A SIMULATION MODEL FOR PREDICTING THE EFFECT OF ADVERTISEMENT SCHEDULES

R. Balachandra
Boston College
Chestnut Hill, MA 02194

ABSTRACT

Advertisement Scheduling refers to the detailing of where advertisements for a product/idea should appear in different media vehicles (TV shows, magazines, etc.) so as to maximize the effect on the target population. The effect is usually hoped to be favorable and influence the population to go out and buy the product/idea.

Media scheduling models provide solutions that describe the number of advertisements to be inserted in different media vehicles over a given time horizon. These models do not specify the precise timing of the insertions, as it is usually assumed that within a given time interval there are not many choices for any one media option. (e.g. there is only one Reader's Digest each month).

But if TV or radio is considered the situation becomes more difficult. The number of choices even within a week is large (e.g. there are five "To-night" shows in a week); and many of the desired spots may not be available because of high demand. Therefore the detailed schedules which take into consideration the availability of specific spots must be evaluated. As the relationships that exist between exposure to advertisements, their timing and the effect on the individuals are complex analytical methods are not very useful.

The simulation model described below evaluates different advertisement schedules for their effect on the target population. The effect can be in terms of any one of a number of variables - probability of purchase, awareness of brand's advertisement, intention to purchase etc. In this model awareness of the brand's advertisement is used as the desired variable. Although not directly influencing the market share of the brand, this variable is considered quite important by marketing managers.

The model is a micro-simulation model - it stimulates the responses of individuals from the sample population. It can simulate single period and multi-periods.

INTRODUCTION

Media Scheduling or advertisement scheduling refers to the details of an advertising campaign. These details include decisions about which media to use (TV, radio, magazines, etc.) and when to insert the advertisements in the media chosen (e.g. December issue of Reader's Digest, December 17th "Saturday Night Live" show, or the ABC National News program in the last week of December).

Media scheduling is still considered an art. Until recently even the objective was not clear. It was thought for example, that if the advertisements reached a larger number of people with a larger frequency it would be effective. Any schedule which accomplished this within the given budget was considered good. However, it has recently been acknowledged that a large number of exposures is not necessarily good as there is likely to be diminishing returns in the effect these exposures produce.

Little and Lodish [10] suggested that the proper function to maximize was neither reach or frequency of exposures, but some measure which reflects the gain to the advertiser as a result of the exposures. They hypothesized an exposure value of a media schedule which was to be maximized over the given time horizon. The exposure value decreased with time and increased with exposures. They developed a heuristic method to solve this problem.

The real world media scheduling problem is very complicated because of a number of factors such as:

i) Non-linear functions of advertising effectiveness

ii) Interactions between different media

iii) Carry over effects and forgetting

These factors make it almost impossible to solve the media scheduling problem by analytical optimizing methods. Therefore most models are solved by using heuristics or simulation. [1] [6] [10] [15]

These models specify the media to be used and the number of insertions in the media chosen in a given period. However the results can not be considered reliable as most of the parameters used in these models are based on subjective estimates of the users and not derived from data. Additionally, the effects of variations in the insertion schedule within a period are usually ignored.

This paper describes a few of the earlier simulation models and presents a simulation model which overcomes some of the weaknesses of the earlier models. The model also incorporates response functions based on "holding" and "switching-in" which seem to capture real-life phenomena much more accurately. These functions have been estimated from an actual survey, so that the model can be more realistically applied. As with most simulation models, this too does not lead to any "optimum" solutions but helps one to choose from among a number of alternative media schedules by evaluating their effectiveness.

SIMULATION MODELS IN MEDIA SCHEDULING

One of the earliest attempts in using simulation for media scheduling was promoted by a company called Simulmatics Corporation. [9]. The Simulmatics model consisted of an imaginary population of 2944 individuals representing a cross section of U.S. population over four years of age. The model moved at certain predetermined intervals of time and scanned all media to determine whether there was an advertisement. If so, then the population was cycled through using Monte-Carlo technique to determine who received advertisements during that interval. The probability of exposure was supposed to be continuously modified to take into account such factors as habit forming, competition from other media and so on. At the end of the simulation the number of exposures received by each individual were summed to get a reach and frequency distribution for each of the schedules tested.

