

PROBABILISTIC BUDGETING: ONE PRACTICAL EXPERIENCE

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ABSTRACT

Probabilistic budgeting has been recommended in the accounting literature for nearly fifteen years as a basis for enabling organizations to better plan for and cope with uncertainty. Unfortunately, while it has been argued that the information contained in a probabilistic budget is essential for improved planning and management neither the feasibility or relevance of probabilistic budgets has been demonstrated.

An attempt to develop a probabilistic budget utilizing Monte-Carlo Simulation is reported in this paper. It describes how the model of a small transport firm was developed and how the data necessary to support the model was gathered. It then discusses the apparent implications of this probabilistic budget for the firm, for the practical application of simulation techniques to the budget development process, and for the broader field of planning and management control.

INTRODUCTION

During the past fifteen years probabilistic budgeting has been recommended in the accounting literature to:

- help management better understand the risk it faces (7)
- help management better analyze its opportunities and options (8)
- improve performance evaluation models (13)
- decide how much it is worth to attempt to change the future (11)
- prepare doomsday budgets to determine if proposed projects will or will not fail (4)
- provide better planning and control (9)
- show when and where contingency plans must be developed and implemented to meet profit objectives (3)

Although it has been argued that the information contained in a probabilistic budget is essential for good management, there is practically no information available on its current use (3). It has not been shown that a probabilistic budget can be prepared within a firm, nor that the preparation of such a budget will change the actions or activities managers undertake.

The purpose of this paper is to report one current attempt to establish the feasibility and relevance of probabilistic budgeting. The paper incorporates the probabilistic budget of one small firm; describes the process and difficulties overcome in its preparation; and then discusses its apparent implications for the firm, and its implications for the broader field of planning and control.

BUILDING A PROBABILISTIC BUDGET

During the past year, the author has been concerned with the planning problems of Floral Transport, a small Canadian firm. Floral Transport specializes in the movement of perishable produce to Canada from the United States. The firm operates a fleet of modern highway tractors and temperature controlled trailers between the grower's fields, the firm's southern consolidation terminal and its customers in Canada.

Recently, in the face of growing volume the firm substantially expanded its fleet. As a result of this expansion and volatility of past earnings, the firm's president has been seeking a more reliable process for estimating future operating results so that: "I'll know if we're going to have problems meeting our commitments on the new equipment."

Answering this concern required forecasting what the year's operating results might be. More importantly it required identifying potential problems which would have to be overcome if the business was to be as profitable as the projections provided the firm's financiers had indicated. Floral faced significant uncertainties: growth in volume; changes in prices and freight mix, direct operating and overhead costs.

Forecasting the firm's results required that these uncertainties be considered. To accomplish this objective the President was asked to think about the uncertainties his business faced, and to quantify his feelings about these uncertainties. The argument for this process was that if anyone could assess the likely future the firm would face, it was the President who was intimately involved in the firm's operations. He should have the best available information concerning the operations of

PROBABILISTIC BUDGETING (continued)

the business, competitive and operating trends and the implication of these trends for the firm. More importantly, he made policy and operating decisions based on what he believed the future would be.

To establish what estimates were to be gathered a model of Floral Transport was developed (Table 1). This model was structured in accordance with the firm's chart of accounts and its financial reports. This structure was chosen, because management was familiar with the components of the financial system; historical data was available for estimating some of the parameters of the model; the relationships between the accounts were either known or could be estimated from prior operating results, and finally, because it was intended to produce a plan which could be compared to actual results as a basis for assessing its validity, and as a basis for operational control.

Gathering Estimates of Uncertainty

Estimates of uncertainties facing Floral Transport model (Table 2) were developed by means of a successive subdivision protocol (10,12). The President was asked:

- 1) What do you expect the value of X (the variable being elicited) to be?
(This answer was taken to be the President's .50 fractile on a cumulative density function)
- 2) You would be greatly surprised if X was greater than what value?
(This answer was taken to be the President's 1.00 fractile on a cumulative density function)
- 3) You would be greatly surprised if X was less than what value?
(This answer was taken to be the President's .00 fractile on a cumulative density function)
- 4) If I told you for certain X was between the .5 fractile (answer to question 1) and the .00 fractile (answer to question 3) would X be more or less than Y (a number chosen by the interviewer between the .5 and the .00 fractiles)? This question was repeated with the value of Y changing until the President became indifferent.
(This answer was taken to be the President's .25 fractile on a cumulative density function), and finally
- 5) If I told you for certain X was between the .5 fractile (answer to question 1) and the 1.00 fractile (answer to question 2) would X be more or less than Y? (Again repeated until the indifference point).
(This answer was taken to be the .75 fractile of the cumulative density function)

