CONTROLLING FOR VARIABILITY IN DEMAND FOR AN s,S INVENTORY SYSTEM

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ABSTRACT

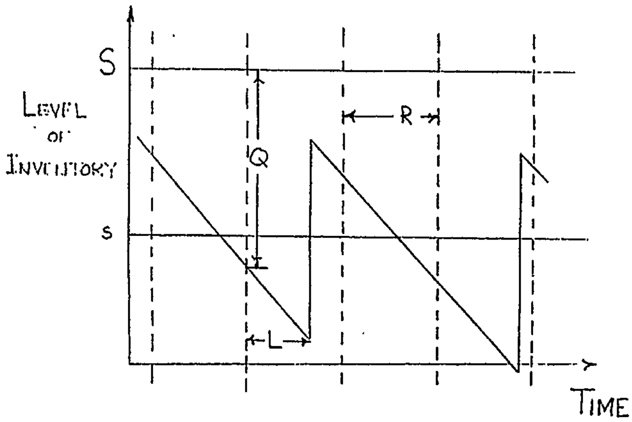
In the establishment of an s,S inventory system, emphasis is given to determining the review period, the nominal order point (s) and the maximum level of inventory (S), which aids in the calculation of the order quantity. The level of safety stock, which is predicated on the demand during the lead time and review period will be affected by the variability present in the demand. This paper investigates the operation of the s,S inventory system under the condition of each of four different demand distributions; the Uniform, Normal, Poisson and Beta. Demand relative to each distribution is randomly generated, a review period is chosen and the levels of s and S are varied in a simulation model of the inventory system. The basic costs - inventory, order, stockout - are calculated to show the effect of the demand distribution.

INTRODUCTION

Whether the concern is for raw materials, in process or finished goods, the maintenance of inventories is an area that receives considerable time from the management of most firms. Turning over one's inventory rapidly while avoiding a no-stock condition is a prime goal in inventory control. Several system procedures are available to the inventory control manager. These would include the fixed quantity system, the fixed interval system, the s,S system, and others that represent ramifications of these three basic types. Each of the three basic systems are unique with respect to order quantities, type of review, level of safety stock and order points. The selection of the system may be predicated on the type of product, the amount of usage, the costs of maintenance, obsolescence, space efficiency and other factors unique to a particular firm or industry.

Greater coverage is generally given to the fixed quantity system because the parameters of the system are more clearly defined. It is the purpose of this paper to investigate the s,S inventory control system and particularly the operation of the system relative to the distribution of demand. The s,S inventory control system is graphically depicted in Figure 1. The basic parameters of concern are the review period (R), the maximum level of inventory (S) and the nominal order point (s). The order quantity (Q) is a variable and has a minimum value

FIGURE 1



s,S Inventory Control System

of (S-s). The length of the review period may be selected for a variety of reasons, including the desire to consolidate orders for discounts, reduce rates for shipping carload quantities, and to order families of items. The maximum level of inventory can be approximated by the equation:

$$S = Q_o + s - \frac{R\bar{d}}{2},$$

where Q_o is the economic order quantity and $\bar{d}$ is the average daily demand. The nominal order point s is determined by the general equation, $s = d(L + R)$, where L is the lead time for the receipt of the order. Manipulation of R and the value selected for d, present opportunities to affect the costs of the system. Safety stock levels, inherent in the determination of the nominal order point, will be predicated on the extent to which management is willing to confront a no stock condition. In most instances modest discussion is given to the demand distribution and lead time is generally treated as a constant. Failure to recognize that demand is not necessarily a constant and that there is variability in lead time may be unrealistic. However, difficulties arise when one considers uncertainty in both the demand and lead time, because to account for this variability it would be necessary to establish a joint probability distribution. If the distributions for both demand and lead time are changing over time, it would be necessary to continually update the joint probabilities to make the decision-making process more viable. An alternative to the establishment of a joint probability distribution is a Monte