The model was doomed to failure from the start because of scarcity of data. Also many of the functions used to represent habit formation and other factors could not be justified with empirical data. [4]

Gensh [6] developed a model which was basically similar to the Simulmatics model in its method for computing the reach and frequency distribution of exposures, except that it used 275 media vehicles, and a sample of 500 people. The data was claimed to be more reliable. The exposures were then converted to "impact units" to reflect the overall effect of the media schedule. The impact units were weighted sums of exposures, the weights depending upon the media type, size, color of the advertisements, the patterns of exposure frequency and the value of exposures to different types of individuals in the target population.

As with Simulmatics model, many of the relationships seemed to be arbitrarily arrived at, though of course based on expert judgement. The large number of sets of weights introduced at various stages into the model were also subjectively estimated.

As seen from the brief descriptions above, one of the major problems in media scheduling is the development of a reliable advertisement response function - a function which describes the effect of different numbers of exposures upon an individual. In the next section, a brief description of the response function is presented.

ADVERTISEMENT RESPONSE FUNCTION

An advertisement response function relates the effect on a variable of interest (e.g. purchase probability of an individual) to the number of exposures received by that individual. The functional form may be explicitly stated [1], [2], [10], or implicit [6]

Most of these functions deal with aggregate responses, i.e., responses of a segment of the population. They are generally supposed to be concave downwards, monotonically increasing and approaching a maximum asymptotically. [14]. Considering the short durations of time involved in media scheduling situations, there may be no significant changes in the aggregate probabilities, although there may be changes in individual probabilities. It has been shown that even for a variable which is affected to a high degree by advertisements the aggregate probabilities do not provide sufficient information to estimate the advertisement response function [2]

Such problems suggest a different approach based on the concept of "holding" and "switching-in" behavior. The basic assumption implied by this concept is that the response process is a first order Markov process.

Consider an individual in period 1 responding favorably. There is some probability that this individual will respond favorably again in period 2. This probability is called the probability of "holding" (h). Similarly if the individual did not respond favorably in period 1, there is some probability (probability of "switching-in", s) that the individual will respond favorably in period 2.

(This can be represented as in figure 1.)

Figure 1

		Period 2	
		Favorable Response	Unfavorable Response
Period 1	Favorable Response	h	1-h
	Unfavorable Response	s	1-s

If the probability of getting a favorable response in period 1 was p, then the probability of getting a favorable response in period 2 is given by

$$p_2 = p_1 \cdot h + (1 - p_1) \cdot s$$

Of course, the probabilities h and s are not constant but depend on the number of exposures received by the individual.

The data for estimating the values of h and s for different numbers of exposures and different media can be obtained without too much expenditure. A suggested method of data collection and analysis is described in detail in [2]

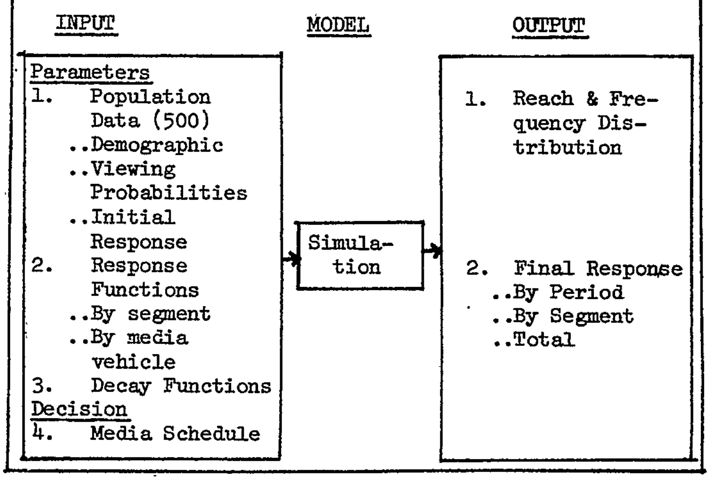
When the data is available for the desired response-purchase, recall of brand's advertisement, feel favorable about the brand etc. - the appropriate function can be derived and used in the simulation model described below.

The ADSIM model described below employs the advertisement response functions based on the "holding" and "switching-in" concept. It will be shown later on that this model provides a more realistic evaluation of media schedules.

THE SIMULATION MODEL (ADSIM)

The Advertising Simulation Model's (ADSIM) main objective is to estimate for a given advertisement schedule the distribution of exposure frequencies for the target population segment and to compute the changes in the response probability of that segment. In this version of the model, the response probability considered is the probabaility of recall of the brand's advertisement, and the media is the television and its different programs. For companies in the consumer packaged goods industry, the expenditure for TV advertising is more than 95% of their advertising budget.

Figure 2



The different phases of the model and the working of the various stages are described below.

