A SIMULATION ANALYSIS OF DYNAMIC INVENTORY POLICIES IN A GENERALIZED STOCHASTIC ENVIRONMENT

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ABSTRACT

This paper presents a simulation-based comparative analysis of several of the more prominent inventory lot-sizing models under multivariate stochastic conditions. Specifically included as stochastic system variables are lead time variation, demand forecast error, and inflation rate. The six rules, classic EOQ, periodic order quantity, least unit cost, part-period balancing, lot for lot, and Wagner-Whitin, are adjusted to deal with a dynamic situation.

Each dynamic lot-sizing rule was simulated for a period of three years with each stochastic variable introduced at three levels. The results were then analyzed on the basis of a complete factorial design and, secondly, as a series of randomized block designs.

INTRODUCTION

There are many unique details which require careful consideration during the design of an inventory control system. However, there are two fundamental decisions which are common to the design of all inventory control systems. The first basic decision regards the degree of control applied (or administered) to each item, or item class, inventoried in the system. Typically, this decision problem is resolved by utilizing the well-known technique referred to as ABC Analysis, although this is not the only available alternative; e.g., see Mayer (3). Subsequent to the classification of inventoried items for control purposes, a second basic decision problem must be resolved. This decision encompasses the selection of an inventory policy (or control mechanism) to be utilized in making operational inventory decisions for items contained in the different control classifications. Obviously, the most important items, i.e., those comprising the highest total dollar usage class, should receive (or be subjected to) the most sophisticated level of control. These control mechanisms typically include complex lot-sizing and buffering rules and also the use of computerized real-time systems to maintain perpetual records and perform updates and reviews.

The essential functions of an inventory policy are to determine <u>when</u> (or at what time) an order should be placed (or a setup started), and <u>what quantity</u> should be ordered (or produced). Ordinarily, the selected policy is expected to satisfy some specified management objective(s), such as minimizing total inventory costs, maximizing return on investment, maintaining a specified service level, and so on.

Most of the currently available inventory lot-sizing models have been developed under assumptions of deterministic conditions. These models must be combined with a buffering technique to protect against uncertainty if they are adapted for use under stochastic conditions.

Some research has been conducted regarding the performance of inventory lot-sizing models under conditions involving limited stochastic variation; e.g., see 6, 7, and 9. The purpose of this study is to perform a simulation-based comparative analysis of the well-known models in a generalized stochastic environment.

OBJECTIVES

The primary objective of this study is to investigate the performance of the most prominent inventory lot-sizing models under multivariate stochastic conditions. Specifically included as stochastic system variables are lead time, variation, demand forecast error, and the rate of inflation.

Two criteria are utilized to evaluate the performance of the lot-sizing models over an arbitrary time horizon. They are:

(a) the total net present cost (TNPC) of a policy, where TNPC = PV (unit costs + ordering costs + carrying costs + shortage costs), and
(b) the shortages experienced under a policy, measured as:
    (1) average shortage in unit-periods, and
    (2) the standard deviation of unit shortages.

There are two secondary objectives. The first is to identify, if possible, the general stochastic conditions for which each of the lot-sizing models is "best suited." The second is to analyze the effect of buffering methods on the performance of the total inventory policy. The overall intent is to make a significant step towards providing general guidelines for inventory systems designers and managers faced with decisions regarding the selection of comprehensive inventory policies.

ASSUMPTIONS REGARDING ENVIRONMENTAL CONDITIONS

The development of mathematical inventory decision models has provided the practitioner with a large number of useful tools for analyzing inventory problems. The need to formulate these models in operational form has necessitated the abstraction of the real-world environment by specific limiting assumptions, which may vary from model to model. As one closes the modeling loop via utilization of a particular decision model in an actual situation, it becomes necessary to investigate the effects of these simplifying assumptions on the "optimal" solution determined by the application of the model under "nonideal" conditions.

The purpose of this study is to compare inventory lot-sizing models in a "realistic" operational environment, i.e., under generalized stochastic conditions. However, the assumption of a stochastic environment violates many, if not all, of the assumptions under which the lot-sizing models were developed. Therefore, it seems reasonable to consider what effect "relaxing" these assumptions has on the performance of the models.

In the stochastic environment lead time, demand fluctuation and demand forecast error are represented as random variables. Further, prices and costs are allowed to vary by coupling them to a variable rate of inflation.