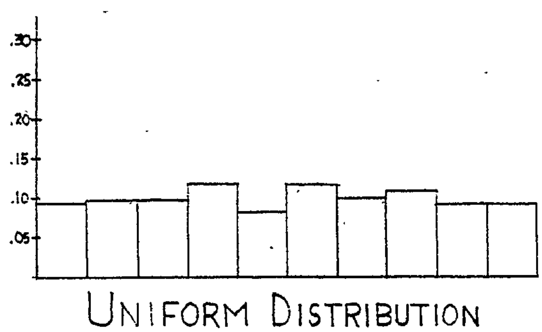
Carlo simulation. This procedure approximates the solution to the problem by sampling from a random stream. Thus, several replications can be made with the opportunity to change the review period, the maximum level of inventory and the nominal order point. The lowest cost situation establishes the values for these parameters which will be implemented in the actual system. The investigation undertaken in this paper will utilize the simulation approach.

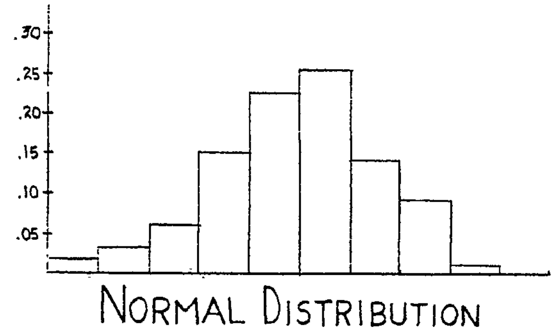
RESEARCH METHODOLOGY

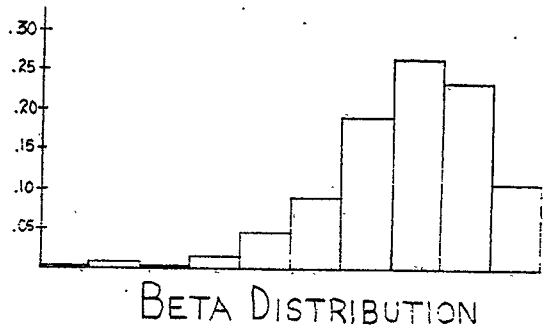
In a review of the literature several articles were encountered that dealt with variability in demand. In general, the uncertainty in demand was on a week to week basis. However, when one considers the varieties of products produced by industries today and further, the different models of those products, consideration of weekly demand may not be satisfactory. In addition, the sophistication of manufacturing facilities today permit rapid changeovers in the product lines to reflect changing demand in the market place. Thus, the daily demand for a product may be the more appropriate figure. It was with this viewpoint in mind that an inventory model was constructed. The language utilized was APL, which permitted immediate reaction to successive runs as well as allowing the alteration of the demand distribution. Each simulation was run for a period of 240 days.

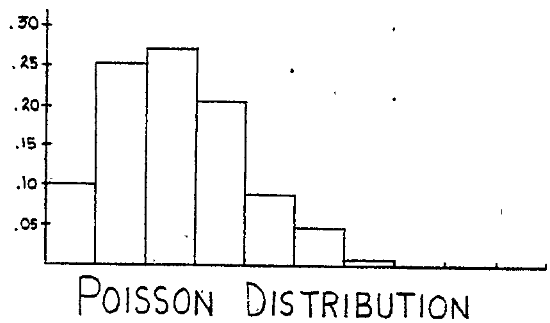
The daily recording and review of the demand for a product is generally not a commonplace procedure of most firms. Basic guidelines of average demand and maximum and minimum levels are noted, for the purpose of establishing a likely distribution so that parameters can be established for the control system. In an effort to investigate the effect of daily demand on the inventory control system, four distributions were established, with the levels of demand common to each and ranging from ten to nineteen units in increments of one. A string of randomly generated numbers was used to select the level of daily demand for each of the theoretical configurations and produced the actual demand distributions that are shown in Figure 2.

