JOINT COST ALLOCATION - A SIMULATION APPROACH

Morcos F. Massoud, Ph.D. and Ralph T. Hocking, D.B.A.
Shippensburg State College
Shippensburg, PA 17257

ABSTRACT

Joint products are produced together because the production process is essentially the separation of natural raw materials into component elements: for example, the conversion of coal into gas and coke, and milk into cream and skim milk. Sometimes joint products are produced together as a result of the technology adopted in a particular production process, as with electricity and water for irrigation from a hydroelectric dam. The essential characteristic of all the preceding examples is that neither product of each pair could be produced independently of the other once the technological setup has been decided.

Often the relative proportions of joint products emerging from a production process can be varied to some extent by changing the quality of input materials or the extent of processing. Such products are said to be joint products in variable proportions.

The accumulation of cost data in joint product situations can provide useful information for analysis of overall cost and revenue changes arising from variations in input quality and processing effort.

Interim incremental analysis of any production change would be profitable where the revenue (ΔR_x) from the enhanced joint products exceed the cost (ΔC) of undertaking the change and any loss in revenue (ΔR_y) from any reduced joint products. Thus a production change would be profitable as long as $(\Delta R_x - \Delta R_y) > \Delta C$.

However, the accounting model is concerned with determining the cost per unit of each joint product. Therefore, accounting for joint cost as traditionally developed through the use of traditional methods is concerned with finding the separate costs of products which cannot be produced separately. The division of joint product costs cannot give meaningful data on the relative profitability of joint products because joint products have no relative profitability: either they are profitable together or they are not profitable at all. The accounting model is useful only to value inventories; it was recommended because it minimizes distortion in the income statement.

The authors intend, through a linear programming simulation model, to show that the traditional accounting models tie into and support the legal-financial data of balance sheet and income statement instead of tying into and supporting the activities of management, and that post optimal analysis of the LP model provides cost data most useful for management decisions.

INTRODUCTION

A great number of products or services are linked together by physical relationships which necessitate simultaneous production. To the point of split-off or to the point where these several products emerge as individual units, the costs of the products form a homogenous whole - a classic example of joint product manufacturing is found in the production of gasoline where the derivation of gasoline inevitably results in the production of such items as kerosine and distillate fuel oils. Another example is found in the meat-packing industry where various cuts of meat and numerous by-products are produced from one original carcass with one lump sum cost.

The main characteristic of joint products is the fact that they have a common inseparable origin. The production of one is contingent upon the production of another product. Thus, one joint product may be said to be complimentary to another joint product. An increase in one joint product will normally result in an increase in a complementary joint product. The increase is not necessarily proportionate. For example, to obtain an increased amount of gasoline from crude oil, a proportionate increase in kerosine would result if the same distilling and cracking procedures were followed. However, by altering processing procedures a proportionately larger or smaller amount of gasoline may be obtained.

In fact, in certain limited cases more of one joint product may be obtained at the expense of a complementary joint product. For example, more gasoline may be obtained from a given quantity of crude oil than from a similar quantity by altering the processing procedures so as to get less kerosene, but the range of variation is limited.

ACCOUNTING AND JOINT PRODUCTS

Since joint products are inseparable products up to the split-off point, it follows that the costs of such joint products are also inseparable and cannot be identified directly with the various joint products. It is impossible, for example, to determine through any form of precise measurement what part of the cost of processing a barrel of crude oil should be applied to a given quantity of gasoline, kerosene, or paraffin.

The accounting motivation for developing product costs from joint costs has been primarily periodic financial reporting in accordance with "generally accepted accounting principles." The determination of inventory values and cost of goods sold requires that the joint costs be split between accounting periods and products. While the results from an approximate method of cost allocation are not ideal, they can be tolerated where it is believed that the actual results will have a minimum variance from the ideal situation. This will happen when the method is applied consistently over time periods and the inventories experience relatively small fluctuations. Where the beginning and ending inventories are zero, the allocation will have no effect upon periodic income. Warnings are given, however, in using these cost allocations for managerial decision purposes. "Where costs are wanted by internal management for evaluating alternative courses of action, it is often preferable to work with cost differentials and thereby to avoid allocating joint costs." (1) Two broad methods to allocate joint costs have been used by accountants.

ALLOCATION BY PHYSICAL ATTRIBUTES

The allocation of costs by physical attributes assumes that the products should receive costs relative to the benefits that the products received from the production process. Numerous physical factors are used including weight, volume, production run time, and qualitative weighted factors. In the petroleum industry, the barrel-gravity, gravity-heat, and BTU are three methods which use physical measurements.

The Barrel-Gravity Method: The specific gravities of the products refined are used as weights in the apportionment of new stock costs. The method seeks to recognize an alleged correlation between the gravity of petroleum products and their commercial value. However, this correlation is regarded as slight and in some cases, it is a negative relationship.