It was decided that the firms overhead costs were basically discretionary--that their amount would be decided upon by management, and that the only real uncertainty relative to these items would be due to

price changes. For this reason overhead costs, estimated on a quarterly basis (Table 3) were incorporated in the model along with a normally distributed spending variance with a mean of zero and standard deviation of ten percent.

All data gathered was assembled and processed by means of a computer model written in SIMPAK. SIMPAK is a special-purpose Fortran based computer language for the Monte-Carlo simulation of complex problems (1). Briefly, SIMPAK provides subroutines which convert the summary descriptions of uncertain inputs into detailed cumulative density functions available to the logic model provided by the analyst. The logic model describes the relationships between the various uncertainties, and the outputs to be generated by the model. As the model executes additional SIMPAK subroutines keep track of the results of each trial and generate output reports--the probabilistic budget presented in Table 4.

WHAT DOES IT MEAN

Implications For Floral Transport

The Floral Transport probabilistic budget provides information not available from the more traditional single figure, or point estimate budget. It describes the range of possible outcomes for each element of the budget, as well as indicating the likelihood or probability that various levels of revenue, expenses and profitability will be achieved. For example, while the Floral budget indicates that a profit of \$75,462 is expected for 1977, the probabilistic forecast indicates operating results could range anywhere from a loss of \$45,000 to profits of \$182,000. Based on the simulation results, we can make the following statements about this range of profits:

- 1) There is a 75% probability that the firm's profit will be at least \$50,000, but,
- 2) There is only a 25% chance that the firm's profit will exceed \$106,000.

Perhaps more significantly, plotting the cumulative probability for cash flow (Figure 1) indicates that there is a 48% chance operations will generate a cash flow less than the required, \$130,000 Floral must repay on its new equipment during the year. Or in other words there is about only 1 chance in 2 that operations will generate enough cash to meet the firm's commitments. For Floral Transport, the probabilistic budget indicates a potentially serious problem -- a problem not indicated by a more traditional point estimate budget.

The important challenge facing Floral's management is determining what operational or policy changes the firm might undertake to reduce the variability in its results. This challenge implies an understanding of the uncertainties Floral faces -- their causes, and the extent to which they can be controlled.

The greatest variability in Floral's transport expenses is in the "hired vehicles" expense account.

TABLE 1

FLORAL TRANSPORT MODEL DETAILS

1) Annual Volumes:	$\tilde{V}_i = V_{t-1,i} \times \tilde{G}_i$, Where \tilde{V} = annual \$ volume i = 1 to 7 product lines \tilde{G} = uncertain anticipated growth rate
2) Transport Revenue	$\tilde{R} = \sum_{i=4}^7 \tilde{V}_i$,
3) Mileage	$\tilde{M} = \tilde{R}/\tilde{r}$, Where \tilde{M} = annual transport miles \tilde{r} = uncertain revenue per mile for transportation.
4) Direct Costs	$\tilde{D}_j = \tilde{M} \times \tilde{C}_j$, Where \tilde{D} = the direct cost per annum j = 1 to 7 direct operating costs \tilde{C} = uncertain operating cost per mile
5) Total Transport Cost	$\tilde{T} = \sum_{j=1}^7 \tilde{D}_j$,
6) Transport Margin	$\tilde{K} = \tilde{R} - \tilde{T}$,
7) Sales Margin	$\tilde{S} = \sum_{i=1}^3 V_i \times \tilde{m}_i$, Where \tilde{m} = uncertain margin rate
8) Total Margin	$\tilde{L} = \tilde{K} + \tilde{S}$,
9) Operating Costs	$\tilde{O} = \sum_{i=1}^8 o_i \times (1+\tilde{v})$, Where o = operating cost planned \tilde{v} = uncertain spending variance
10) Administrative Costs	$\tilde{A} = \sum_{i=1}^5 a_i \times (1+\tilde{v})$, Where a = administrative cost planned \tilde{v} = uncertain spending variance
11) Depreciation	W, is given	
12) Profit	$\tilde{P} = \tilde{L} - (\tilde{O} + \tilde{A} + W)$	