In this study, it is assumed that:

(a) the lead time is uniformly distributed around a mean value of 15, with the range expressed as a percentage of the mean value;
(b) the demand forecast is generated from a uniform distribution with a mean = 100, and a range = 50, i.e., uniform (75, 125).
(c) the forecast error (actual demand - forecasted demand) is normally distributed about the forecasted demand value.
(d) the rate of inflation is normally distributed about a mean value, which is related to the discount rate.

Other assumptions regarding procedural details include:

(e) orders received during a given time period (day) are not available for use until the following time period (day).
(f) inventory carrying costs are computed on the average number of units on hand during a given period, i.e., (BOP + EOP)/2.0, where BOP and EOP represent the inventory on hand at the beginning and end of a period, respectively.

LOT-SIZING MODELS AND INVENTORY POLICIES

As was previously mentioned, the principal function of an inventory policy is to determine when an order is to be placed, and what quantity is to be ordered. Under various assumptions and deterministic conditions, the most prevalent inventory lot-sizing rules readily provide the required decision values. (The when decision is implicitly specified once the quantity decision is known, or vice versa.) However, in a stochastic environment, the when decision requires a separate analysis apart from the quantity decision. Uncertainty regarding the demand during lead time is the central issue, and the two main approaches utilized in confronting this problem are safety stocks and safety lead time, as suggested by Whybark and Williams (9).

As a matter of convenience, we have arbitrarily chosen to combine a safety stock buffering method with each of the inventory lot-sizing models under investigation in order to create operational inventory policies. The six lot-sizing models selected for comparison are taken from those suggested by Orlicky (4). They are listed here and described in the next section, along with the procedure utilized to determine the safety stock factor.

(a) Economic Order Quantity (EOQ),
(b) Periodic Order Quantity (POQ),
(c) Lot for Lot (LFL),
(d) Least Unit Cost (LUC),
(e) Part-Period Balancing (PPB) (or least total cost with extensions), and
(f) Wagner-Whitin (WWM), a dynamic programming approach.

The following notation is utilized in the description of the lot-sizing models:

Symbol		Represents
C_1	=	cost to place an order
C_2	=	carrying cost per unit-period
C_3	=	backordering cost
d_i	=	demand during period i
H	=	number of periods in the planning horizon
n	=	last period covered by previous order(s)
P	=	unit price
Q	=	order quantity
Q*	=	"optimal" order quantity
R	=	the demand over the planning horizon

A. Economic Order Quantity (EOQ)

There have been numerous versions of the economic order quantity model presented in the literature; e.g., see 2, 3, and 5. For the purpose of this investigation, the classical EOQ model was used. In this model, the total cost (TC) is

$$TC = R \cdot P + (C_1 \cdot R)/Q + (Q \cdot H \cdot C_2)/2.0$$

where $R = \sum_{i=n+1}^{n+H} d_i$

and the optimal order quantity is given by:

$$Q^* = \mathrm{SQRT}[(2.0 \cdot C_1 \cdot R)/(H \cdot C_2)]$$

In a stochastic environment, it is likely that the value of Q* will vary each time an order is placed because the demand over the planning horizon (R) is subject to variation.

B. Periodic Order Quantity (POQ)

The development of the periodic order quantity (POQ) model is quite similar to that of the EOQ model. The emphasis, however, is on the order interval (or time between orders) rather than on the quantity ordered.

In a planning horizon of length H, the number of orders placed (according to EOQ) is R/Q. Thus, the order interval will be (HQ)/R. In the dynamic sense, this rule indirectly specifies the order quantity by determining the time interval the order is to cover. Thus,

$$Q^* = \sum_{i=n+1}^{n+I} d_i \text{, where } I = (HQ)/R$$

C. Lot for Lot (LFL)

The lot for lot model is similar to the POQ model in that the order quantity is indirectly specified by first determining the time interval to be covered by the order. The difference is that in the lot for lot model the order interval (how often to order) is rather arbitrarily set by management. Given a management decision to order for G periods, the quantity ordered would be

$$Q^* = \sum_{i=n+1}^{n+G} d_i$$

This model has no theoretical foundation and is included in this study for control purposes.