The Gravity-Heat Unit Method: It is a special adoption of the barrel-gravity method in which separate allocations are provided for crude and processing costs. Crude costs are assigned to products using the barrel-gravity method and processing costs are distributed to products on the basis of the number of heat units requisite to the various refined products manufactured. Since the crude costs bulk are so large, this method carries the same disadvantages as the first and neither one is in common use now.

The BTU Method: It uses the relative heat content of oil and gas, expressed in British Thermal Units. Equivalent BTU content ratios of gas to oil used range from 4.8 to 6 mcf to 1 bbl of oil. Joint costs are allocated in proportion to BTU content of the products and can be applied to the output of the test year as well as to the average remaining reserves. This method has been criticized for not properly reflecting the value of all the joint products produced. Only a part of the products resulting from crude oil are valuable for BTU content alone. Gasoline is valuable because of its form.

The three methods suffer from two potential weaknesses. First, there is the underlying assumption that the costs incurred vary in direct proportion to variations in the physical attributes. Second, all physical units are treated as homogenous in nature, which may not be true.

ALLOCATION BY ABILITY-TO-ABSORB COSTS

Probably the most prevalent method used is the allocation of costs to the products on the basis of their ability-to-absorb costs. The allocation of costs by the sales realization method results in costs for each product that are proportional to the sales value. It assumes that all products refined from crude oil yield the same rate of profit. This assumption necessarily implies that the profit ratios for gasoline, kerosene, middle distillates and lubricants are equivalent. Thus, it does not provide a reliable yardstick for planning and decisionmaking. Changes in the market value of one or more of the products automatically cause a change in the cost-allocation basis. Yet, it is likely that any one product will cost no more or less because of the market change. The method can cause distorted income measurement as well as inaccurate managerial decisions when some of the products' market value remains stable while others fluctuate.

The liminations of multiple product costing are clearly apparent from the many different unit costs which can be derived from a given set of data. Although there is always a cost that can be determined from any set of figures after certain presumptions have been made, there is no way of finding the cost of any one of several products emerging from joint processing. Without a clear understanding of the assumptions which were made in the allocation of joint cost, unit costs as managerial tools are not only uninformative, but worse yet they can be misleading.

However, management is constantly faced with deciding between alternative courses of action. What products should be manufactured and in what qualities? Should crude oil be processed to yield a higher percentage of gasoline? None of these questions is answered by unit costs computed after the conventional cost allocations have been made.

Management is not interested in the income being reported on individual products. Its objective is to earn a reasonable return on its investment in production facilities. To accomplish this goal, it must find a profitable combination of products. In making an evaluation, each product must be measured in terms of its contribution to joint cost recovery, not in terms of whether it is bearing its "fair share" of the total cost incurred.

LINEAR PROGRAMMING AND JOINT PRODUCTS

A more fruitful method for managerial decision-making, as well as for income determination in joint cost allocation, is using linear programming. At first glance, LP does not appear to be useful since it is not possible, due to the existence of joint costs, to compute the contribution toward profits of each product. However, as will be demonstrated, the objective equation does not have to be constructed in terms of the product's contribution toward profit.

The LP formulation which follows is structured so as to simulate a decision which management hopes will never need to be made -- that of ceasing operations because of unprofitability. The example is a simplified version of the joint products costing problem for a crude oil refining process.(2) The decision, in this case, is whether to refine the barrel of crude oil or not. While the decision may be obvious, the results from post-optimality analysis can contribute the desired joint product costs.

In the example, a barrel of crude oil costs $3.45 delivered to the refinery and $.95 to process it for a total cost of $4.40 per barrel. The decision to not refine a barrel of crude will, therefore, be worth $4.40 to the firm. If refining this barrel of crude yields .50 barrels of gasoline, .10 barrels of kerosene, .25 barrels of distillate fuels and lubes, .10 barrels of residual fuel oil and .05 barrels of losses, then not refining this barrel removes these products from the "inventory" of finished products. The constraints on the problem are the amounts of finished products refined (for "inventory") from a barrel of crude. If just one small quantity of one finished product is sold from inventory, the constraint equations are structured so as to prohibit the "do not refine" decision.

Market prices for the finished products in this example are: $5.20/ββl for gasoline, $4.40/ββl for kerosene, $5.20/ββl for distillate fuels and lubes, and $2.60/ββl for residual fuel. (These prices are, admittedly, many years out-of-date, but more about that later.) If the variable, C, is the barrel of crude not refined, G the barrel of gasoline sold, K the barrel of kerosene sold, D the barrel of distillate fuels and lubes sold, and F the barrel of residual fuel sold, (Losses are not included since they produce no revenue.) then the LP model can be formulated as:

Max: revenue =

$$4.40C + 5.20G + 4.40K + 5.20D + 2.60F$$

Subject to:

$$.50C + 1.0\,G \le .50 \text{ gasoline}$$
$$.10C + 1.0\,K \le .10 \text{ kerosene}$$
$$.25C + 1.0\,D \le .25 \text{ dist.\&lub}$$
$$.10C + 1.0\,F \le .10 \text{ resid.Fu.}$$

The solution to this simple model is almost trivial and, in fact, does require the barrel of crude to be refined (C not in the basis). .50 barrels of G, .10 barrels of K, .25 barrels of D, and .10 barrels of F are the respective levels of production in the optimized solution.