TABLE 2

FLORAL TRANSPORT - CUMULATIVE DENSITY FUNCTIONS DESCRIBING MAJOR UNCERTAINTIES IN THE FLORAL TRANSPORT MODEL

	Points on Cumulative Density Function					
	.00	.25	.50	.75	1.00	
Growth - product line 1	0.0	12.5	20.0	25.0	30.0	%
- product line 2	-10.0	-3.5	0.0	2.5	5.0	%
- product line 3	0.0	25.0	50.0	100.0	200.0	%
- product line 4	0.0	17.5	25.0	32.5	50.0	%
- product line 5	15.0	23.5	25.0	26.5	35.0	%
- product line 6	25.0	30.0	35.0	41.0	50.0	%
- product line 7	10.0	17.5	25.0	35.0	100.0	%
Margin - product line 1	40.0	43.0	45.0	50.0	60.0	%
- product line 2	0.0	12.0	15.0	17.0	20.0	%
- product line 3	10.0	20.0	22.5	25.0	40.0	%
Transport Revenue Per Mile	.80	1.10	1.15	1.20	1.25	\$
Direct Cost Per Mile - manpower	.18	.21	.225	.24	.30	\$
- maintenance	.05	.075	.09	.10	.125	\$
- fuel	.115	.145	.16	.17	.20	\$
- licenses, permits, insurance	.04	.05	.055	.06	.07	\$
- hired vehicles	.10	.17	.20	.23	.30	\$
- redelivery	.40	.50	.55	.60	.75	\$
- communications	.020	.022	.024	.028	.035	\$

PROBABILISTIC BUDGETING (continued)

TABLE 3

FLORAL TRANSPORT - OVERHEAD COSTS PER QUARTER

Utilities	\$2,000	Office Wages	\$20,000	Running Supplies	\$1,000	Property Taxes	\$250
Facilities	\$2,000	Office Expenses	\$4,000	Warehouse Expenses	\$1,000	Salaries	\$7,500
Professional Fees	\$2,000	Interest	\$6,500	Travel	\$2,500	Advertising	\$500
		Depreciation	\$15,000				

TABLE 4

FLORAL TRANSPORT FISCAL 1977 BUDGET (Based on 500 Trials)

	Expected		Cumulative Probabilities			1.00
	Value	.00	.25	.50	.75	
			(budget amounts in 000's)			
Mileage	880312	778	846	874	908	1058
Revenue Per Mile	\$ 1.260	\$1.098	\$1.226	\$1.27	\$1.307	\$1.375
Transport Revenue	\$ 1109870	1040	1086	1108	1132	1191
Manpower	\$ 199139	161	189	197	207	253
Maintenance	77450	55	71	77	83	102
Fuel	139593	109	131	139	146	172
License & Insurance	48482	38	45	48	50	59
Hired Vehicles	201288	145	185	199	215	275
Redelivery	100079	71	90	99	109	145
Total Transport	\$ 766031	666	734	761	794	925
Transport Margin	\$ 343839	223	317	347	374	446
Sales Margin	28267	21	26	27	30	38
Total Margin	\$ 372106	249	344	376	402	472
Utilities	\$ 7997	7	7	7	8	8
Telephone	30156	24	28	29	31	38
Facilities	7982	7	7	7	8	8
Office Wages	90483	69	86	90	95	106
Office Expenses	16998	15	16	16	17	18
Running Supplies	4650	4	4	4	4	4
Warehouse Expenses	6199	5	6	6	6	6
Property Taxes	900	1	1	1	1	1
Total Operating	\$ 165365	144	160	165	170	183
Salaries	\$ 29230	27	28	29	29	31
Professional Fees	10010	9	9	10	10	10
Interest	25527	24	25	25	25	27
Travel	9510	8	9	9	9	10
Advertising	2002	1	1	2	2	2
Total Admin.	\$ 76729	71	75	76	77	80
Depreciation	\$ 55000	55	55	55	55	55
Total Expenses	\$ 296644	273	291	296	301	314
Profit	\$ 75462	-45	50	78	106	182
Cash Flow (profit plus depreciation)	\$ 130462					

The question facing the President is why is this expense uncertain, and what can be done about it?