FIGURE 2



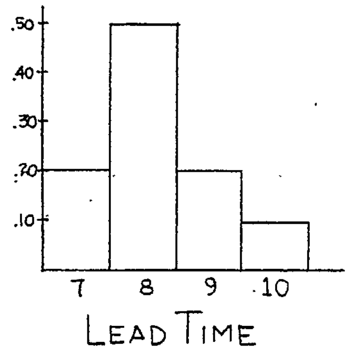






A separate string of random numbers was used to generate lead times according to the theoretical distribution shown in Figure 3.

FIGURE 3



Since the alteration of the review period would change the number of orders to be placed for any one run, no actual distribution was determined.

In the search to minimize the total cost to maintain an inventory of this theoretical product, figures of $20.00 and $50.00 were established as the cost per order and the stockout rate per day, respectively. A holding rate of 20% was applied to items maintained in stock and valued at $25.00 per unit.

RESULTS

The randomly generated distributions produced the statistics that are shown in Table 1. Utilizing these basic statistics, the maximum level of inventory and the nominal order point were established.

TABLE 1

	DISTRIBUTION			
	UNIFORM	NORMAL	POISSON	BETA
MEAN	14.517	14.483	12.163	16.804
STD. DEV.	2.809	1.652	1.391	1.576
MAXIMUM	19	18	16	19
MINIMUM	10	10	10	10

In addition, the review period was arbitrarily selected predicated on the desire to maximize inventory turnover. Several cases were investigated and are shown below. It should be noted that an optimal solution was not the prime goal of this investigation, but rather the effect of the distribution of demand relative to the parameters of the control system.

Case 1

The initial simulation was conducted with a review period of 10 days. The values for S and s were 440 and 346 respectively. These figures were arrived at using a d value of 19, the maximum daily demand. The total costs resulting from these runs are shown in Table 2. Since no stockouts were incurred, the

TABLE 2

	DISTRIBUTION			
COST	UNIFORM	NORMAL	POISSON	BETA
TOTAL	1875.02	1879.13	2023.00	1735.60

total cost is comprised of $600.00 for ordering and the remainder is inventory expense. It is deemed that the high level of the nominal order point has caused the high inventory expense. Thus, a reduction would in turn lower the total cost.

Case 2

The initializing conditions for R and S were maintained and the nominal order point was determined for each distribution where d was represented as the average daily demand. For the four distributions, these values were 264, 264, 221 and 306 respectively. The reduction of s in essence reduces the amount of inactive inventory below the level of s and places it above the level of s where there will be an increase in the turnover. The resulting costs are shown in Table 3. With the exception of

TABLE 3

	DISTRIBUTION			
COST	UNIFORM	NORMAL	POISSON	BETA
INVENTORY	931.71	936.85	1132.21	1135.60
TOTAL	1231.71	1236.85	1432.21	1735.60

the Beta Distribution, two significant observations are noted. The number of orders is cut in half and the cost to carry inventory is significantly reduced. For the Uniform and Normal Distributions, it represents a 33% reduction in total cost, while the POISSON Distribution produces nearly a 40% reduction in total cost. Because of the high level of daily demand, the Beta Distribution was uneffected by the alteration of the nominal order point.

Case 3

Since the order quantity is predicated on the difference between the maximum level of inventory and the inventory level at the time of review, it was felt a further reduction in cost could be achieved by lowering S in accord with the value of s determined in Case 2. Thus, maintaining R and s, the values for the maximum level of inventory were determined for the four distributions as follows: 358, 358, 313, and 402. The cost figures are shown in Table 4. As the figures indicate, there

TABLE 4

	DISTRIBUTION			
COST	UNIFORM	NORMAL	POISSON	BETA
INVENTORY	869.19	873.25	809.58	952.73
TOTAL	1444.19	1448.25	1389.58	1552.73

has been a further reduction in the cost to carry goods in inventory, but the reduction has caused an increase in the number of orders and the net result is an overall increase in cost for all distributions except the Beta.

