephens, "Asymptotic Results for Goodness-of-Fit Statistics with Unknown Parameters," Annals of Statistics, V.4, no.2, 1976, pp. 357-369.

9. U.S. Department of Commerce Bureau of Economic Analysis, Input-Output Structure of the U.S. Economy 1967, U.S. Government Printing Office, Washington, D.C., 1974.

10. U.S. Department of Commerce Bureau of Economic Analysis, Definitions and Conventions of the 1967 Input-Output Study, (mimeo) 1974.

TABLE 1

90 ORDER SECTOR NAMES

1. Coal Mining
2. Crude Petroleum & Natural Gas
3. Petroleum Refining & Related Products
4. Electric Utilities
5. Gas Utilities
6. Livestock & Livestock Products
7. Other Agricultural Products
8. Forestry & Fishery Products
9. Agricultural, Forestry & Fishery Services
10. Iron & Ferroalloy Ores Mining
11. Nonferrous Metal Ores Mining
12. Stone & Clay Mining & Quarrying
13. Chemicals & Fertilizer Mineral Mining
14. New Construction
15. Maintenance & Repair Construction
16. Ordnance & Accessories
17. Food & Kindred Products
18. Tabacco Manufactures
19. Broad & Narrow Fabrics, Yarn & Thread Mills
20. Miscellaneous Textile Goods & Floor Coverings
21. Apparel
22. Miscellaneous Fabricated Textile Products
23. Lumber & Wood Products, Except Containers
24. Wooden Containers
25. Household Furniture
26. Other Futniture & Fixtures
27. Paper & Allied Products
28. Paperboard Containers & Boxes
29. Printing & Publishing
30. Chemicals & Selected Chemical Products
31. Plastics & Synthetic Materials
32. Drugs, Cleaning & Toilet Preparations
33. Paints & Allied Products
34. Paving Mixtures & Blocks
35. Asphalt Felts & Coatings
36. Rubber & Misc. Plastics Products
37. Leather Tanning & Industrial Leather Products
38. Footwear & Other Leather Products
39. Glass & Glass Products

Table 1 (continued)

40. Stone & Clay Products
41. Primary Iron & Steel Manufacturing
42. Primary Nonferrous Metals Manufacturing
43. Metal Containers
44. Heating, Plumbing & Fabr. Structural Metal Prod.
45. Screw Mach. Prod., Bolts, Nuts, etc. & Metal Stampings
46. Other Fabricated Metal Products
47. Engines & Turbines
48. Farm Machinery
49. Construction, Mining, Oil Field Mach., Equip.
50. Materials Handling Machinery & Equipment
51. Metalworking Machinery & Equipment
52. Special Industry Machinery & Equipment
53. General Industrial Machinery & Equipment
54. Machine Shop Products
55. Office, Computing & Accounting Machines
56. Service Industry Machines
57. Elec. Transmission & Distrib. Equip. & Elec. Industrial Apparatus
58. Household Appliances
59. Electric Lighting & Wiring Equipment
60. Radio, Television & Communication Equipment
61. Electronic Components & Accessories
62. Misc. Elec. Machinery, Equipment & Supplies
63. Motor Vehicles & Equipment
64. Aircraft & Parts
65. Other Transportation Equipment
66. Prof., Scientific & Controlling Instru. & Supplies
67. Optical, Ophthalmic, & Photographic Equip & Supplies
68. Miscellaneous Manufacturing
69. Railroads & related Services
70. Local, Suburban & Interurban Highway Pass. Trans.
71. Motor Frieght Transportation & Warehousing
72. Water Transportation
73. Air Transportation
74. Pipe Line Transportation
75. Transportation Services
76. Communications Except Radio & TV Broadcasting
77. Radio & TV Broadcasting
78. Water & Sanitary Services
79. Wholesale & Retail Trade
80. Finance & Insurance
81. Real Estate & Rental
82. Hotels & Lodging Places; Personal & Repair Serv.
83. Business Services
84. Automobile Repair & Services
85. Amusements
86. Medical, Educ. Services & Nonprofit Organization
87. Federal Government Enterprises
88. State & Local Government Enterprises
89. Business Travel, Entertainment and Gifts
90. Office Supplies

Figure 4-1. Maximum Error Tolerances on Results

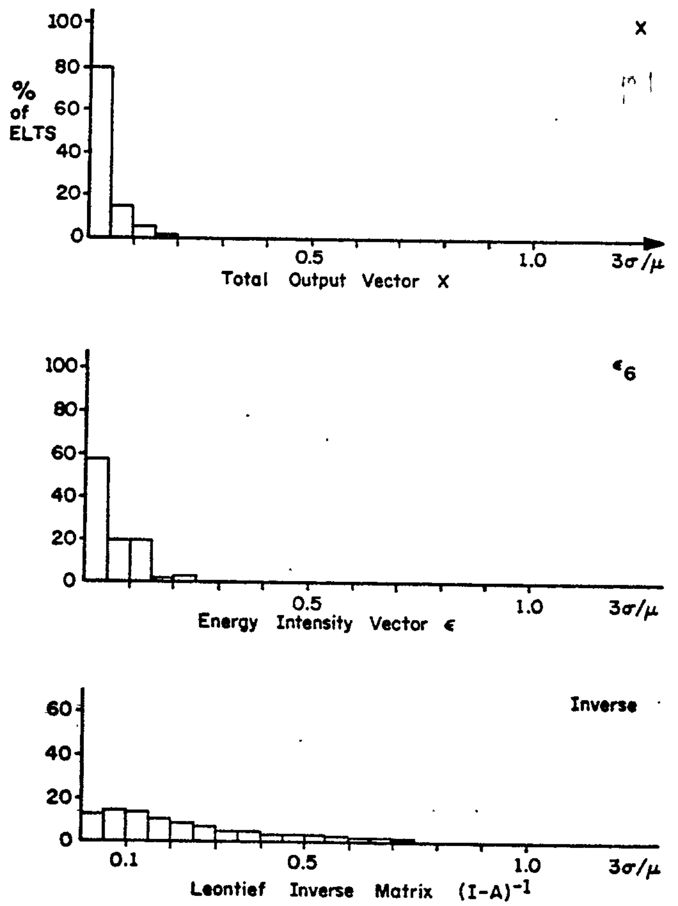

Figure 4-2. Mean Value Bias