The value of this example lies in the next step -- post optimal analysis. The appropriate "what if" question concerns the market prices of the finished products. If the market price for one of these products were to decline, how far down must it go before the total situation reaches the point where that barrel of that product will not be sold. For example, if the market for residual fuel declined from the $2.60/barrel assumed in the example (as, by the way, it did in the early 1960's), at what point would the basis change so that residual fuel is no longer sold? This point is taken as an obvious break-even point at which the product cost equals the revenue from that product.

A question such as that just stated can be answered by ranging the objective raw coefficient for the product under analysis. The low end of the range for that coefficient is the break-even price for that product as it exists and jointly interacts with the other products and the alternative of not refining the barrel of crude in the first place.

The process of deriving these revenue coefficient ranges is described in most operations research texts and need not be detailed here. This particular example was run on the MPS package supplied by Univac for the Univac 3 at Shippensburg State College.

The results of ranging the objective raw revenue coefficients appear below:

Variable	Coefficient	Coefficient Range Lower	Upper
Crude	4.40	$-\infty$	4.60
Gasoline	5.20	4.80	∞
Kerosene	4.40	2.40	∞
Dist. & Lube	5.20	4.40	∞
Resid. Fuel	2.60	.60	∞

These results indicate that managerial decisions can be made to meet, competitively, a deterioration of the price of gasoline down to a low of $4.80/ββl, so long as everything else remains the same. similarly, the firm can compete in the kerosene market at prices down to nearly $2.40/ββl; for distillate and lubes down to $4.40/ββl; and for residual fuel customers as low as 60¢/ββl. Interestingly, it was during the period when these market prices were operational that at least one major oil company considered $1.75/ββl to be the break-even price for residual fuel sold to the Queen Mary. The results of the coefficient ranging indicate that such a price was more than a dollar a barrel too high.

The range on the crude oil "revenue" coefficient is of further interest. This range indicates that cost of purchasing and refining a barrel of crude could increase to only $4.60 before refining operations cease. As mentioned earlier, the values used in this example are out-of-date and the question now might center on the four-fold increase in crude costs in recent years. Obviously, the market prices for the refined products has increased drastically also and, had they not, the oil companies should not be operating. Drastic changes in market prices for the raw materials and joint finished products such as have occurred in the oil industry demand that accurate means be available upon which to base managerial decisions. The authors regret not having complete cost data for the recent oil situation, so that the model could be re-run. Clearly, with crude prices in the $15/ββl range instead of the $3.45 used in the example, and refining costs assumed to increase by some large amount, results such as this example present could change completely.

The changes in the separate market prices for the finished products could have an even more interesting influence on the results. While the average motorist has seen gasoline prices more than double in the past decade, the steamship France saw its residual fuel cost increase to nearly $15.00/ββl before it was retired from service. Based on the $2.60/ββl used in the example, this is nearly a <u>six-fold</u> increase.

When the changes in costs and prices are not proportional to each other, results showing break-even prices can change drastically. With crude prices increasing by a factor of four, gasoline more than doubling, and residual increasing nearly six times, a re-running of a linear programming analysis such as the one presented could help managers re-direct their attention toward those components of the barrel once thought of as by-products (residual fuel) instead of assuming gasoline as the only important component of the barrel. Those experiences with linear programming know that there is a real possibility that the break-even price for gasoline could, in fact, be <u>lower</u> as indicated by ranging objective raw coefficients than major oil companies think. The example shows that the break-even price for residual fuel was, in fact, much lower than was thought in the past. As the value of residual fuel increases at a more rapid rate than gasoline, results showing over-pricing of gasoline could easily be imagined.

Regardless of how results in present situations may turn out, costs can be derived by ranging the objective coefficients of an LP model simulating the decision problem of refining crude or not. Costs derived in such a manner appear to satisfy the difficulties encountered in joint product cost allocations.

SUMMARY

As has been demonstrated, the process of deciding whether or not to refine the crude oil is not as simple as accountants assume through conventional accounting methods. Linear Programming can be used in these situations as long as the production relationships remain relatively constant. However, in applying linear programming it is necessary to allow for inventories of unused intermediate outputs. It is not possible to construct a general model, but through simulation, many decisions models could be formulated when joint products are involved.

REFERENCES

1. Costing Joint Products, National Association of Cost Accountants, Research Series, 31 (April 1, 1957), p. 7.
2. The example is borrowed from Charles H. Griffin, "Multiple Products Costing In Petroleum," <u>The Journal of Accountancy</u>, March, 1958.
3. Wagner, Harvey, <u>Principles of Management Science</u>, Prentice-Hall, Englewood Cliffs, N.J., 1970, pp. 112-113.