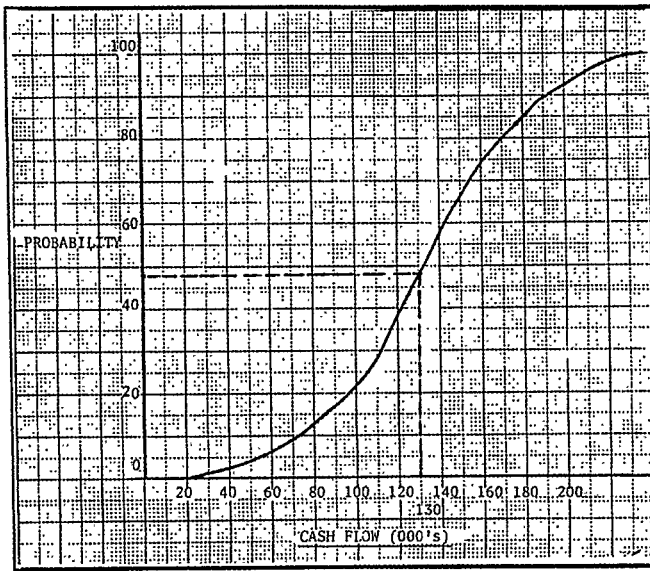
Floral rents equipment from other truckers, and from rental firms like Ryder when volume is greater than the firm's own capacity, or when breakdowns reduce the firm's capacity. If management could foresee extra demand additional equipment could be added permanently to the fleet -- capital investment could reduce the necessity to utilize hired vehicles and could reduce the likelihood of major breakdowns. To date management has not had enough confidence in its ability to predict future volume to make this investment. Perhaps under probabilistic budgeting where uncertainties can be explicitly

examined this decision will be better addressed.

The next major variation is attributable to the manpower account. Partially this variability is due to a multiplicity of pay plans and payment schedules utilized by Floral. A change in management policy to reduce the number of pay schemes could reduce this variability.

Fuel costs also vary. Management has already taken steps to reduce this variability and the total fuel cost. The installation of fuel supplies at its depots means Floral's trucks will no longer be as susceptible to the vagaries of the retail highway fuel market, and the new "fuel-economy" tractors

FIGURE 1
FLORAL TRANSPORT - CASH FLOW



should reduce actual fuel consumption.

The driver's behavior can significantly effect the fuel consumption of Floral's equipment over the long distances the company travels. By recording, monitoring and perhaps rewarding acceptable consumption fuel costs/variability may be reduced.

In summary, the information generated by the probabilistic budget, and the analysis of the results it projects raise serious questions about Floral's operations. If management responds appropriately to these questions the variability (riskiness) of the firm's operations can be reduced.

IMPLICATIONS FOR DEVELOPING PRACTICAL PROBABILISTIC BUDGETS

The variability which exists in a Monte Carlo simulation may be attributed to several factors:

- 1) Errors in the determination of the subjective probability functions.
- 2) The nature of data gathered.
- 3) The manner in which the model is constructed.
- 4) The number of trials which the model is simulated.
- 5) Uncontrollable causes as a result of changes in the economy or other similar factors over which management has no control.
- 6) Those causes which management can control.

What portion of the variability is assignable to each of these factors is difficult to say. It would seem, however, that the first four technical

factors alone can have a considerable effect on the validity of results reported by any Monte-Carlo forecast.

Elicitation Errors

The effect of errors in the estimation of the probabilities upon which the simulation is developed may be small on the estimation of means, but is typically large on the estimation of the standard deviation, or the range of outcomes reported. Only minor changes in the estimates provided the model change the degree of variability significantly. The picture of risk presented is highly susceptible to the accuracy with which we are able to develop management's initial assessments of uncertainty.