THE INPUT PHASE

The input phase of the ADSIM model consists of four stages as shown in figure 2. The first stages (which are called the PARAMETER INPUT in the figure) define the model characteristics and set it up for the desired target group and the media vehicles being considered. The last input stage, called the DECISION INPUT, consists of the actual media schedule being considered for the simulation. For a given situation of media scheduling, the first three stages are fixed. The fourth or the Decision Input stage is manipulated to evaluate the different scheduling strategies in this situation.

In the three stages consisting of the parameter input, all the relevant parameters describing the target population and the response functions for the population when subjected to exposures from the media vehicles (which are already decided upon) are entered.

In the first stage the population characteristics are entered. The current design of the model (the computer program) can accept a total of five hundred individuals. This number can be increased easily by suitable redimensioning in the computer program.

As most segment classifications are based on demographics [3], the population demographic characteristics should be included. Some of these are:

i) Age, ii) Sex, iii) Family Income, iv) Education, v) Number of children and their age distribution, vi) Occupation, vii) Own home/rented

The above list is given only as an indication of the type of information that may be included. For specific applications, the proper choices of the demographic variable has to be included. For example, if dog food is being considered, information about ownership of dogs and the number of dogs owned has to be included.

The viewing and reading habits of these individuals is included next. This data is generally available from professional data gatherers--A.C. Nielsen and Co., W.R. Simmons and Associates, for example--who supply computer tapes providing information on a very large sample of individuals (16,000 to 20,000 individuals). Many large organizations conduct their own surveys periodically from selected consumer panels. These studies include the exposure probabilities in addition to the demographic characteristics. They also contain information about the response of the individual with respect to the specific habit of interest. If purchase propensity is being studied, the last purchase information would be included (1 if the brand was bought, 0 othewise).

If this information for each individual is not available, then it can be generated if the average value of the response probability (e.g. market share for purchase propensity) is known. Thus, if the market share for the period is 10%, then 10% of the individuals in the sample population are randomly assigned to have purchased at the start of the period.

In the second stage of the input the response functions for holding and switching are entered. These functions are derived from actual data as described in [2]. For any specific response a separate study has to be conducted to determine the advertisement response functions. These functions describe how an individual responds at the next response occasion given the fact that he or she responded in a particular way in the earlier occasion and has received some known number of exposures from some media vehicle or a combination of media vehicles.

A typical response function (whose values are observed, and not smoothed) is given in Table 1. Such function tables are provided for all media vehicles and for all segments that are to be included in the simulation. For combinations of media vehicles, a weighted average of the response function values will be computed.

These response functions describe the probability with which an individual will change his/her response state. As an example, consider an individual of segment W1 who recalled the advertisement for the brand initially. If this individual was exposed to six exposures of TVA, then the probability with which the individual will recall the advertisement for the brand again is given by the value of the "holding" function corresponding to six exposures (0.60, see Table 1). This probability will be used to simulate the actual response of the individual.

Table 1

Examples of Response Functions

Message Type: TVA*

Segment: W1*

# of Exposures	Prob. of Holding	Prob. of Switching-In
0	.4	.02
1	.49	.11
2	.55	.13
3	.58	.16
4	.60	.16
5	.61	.19
6	.60	.19
7	.66	.20
8	.67	.22
9	.66	.21
10	.63	.23
11	.63	.25
12+	.66	.27

*The actual specifications are not given in accordance with the wishes of the organization which provided the data.

The third input stage consists of the decay or forgetting function for the specific response. Thus, if brand advertisement awareness is being considered, this function will describe how the recall decays over time in the absence of exposures. The decay rate may not be large for habits which are deeply rooted, like purchase propensity. But it is large for recall of advertisements [13].

For developing the forgetting functions within the period, we have to assume that there exists a length of time during which the actual positioning of the exposures has no discernible effect. This interval will of course depend upon the nature of the habit or response. It may be as long as a month or more for some responses like purchase or expressing a favorable opinion about a brand, or as short as a day for others such as recall of the specific copy theme of a commercial. In the MEDIAC model [10] it is implicitly assumed that the specified period (say one month) meets this criterion. Gensh [6] uses periods of two weeks or five weeks depending on the exposure level.

The length of the longest non-exposure period determines the actual response probability. Thus, if there was exposure during all the periods, then the actual response probability will be the same as the response probability given by the response functions. If, however, there were one or more periods of non-exposure, then the response probability will be modified by a factor, which will be called "forgetting". The "forgetting" factor is a function of the number of sub-periods (weeks) during which no exposures are received.