D. Least Unit Cost (LUC)

The least unit cost model is based on the premise that it is most economical to order a quantity which is expected to result in the lowest average cost per unit for the period covered by the order. In essence, it is another trade-off between ordering costs and carrying costs. The procedure to determine the order quantity (Q) is conducted as follows:

(1) Set $A^* = C_1$

(2) For $k = n + 1, n + 2, \ldots, n + H$

(3) Compute
$$TC(k) = C_1 + \sum_{i=n+1}^{k} [d_i \cdot (i - n - 0.5) \cdot C_2]$$

(4) $$R(k) = \sum_{i=n+1}^{k} d_i$$

(5) $A(k) = TC(k)/R(k)$

(6) Is $[A(k) < A^*]$?
Yes - Stop: Period k-1 is the end of the current order interval; place order for R(k-1).
No - Set $A^* = A(k)$, increment k; Go to 3.

E. Part-Period Balancing (PPB)

The Part-Period Balancing model is an extension of the least total cost (LTC) model. The least total cost model is based on the premise that the total inventory cost, i.e., the sum of the ordering and carrying costs, for an entire planning period will be minimized if these costs are as nearly equal as possible. (Note the similarity to the EOQ results.) The LTC method attempts to achieve this objective by ordering quantities such that the ordering and carrying costs are approximately equal. The part-period balancing model extends the least total cost model by incorporating a "look-ahead, look-back" adjustment feature. The adjustment is made by computing the savings and costs associated with including (or excluding) the requirements for an additional period. The procedure begins with the "look-ahead" adjustment after the LTC interval and quantity has been determined. If savings exceed costs, the adjustment is made, and the procedure continues. If costs exceed savings, the procedure either terminates or switches to the "look-back" adjustment if no "look-ahead" adjustment has been made. Mathematically, the model is developed as follows:

An order of Q_{n+1} units is placed so as to arrive in period n+1, where,

$$Q_{n+1} = \sum_{i=n+1}^{n+k} d_i$$

and k is determined by the following series of calculations.

(a) Let k_1 be the largest integer, such that

$$C_2 \cdot \sum_{i=n+1}^{n+k_1} [d_i \cdot (1 - n - 0.5)] = C(k_1) < C_1$$

and k_2 be the smallest integer, such that

$$C_2 \cdot \sum_{i=n+1}^{n+k_2} [d_i \cdot (1 - n - 0.5)] = C(k_2) > C_1$$

(b) if $C_1 - C(k_1) < C(k_2) - C_1$, then $k = k_1$; otherwise $C(k_2) - C_1 < C_1 - C(k_1)$, and $k = k_2$.

The "look-ahead," "look-back" adjustments are made as follows:

(A) Look-ahead procedure:

(1) The objective is to determine Min [Costs, Savings] of including d_{n+k+1} in the order for period n+1.

(a) Cost = $d_{n+k+1} \cdot (k+1/2) \cdot C_2$

(b) Savings = $d_{n+k+2} \cdot C_2$

(2) If Costs < Savings: Then set k = k+1 and Repeat 1. Otherwise: Go to "look-back" (if k has not been altered); or STOP.

(B) Look-back procedure:

(3) The objective is to determine MIN

[Cost, Savings] of excluding d_{n+k} from the order for period n+1.

(a) Costs = $d_{n+k+1} \cdot C_2$

(b) Savings = $d_{n+k} \cdot (k - 1/2) \cdot C_2$

(4) If Costs < Savings: Then set k = k - 1 and Repeat 3. Otherwise: STOP.

F. Wagner-Whitin Model

The Wagner-Whitin model is a dynamic programming approach to lot-sizing, which guarantees an "optimal" solution for deterministic problems. This approach performs an implicit enumeration of all possible solutions in a given planning horizon. It begins by determining the optimal solution for the first period (a one-period problem), then for the first two periods (a two-period problem), and so on until an optimal solution is found for the first H periods - the planning horizon. Mathematically, the Wagner-Whitin Method is expressed in the following fashion:

Let $C^*(k)$ = the cost of the optimal k-period solution

$B_j(k)$ = the cost of the k-period solution which orders for period k in period k - j + 1

The following recursive relationships are repeatedly solved to determine the optimal H-period solution.