Case 4

In an attempt to again increase the difference between S and s, which made the costs of Case 2 better than Case 1, the nominal order point was determined on the basis of $d = (\bar{d} - 1SD)$. The values for R and S were the same as Case 3 and the new nominal order points were determined to be 213, 234, 196, and 277 respectively. Table 5 shows the results of these runs. There is no change in the costs

TABLE 5

	DISTRIBUTION			
	UNIFORM	NORMAL	POISSON	BETA
INVENTORY	733.81	837.25	740.13	952.73
ORDER	400.00	575.00	475.00	600.00
STOCKOUT	650.00	0	350.00	0
TOTAL	1783.81	1448.25	1565.13	1552.73

for the Normal and Beta Distributions. In noting the Uniform and POISSON Distributions, we again have a reduction in inventory costs as well as the order costs. But, the variability in demand for these two distributions coupled with the lower s value has caused a no condition resulting in an increase in the total cost.

Cases 5, 6, and 7

Three additional simulations were conducted, where the review period was increased to 12 days. The values for S and s for each run are shown in Table 6. In Case 5, the parameter values are exactly

TABLE 6

	Case					
	5		6		7	
	S	s	S	s	S	s
UNIFORM	358	264	358	293	378	293
NORMAL	358	264	358	293	378	293
POISSON	313	221	313	244	323	244
BETA	402	306	402	339	418	339

the same ones used for Case 3, i.e. determined on a 10 day review period. The results of this run are shown in Table 7, and it is noted there is an across the board reduction in total cost, ranging from 8 to 10 percent, when compared to Case 3. It is logical that this would occur because the longer review period would reduce the number of orders and would also reduce the amount of product used between review periods. The lone stockout shown essentially resulted because of initializing conditions.

TABLE 7

	DISTRIBUTION			
	UNIFORM	NORMAL	POISSON	BETA
INVENTORY	813.85	815.23	759.85	887.48
ORDER	475.00	475.00	475.00	500.00
STOCKOUT	50.00	50.00	0	0
TOTAL	1338.85	1340.23	1234.85	1387.48

The last two tables (8 and 9) represent the alteration of s and then s and S, respectively to incorporate the change in the time period. As noted in Table 8, there is a modest improvement in the costs for the Uniform and Normal distributions. When

TABLE 8

	DISTRIBUTION			
COST	UNIFORM	NORMAL	POISSON	BETA
INVENTORY	826.38	828.38	759.85	887.48
ORDER	500.00	500.00	475.00	500.00
TOTAL	1326.38	1328.38	1234.85	1387.48

TABLE 9

	DISTRIBUTION			
COST	UNIFORM	NORMAL	POISSON	BETA
INVENTORY	922.63	924.63	805.49	964.48
ORDER	500.00	500.00	475.00	500.00
TOTAL	1422.63	1424.63	1275.49	1464.48

the maximum level of inventory is increased there is an across the board rise in costs as seen in Table 9.

DISCUSSION AND CONCLUSION

It is important to realize that the goal of this simulation was not to reach an optimum cost for each of the distributions under the s,S Inventory System. It was designed to investigate the overall effect of the variability in demand coupled with the variability in lead time and to determine the effect the costs to maintain the system are altered. Since the relationship of the cost to review and place an order and the cost to hold inventory are integral to the total cost, it is difficult to utilize general rules for each of the distributions. In addition, the costs will also be pertinent to the levels we establish for the s and S. Generally the results would indicate that where a marked difference exists between s and S, and the cost to hold inventory is a fractional part of the order cost, the overall cost can be near a minimum as shown in Case 3. As s is reduced the variability in demand, coupled with lead time variability, may cause stockout as exhibited in Case 4. Thus, it would appear that there is a necessity to accurately determine the variable costs involved and then apply them to the distributions of demand determined from historical records.

As a concluding note, further work is necessary in extending the distributions, that is a wider range of values. Real data should be investigated to ascertain if such precise distributions actually exist, or is the variability for greater than evidenced in the simulation model.