Thus, an individual receiving all exposures in the last week has a lower response probability than an individual receiving the exposures evenly throughout the four weeks or an individual receiving it during three weeks out of four. Such a phenomenon has been reported in the case of recall of advertisements by Zielski [15].

The values for the forgetting functions have been collected from actual data. The method of data collection and its analysis for deriving the forgetting function values are described in [2].

The fourth stage of the input consists of the actual insertion schedule to be evaluated. The budget and availability lead to the number and type of insertions in each media vehicle to be included in any period. This list has to be converted into a time-ordered sequence of insertions for introducing into this stage.

The first three stages have identified the environment in which the advertisements or insertions are going to operate. This stage, which defines the actual insertion schedule to be evaluated, can be varied to suit the subjective and media scheduling constraints.

Consider the situation where the list consists of the following schedule of insertions.

Period 1

No.	Show/Program	# of insertions
(1)	TV-Sunday Evening "Cher"	2
(2)	TV-Monday "Monday Night Football"	2
(3)	TV-Weekday "Soap Operas"	8

Since there are four slots for the "Cher" Program, four slots for the "Monday Night Football", and eighty (20 days x 4 per day on an average) slots for the soap operas, there are a total of $\frac{(4)(4)(80)}{(2)(2)(8)} = 1.04 \times 10^{12}$ combinations. Fortunately not all of these combinations are available or feasible. Additionally, the individual's exposure probabilities may not differ much between the different soap-operas, so that the final selection between any two does not make much difference as far as exposure is concerned. These factors reduce the total combinations to a smaller number which is still too large for evaluation.

The actual availability of the time slots is also restricted by organizational constraints. For example, many large corporations buy time for the corporation as a whole, which is then allotted to individual brands. Knowing the actual availability of time slots and the insertion mix decided upon by the budget, it is possible to develop a number of alternative schedules, using the methods of job shop scheduling. (The Gantt chart is a very useful device for developing such schedules.)

Using these scheduling techniques several alternative detailed insertion schedules can be developed. Each of these is different in terms of the actual positioning of the advertisements.

THE SIMULATION

The simulation consists of four steps performed in sequence. These steps are:

(1) Exposure computation for each individual.

(2) Computation of total number of exposures for each individual and the time pattern

(3) Aggregation for Reach and Frequency

(4) Computation of new response of each individual and the resulting habit strength

These steps will be described in detail below:

EXPOSURE COMPUTATION

The input stage 4 (media schedule) would have arranged the advertisements in a chronological sequence. The program selects the first advertisement in the sequence and for each individual in the sample performs a Monte-Carlo type simulation by generating a random number n ($0 < n < 1$). If the number generated is less than the value of the probability of receiving that commercial by the individual (data from input stage 1), then the individual is said to have seen the commercial, and is scored as such. For those who saw the commercial, the actual time is also noted (the week in this case). This procedure is repeated for all the advertisements in the schedule so that at the end we have information about the advertisements seen by each individual, their times and the types.

AGGREGATION OF EXPOSURES AND THE TIME PATTERN

The output from the last stage, which simply simulated the advertisements for each individual to determine whether the individual saw the advertisement or not, is fed into this stage. In this stage, the exposures are grouped into their respective types. Additionally the time pattern of the exposures is examined and the weeks (or any other length of period) during which no exposures were received are determined. This is necessary for incorporating forgetting. These aggregations are performed for each individual in the sample population. The frequency of exposures received by each individual is aggregated to provide a reach and frequency distribution chart (Table 2).

Table 2

FREQUENCY DISTRIBUTION

(For Some Schedule A1)

# of Exposures	Percent of Individuals
0	28
1	30
2	16
3	10
4	6
5	3
6	2
7	2
8	1
9	1
10	1
11	0
12+	0

DETERMINATION OF NEW RESPONSE

The previous stage computes and provides the number and type of exposures received by each individual and the time pattern for each individual. In this stage, the program refers to the appropriate response function table and the forgetting function table, corresponding to the individual's segment classification, the type of exposures, and the time pattern. Using these figures, the overall response function value is computed. Of course the appropriate response is used--viz. either "holding" or "switching-in" depending upon whether the individual had initially responded or not.

Knowing the pre-response and using the overall response probability, another Monte-Carlo type simulation is performed again to determine the new response of each individual. Table 3 shows an example of this step.

In this example, individuals #001 and #003 who responded originally will respond again in the same manner. Individual #002 will have had no impact from the six exposures; whereas individual #500 will respond positively, after not responding earlier.