Given: $C^*(1) = C_1 + (d_1 \cdot 1/2 \cdot C_2)$

then, $B_1(2) = C_1 + (d_2 \cdot 1/2 \cdot C_2) + C^*(1)$

$B_2(2) = (d_2 \cdot 3/2 \cdot C_2) + C^*(1)$

or, in general,

$$B_j(k) = C_1 + \sum_{i=k-j+1}^{k} [d_i \cdot C_2 \cdot (j - 1/2)] + C^*(k - j)$$

where j = 1, 2, . . . , k

and, $C^*(k) = \mathrm{Min}[B_j(k),\ j = 1, 2, \ldots, k]$

The efficiency of the model is a result of the Planning Horizon Theorem which eliminates the need to consider alternatives which order for period k in periods previous to the period in which the optimal policy for period k-1 would order the requirements for period k-1. For a complete statement of the theorem, see Wagner and Whitin (8).

Safety Stock Factor

A simulation-based procedure was developed to determine the "safety stock factor" utilized in these experimental runs. In the actual experiment, both lead time and daily demand are random variables. As such, these two random variates jointly determine the demand during lead time (DDLT). To develop a safety stock factor, the simulation model generated a random lead time for each of 1000 sample observations. The next step was to generate both forecasted and actual demand values for these sample lead time periods. Summing these values over the days in each lead time period resulted in 1000 values for forecasted demand during lead time (FDDLT) and actual demand during lead time (ADDLT).

The next step of the process was to rank order the actual demands during lead time, from highest to lowest. The safety stock factor for a stockout probability = 10% was then calculated by dividing the 100th ADDLT value by the mean forecasted demand.

Mathematically, the safety stock factor may be expressed as:

$$SSF(p) = ADDLT(p \cdot NSO)/XDDLT$$

where,

$$XDDLT = \sum_{i=1}^{NSO} FDDLT(i)/NSO$$

and,

p = probability of a stockout
SSF = safety stock factor
ADDLT(i) = ith ranked actual demand during lead time from a sample of NSO observations.
FDDLT(i) = ith forecasted demand during lead time from a sample of NSO observations
XDDLT = mean forecasted demand during lead time
NSO = number of sample observations (=1000)

During the simulation experiments, the forecasted values were adjusted by the safety stock factor to provide a buffer against uncertainty.

EXPERIMENTAL DESIGNS AND RESULTS

In order to satisfy the objectives of this investigation of inventory policies in a stochastic environment, two separate experimental designs were utilized to provide a foundation for analysis. A complete factorial design was employed to analyze the effect of the controllable factors on two distinct measures of performance (net present cost and total unit-backorders). A series of randomized block designs was subsequently applied to analyze the performance of the six inventory policies under all possible combinations of stochastic conditions.

Lead time variation, demand forecast error, and inflation rate were treated as controllable factors, each being varied over three levels. Inventory policies were treated as the fourth factor, with six different policies being tested. Two measures of performance were collected from each simulation experiment--the net present cost of the policy (over a three-year duration) and the number of unit-backorders (representing service level). To determine the effect of the four controllable factors on these measures of performance, two complete factorial designs were analyzed using net present cost and unit-backorders, respectively, as the independent variable. Each combination of factors was replicated five times.

The analysis of net present cost indicated that the inflation rate was the only significant factor at an $\alpha = .05$ level. However, the inventory policies reflected significance at a marginally higher level of confidence. In the unit-backorder analysis, both lead time variation and inventory policies were significant, along with the interactions involving these factors and the demand forecast error.

In an effort to determine the "relative performance" of the inventory policies under varying combinations of stochastic conditions, each of the twenty-seven cells in the complete factorial design discussed above was analyzed as a separate block, with the inventory policies as treatments. This analysis confirmed two intuitively appealing suppositions. The first was that the inventory policies were not significant in the cases in which the demand forecast error was present at a relatively large value (40% of the forecasted value). Secondly, inventory policies became less significant as the lead time variation increased.

SUMMARY AND CONCLUSIONS

This paper has reported on the results of one phase of an investigation having as its ultimate objective the development of an operational inventory policy(s), which is effective in a stochastic environment. The results at this stage of the analysis imply that the net present cost of a dynamic inventory policy may be relatively independent of lead time variation, demand forecast error, and even inventory ordering rules in a stochastic environment. It appears, however, that service level (as measured by units-backordered) is dependent upon these same factors.

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