Recognizing the substantial influence that the accuracy of initial estimates may have upon the model's final results raises questions of how the analyst should evaluate the accuracy of those initial estimates. In developing the Floral model an attempt was made to at least assure the data presented was internally consistent. Other checks suggested in the elicitation literature such as

- i) asking for the same information by means of several elicitation protocols to establish the consistency of the responses.
- ii) repeating the elicitation questions several days later to establish the stability of these estimates.

were not attempted because we were dealing with the President, an extremely involved and busy manager. While eliciting data from this firm's key officer minimized problems of dealing with conflicting responses from multiple respondents, there was a limit to both the time the President could give to the elicitation questions, and to his patience. How do you tell a President -- "I'm sorry sir, but I am not sure I believed your answers yesterday, would you answer my questions again today?"

Until these serious behavioral questions can be answered, the feasibility of establishing accurate estimates of the manager's perceptions remains in doubt. And without faith in these estimates, how can we rely on the results of the probabilistic forecasts.

Structural and Data Errors

Haley and Schall have discussed economic and statistical independence (2). We can say we have economic independence where the occurrence of a specific value of one variable does not affect the probability of the occurrence of a second variable. For example, in Floral Transport, if we have a specific cost per mile for manpower, that should not affect the specific cost for license, permits and insurance per mile. These two variables are, therefore, economically independent. These two variables may, however, be statistically dependent. They are statistically dependent because they tend to move in the same direction along with the general economy.

PROBABILISTIC BUDGETING (continued)

In Floral, while it was realized interdependencies could exist, all data gathered was elicited on an "independence" basis. As it was nearly twenty distributions were elicited for this simple model. Had joint distributions been requested, as might well have done between the growth expectations by product line, or between major operating costs the number of assessments required could have increased to over 60 -- clearly more assessments than we could have readily asked this President.

In addition to the problem of data overload the dependence/independence question poses additional problems relative to the structure of the Monte Carlo model. If one randomly samples from a number of independent, random variables, it would be expected that the correlation matrix between the random variables would not be significantly different from 0. That is, the elements on the off diagonal should be close to 0 and that on the average, half of them would be negative and half of them positive.

The challenge to the modeller is how does his model behave -- does it reflect the intervariable relationships anticipated. Table 5, reproduces the intervariable correlations actually generated by the Floral Transport model. Since the Floral Transport model samples independently from each of the distributions included in the model, the intervariable correlations indicated by the table are not expected. Where do these apparent relationships come from?

Table 1, suggests an answer to this question. All the transport costs are functions of mileage in the Floral model. As mileage increases so will the individual transport costs -- an unexpected relationship has been built into the model because of the way it was structured.

The challenge is to establish if this relationship is appropriate. For this reason, Floral's data for six months selected operations were assembled and analyzed to establish the degree of inter-variable variability. This analysis indicated that with the exception of the manpower, maintenance and fuel costs little dependence exists between Floral's transport costs.

The historic relationship between manpower, fuel and maintenance costs poses special modeling problems. Assuming independence would understate this relationship in the model, while assuming dependence would clearly overstate these relationships. The artificial relationship provided by the structure of the model also understates the historic pattern.

Coping with this problem remains under investigation. Possible answers appear to include attempting to derive joint distributions (i.e. complicate the data gathering problem), or managing the simulation process to generate the correlation patterns desired (5,6). To date no satisfactory answer to this problem has been developed.

Length of Run

The length of the simulation run can also influence the reported results. While changing the length of run does not substantially change estimates of means, differences are apparent in the estimates of the standard deviations and estimates of inter-variable correlations change substantially with changes in the length of run.

For the modeller, the question of length of run implies an important trade off between the accuracy of results he may desire, and the costs of running the simulation. Executing the Floral Transport model incurs substantially greater costs the greater the number of trials required*.

IMPLICATIONS FOR BUDGETING AND CONTROL

For planning, budgeting and control in general, the advent of probabilistic budgeting also appears to raise important questions.

Inadequacies of Current Practices.

Evaluating Uncertainty

A common practice when planning is the use of

*100 trials required 50.52 CPU seconds on a DecPDP 10
 500 trials required 210.04 CPU seconds and
 1000 trials required 486.24 CPU seconds.