AGGREGATION OF THE NEW RESPONSES

The new responses are aggregated in this stage according to the desired classification of segments. Thus, summary figures are printed out to show the changes in response from, say, the segment of housewives (18-34 years of age with family income over $15,000) and other segments. In addition, an overall summary figure is developed and printed.

TABLE 3

Computation of New Response Probability

Individual	Pre-response	Response in value	Forgetting in value	Overall Response in value	Random number	Post-Response
001	1	.60	0.95	.57	.34	1
002	0	.19	0.95	.18	.72	0
003	1	.66	1.0	.66	.09	1
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
500	0	.21	1.0	.21	.18	1

These results are shown in Table 4 for a hypothetical problem.

TABLE 4
Summary Results of Simulation

Schedule: A1

SEGMENT	WGT.[a]	PRE	POST[b]	HOLD(%)	SWITCH(%)
(1) Women					
18-34	2	0.15	0.18 (.012)	0.58 (.045)	0.11 (.010)
(2) Men 18+	0.5	0.06	0.08 (.008)	0.40 (.32)	0.06 (.005)

Notes: a. The weight represents the monetary benfit derived by the advertiser from an individual of this segment.

b. These are the mean and the standard deviation of five simulation runs.

To minimize the high-variance effects of any single Monte-Carlo simulation, this simulation is performed a number of times and the variance of these runs is computed. The program currently repeats each simulation five times, as the reduction in variance between five simulations and ten or more is not very significant.

MULTI-PERIOD SIMULATION

The simulation (ADSIM) can be converted into a multi-period simulation by making very small changes in the input. The computer program is capable of handling up to twenty periods.

The major change is in the input stage 4, where the media schedule is entered. Each media insertion should be arranged chronologically with identification for the period in which it is scheduled.

Secondly, the simulation model assumes that the level of the response probability at the end of the period is the level at the beginning of the next period. With these modifications, the simulation described in the previous section is performed for any number of periods desired by the user.

The output consists of three parts:

(1) The media schedule in each period--a graphical representation of the media schedule input;

(2) The reach and frequency distribution achieved in each period from the media schedule input;

(3) The changes in the levels of the response probability from period to period.

USING THE SIMULATION FOR SENSITIVITY ANALYSIS

The results from the simulation (either the single period or the multi-period version) provide valuable information about the expected results from a specific media schedule. These results are arrived at after considering the effects of the actual positioning of the different insertions.

There are two kinds of sensitivity analysis that can be performed using the ADSIM model:

(1) Determine the effect of re-arranging the various insertions within a given time period (i.e., instead of scheduling throughout the period--four weeks--schedule during only the first half, or during the final three weeks, and so on).

(2) Given a particular solution consisting of the number of different media options that should be used in any given period, determine the effect of marginally increasing or decreasing the number of one or more of the media options.

For each of such variations the ADSIM model produces output which shows:

i) the Reach and Frequency distribution

ii) the expected response level

These results enable one to select the schedule which produces the most suitable contribution and the reach and frequency distributions, according to any criteria set down by the manager of the system.

The actual variations to be tested on the ADSIM can be developed using any of the standard experimental techniques. [7] [8] [12]. An example of one such experimentation is described and the results given in [2].

COMPUTATIONAL CAPABILITIES
AND COMPUTER REQUIREMENTS

The simulation program has been written in FORTRAN IV for the IBM/360/75-91 system. The core requirements are 150K bytes. This large requirement is mainly because the population data is read in and stored in the active area of the computer. This is necessary since the model is situation specific and does not claim to be a general purpose model for any application. But this core requirement is compensated for by the speed of computation and by the use of "real" data base.

The time requirements depend on the number of experiments included in one run. The main simulation program is a subroutine which can be called upon any number of times by the main program. The time for one-period runs can be estimated by the formula

$$T = 20 + 4n \text{ seconds}$$

where n is the number of experiments.

The cost of testing a large number of experimental schedules is, therefore, quite modest ($15-20 for 10 experiments).

VALIDATION OF THE SIMULATION MODEL

Validation of models representing social or economic systems is one of the most difficult tasks. There is no agreement among authors about the methodological procedure to be used.

Naylor et al [12] propose a compromise multi-stage approach, which may be particularly suited to the verification of computer simulation experiments on business economic systems. The first stage deals with the formulation of a set of postulates describing the behavior of the system. This includes the selection of the relevant variables, their specification and the set of relationships that are assumed to exist between these variables. In the second stage, the basic set of postulates are empirically verified using appropriate statistical techniques (ignoring for the moment the philosophical questions of "What does it mean to verify a postulate?). The third stage tests the model's ability to predict the behavior of the system under study.