TABLE 5
 FLORAL TRANSPORT - INTERVARIABLE CORRELATIONS

	<u>Manpower</u>	<u>Maintenance</u>	<u>Fuel</u>	<u>License</u>	<u>Hired Vehcles</u>	<u>Redelivery</u>
Manpower	1.000	.320	.374	.452	.290	.047
Maintenance		1.000	.372	.320	.215	.065
Fuel			1.000	.457	.348	.055
License				1.000	.287	.087
Hired Vehicles					1.000	.018
Redelivery						1.000

sensitivity analysis to explore a project's riskiness. Typically this means evaluating a project under various sets of assumptions -- assuming the absolute worst future will prevail; and then, that the very best future will occur. Based on these assumptions the profitability of the project and its risk are assessed.

Unfortunately, this common practice tends to overstate the riskiness and variability of the project. A best/worst budget prepared for Floral Transport reflected a range of profits from -\$182,000 to +\$293,000, which is substantially greater than the range described by probabilistic budget.

The worst/best analysis overstates the riskiness because it does not consider the co-variability of the elements of the budget*. Co-variance is not considered in the typical best/worst analysis - in fact, without a tool like probabilistic budgeting it would be almost impossible to even estimate.

Managing Uncertainty

If we analyze the basic elements of most current management control systems: a departmentalized or divisionalized structure; a process for setting long range objectives and operating budgets coupled to routine reporting, evaluative and reward systems, it is apparent that their form, their intent and the activities associated with them, remain basically suited to the management of stable predictable situations.

The firm operating in a stable environment is able to establish long term objectives and to forecast anticipated levels of activity and costs and revenues as a basis for the preparation of detailed operating plans, budgets, and exception reporting systems. The firm facing substantial uncertainties has great difficulty predicting any reliable estimates of future activity. The development of long range plans, budgets, standard costs becomes extremely difficult in such dynamic settings.

*While the mean of the transport margin equals the difference of the means of the transport revenue and transport expenses the variance of the transport margin is not equal to the difference of the variance of the transport revenue and the transport expenses because transport revenues and transport expenses are not independent. Instead the variance of the transport margin is equal to:

$\sigma^2x_1 + \sigma^2x_2 - 2\sigma x_1x_2$, where σx_1x_2 is the covariance of the transport revenue and transport expense terms, or

$963,540 + 2,104,330 - 2(627,178) = 1,813,514$
whose square root = 42585, is the standard deviation of the transport margin.

The Way to Go

Probabilistic budgets -- or more generally, the concept of recognizing uncertainty in our plans and our control practices should alleviate these shortcomings.

Recognizing the potential major uncertainties in operating plans should lead to more contingency planning. The recognition that operating results are highly volatile should lead managers to attempt to at least understand, and at best control that volatility. At least, this recognition should lead to a more thorough exploration of the underlying causes of the volatility. At best, managers should create plans to cope with these uncertainties.

SUMMARY

While probabilistic budgets and Monte Carlo simulations have been recommended during the past fifteen years, little has been done to explore the feasibility and relevance of this approach to planning. This paper reports one attempt to establish whether preparing a probabilistic budget is feasible and useful.

Based on this attempt, it is apparent that while possible, the Monte Carlo process is fraught with practical problems. For this reason probabilistic forecasts based on this technique must be carefully prepared and interpreted. Much work still is necessary to improve data gathering techniques, and to improve understanding of how probabilistic models behave and can be utilized.

While there are problems with the Monte Carlo approach, this simulation did generate information beyond the normal budget process -- information of value to the firm's President. The measure of risk identified by the model at least verified the President's concerns for his firm's operations, and perhaps as importantly helped suggest areas of operational change.

The feasibility of preparing a probabilistic budget has been demonstrated for a small, relatively simple firm. Can such a budget be developed for a large, complex, perhaps multi-divisional organization? Work to investigate this question is currently underway. Already, special problems of adapting to current budgeting practices, and of modifying existing deterministic budget models pose serious challenges. It is not that managers are not interested in forecasting uncertainty in the large firm -- the technical problems are just more difficult.

It has been anticipated probabilistic forecasts will pay off for management because contingency plans will be developed and the variability of operating results reduced. To date it has not been demonstrated that managers forewarned about uncertainty can better cope with that uncertainty - this practical pay off of probabilistic budgeting is still to be demonstrated.

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