Therefore, in this multi-stage approach, mere accuracy in prediction is not the sole criterion of verification, as this procedure attaches equal weight to the validity of the assumptions and to the predictive capability of the model.

According to Bowersox [5], however, an important element of validation is the output validity, consisting of long-term stability, congruence between historical and simulated output, and sensitivity to model assumptions.

Gensh [6], while attempting to validate his AD-ME-SIM model, discards all these approaches, and claims that expert opinion regarding the model's assumptions and the output in a number of situations is adequate for his model. He uses this approach mainly because the assumptions made by him are too difficult to test empirically, and the output variable (the impact units) is impossible to measure in practice. He therefore relies on the experts' feel for the relative effectiveness of different media schedules which are simulated in his model. If the simulation gives the highest impact unit value for some schedule, and the experts pick that schedule as the best, the simulation model is verified.

The other advertisement simulation models have no discussion on validation. They presumably assume "synthetic a priorism" [12] as the models are based on "reasonable" hypotheses.

The simulation model described in this chapter (ADSIM) is of the normative or prescriptive type. The first stage of the validation procedure (regarding the internal consistency) has been discussed in detail in [2] where the process and the underlying assumptions are examined; empirical evidence was introduced to validate the functions employed, and the relevant literature was cited.

The second stage of the validation procedure (output validity) is much more difficult. Implementing the policy suggested by the model and comparing the results is too expensive, time consuming and simply not feasible. Using the simulation model to evaluate historical data is also not feasible as the parameter values for the functions have all been obtained by the only data that were available, and using these to validate is tautological.

The procedure that was developed for this situation was therefore similar to Gensh's. A number of alternative schedules were developed in co-operation with the executives of a major consumer products company. These schedules were used as input (decision input) into the simulation model, keeping all other factors constant. The outputs (post-awareness levels) for each schedule were evaluated by experts and were found to agree with their expectations.

An intermediate step in verifying the output validity was more objective. This was in the comparison of the frequency distribution produced by the simulation for a given schedule, and the frequency distribution obtained by using the estimating techniques due to Metheringham [11]. There was a good deal of agreement in the two frequency distributions for the schedules that were tested, within the limits of accuracy predicated by the sample size of the simulation.

* * * * * * *

REFERENCES

1. Aaker, David A., "ADMOD: An Advertising Decision Model", Journal of Marketing Research, XII (February 1975), 37-45

2. Balachandra, R., "Media Selections and Advertisement Scheduling Strategy", Unpublished Ph.D. Dissertation, Columbia University, 1975

3. Bass, Frank M., Pessemier, E.A. and Tigert, D.J., "A Taxonomy of Magazine Readership Applied to Problems in Marketing Strategy and Media Selection," Journal of Business, 42 (July 1969) 357-363

4. Bogart, Leo, Advertising Strategy, New York: Harcourt, Brace and World, 1967

5. Bowersox, D., Dynamic Simulation of Physical Distribution, Division of Research, Grad. School of Business, Michigan State University, 1972

6. Gensh, Dennis H., Advertising Planning, New York: Elsevier Scientific Publishing Co., 1973

7. Hicks, Charles R., Fundamental Concepts in the Design of Experiments, New York: Holt, Rinehart and Winston 1964

8. Jessen, R.J., "A Switch-over Experimental Design to Measure Advertising Effect", Journal of Advertising Research, March 1961

9. Kotler, Philip, Marketing Decision Making: A Model Building Approach, New York: Holt, Rinehart and Winston 1971

10. Little, John D.C., and Lodish, Leonard M., "A Media Planning Calculus", Operations Research, 17, (Jan.-Feb. 1969) 1-35

11. Metheringham, Richard A., "Measuring the Net Cumulative Coverage of a Print Campaign", Journal of Advertising Research, 4:4 (December 1964)

12. Naylor, T.H., Balintfy, J.S., Burdick, D.S., and Chu K., Computer Simulation Techniques, New York: John Wiley & Sons, Inc., 1966

13. Ray, Michael L., and Sawyer, Alan G., "Repetition in Media Models: A Laboratory Technique", Journal of Marketing Research, VIII (February 1971) 20-29

14. Simon, Julian L., Issues in the Economics of Advertising, Chicago: University of Illinois Press, 1970

15. Zufryden, Fred S., "Media Scheduling: A Scholastic Dynamic Model Approach," Management Science, August 1973, 1395-1405