On Missile Simulations under Interrupted Guidance Conditions

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Abstract

Several guidance laws have been proposed in the past for defense systems. Applications vary from tactical missiles (with different ranges) to target motion analysis (TMA) in naval combat control systems. Guidance techniques are typically classified as either classical or modern with the emphasis on modern being placed on techniques derived from optimal control. The motivation behind this paper is to consider the important applications where, an interceptor missile tracks on radar energy that is being reflected or emanating from a victim target. Data transmitted may be periodic, aperiodic or interrupted. Missile guidance when no data is being received is termed blind-mode guidance. This paper will briefly review classifical and modern schemes, then the question of what guidance strategy a missile in blind-mode should pursue that would result in the best probability of target hit will be addressed. This is followed by computer simulations of a tactical missile under interrupted guidance conditions along with a few unclassified results.

I. BACKGROUND

This introductory section will briefly describe the major categorizations of current U.S. Army missile systems. Emphasis is placed on systems which are currently under development, or which are deployed in the field but still undergoing improvement programs.

Missile systems may be categorized according to any of the following criteria:

- Mode of operation (surface-air, surface-surface, air-surface)
- Function (air defense, general support, close support)
- Guidance method (none , command-to-line-of-sight (CLOS), active, semi-active, passive).

Guidance categories are defined in a conventional manner: CLOS will be addressed in detail in section II; active guidance occurs when the missile operates in a self-contained mode wherein the target is illuminated by an energy source carried by the missile and reflections from the target are processed to yield missile guidance; semi-active guidance operates similarly, but the illuminating source is positioned remotely from the missile; passive guidance is designed to use emissions which originate at the target, generally RF, IR or visible light. Both CLOS and semi-active guidance modes require intervention by ground tracking equipment throughout the course of a missile flight. Active and passive guidance methods are "fire and forget" techniques where no outside intervention is required after missile launch. Table 1 describes missile categorization according to guidance type and function.

As an example, the PATRIOT system will be briefly described below; it is a high performance long-range air defense system designed to counter the air threats of the 1980-2000 time frame. The heart of the PATRIOT system is a large-scale digital computer which controls all aspects of the system's operation. Surveillance, target acquisition and target and missile tracking are all performed by a multi-function phased array radar (MFAR) operating under the control of the central computer. Each air defense site has several missile firing

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1979 Winter Simulation Conference

Background (continued)

stations associated with it, linked to the engagement control station by an RF data link. The PATRIOT system has the capability of engaging several targets simultaneously.

The missile round itself is a supersonic airframe controlled by cruciform cropped-delta, all-moving tail surfaces. A gimballed seeker is mounted behind a radome in the nose of the missile.

II. CLASSICAL GUIDANCE TECHNIQUES

Basically, missile guidance may be considered a process of conveying information. In command and beam rider missiles, the missile obtains its guidance information from a source at a launching station. The theory of homing guidance (often referred to as "two point guidance") has been developed for both accelerating and non-accelerating targets. One of the earliest suggested guidance techniques is often referred to as a pure pursuit course. In this case the missile aims directly at the target similar to a dog chasing a rabbit. This technique, typically, requires high missile accelerations and the necessary missile turn rate becomes infinite when the missile-to-target speed ratio exceeds two (1). The constant-bearing navigation (2), (3), as the term implies, is based on the constant bearing of the line of sight (LOS) in space. This is achieved by aligning the relative missile-target velocity with the LOS. Figure 1 shows the geometry for a constant bearing course. The LOS between missile and target is at an angle ψ relative to an inertial space reference for all times of flight. The missile velocity vector is at angle L relative to the LOS while the target velocity vector is at an angle A relative to the LOS. If the length of the LOS is R, the relative closing velocity between missile and target, $\dot{R}$, is:

$$\dot{R} = V_t \cos A - V_m \cos L$$

where $V_t \cos A$ and $V_m \cos L$ are the velocity components along the LOS. The velocity components normal to the LOS are $V_t \sin A$ and $V_m \sin L$, and the rotation rate of the LOS is thus:

$$R\dot{\psi} = -V_t \sin A + V_m \sin L \,.$$

For a constant bearing course, $\dot{\psi}$ will be zero since ψ is constant. Thus, the missile lead angle L is evaluated by

$$L = \sin^{-1}\left(\frac{V_t}{V_m} \sin A\right) \,.$$

It is interesting to note that if the velocity vectors are not aligned with the LOS, the LOS will rotate at ever-increasing angular rates. Thus, collision will not happen unless compensating maneuvers are performed by the missile.

Proportional Navigation Guidance (PNG is used to counter the tendency of the LOS to rotate and therefore approximate a constant bearing course (4-5). PNG is defined as a course in which generated control accelerations are proportional to the measured rate of rotation of the LOS. The PNG law is expressed as:

$$\dot{\gamma}_m = N \dot{\psi}$$

$\dot{\gamma}_m$ is the velocity vector (rate of change of the missile heading),

$\dot{\psi}$ is the LOS rate of change, and

N is the navigation ratio (typically between 3 and 5).

Murtaugh and Criel (2) have also presented two extensions to the true PNG presented above. Proportional navigation with a bias is suggested to overcome problems with seeker tracking noise causing the measured LOS rate to vacillate between positive and negative values as the LOS rate approaches zero. In the scheme of PNG with command acceleration bias, the acceleration is proportional to the difference between the magnitude of the actual LOS rate and a specified bias.

When the acceleration is commanded only when the magnitude of the LOS rate exceeds some given magnitude, the scheme is termed proportional navigation with a dead space. Figure 2 shows the three main schemes of PNG. In Reference (6), Siouris suggested the addition of an estimate of target acceleration to the missile acceleration command to yield an augmented PNG law.

In the above mentioned PNG laws, one needs to consider the additional burden on the interceptor subsystems due to possible nonideal conditions (2). If, for instance, the commanded acceleration exceeds specific acceleration limits, saturation will result. Another practical problem is gyro drift rate. This drift rate will introduce an error in the system as an apparent LOS rate. It is interesting to note in this regard that while an additional increment in velocity is required, a constant seeker gyro drift does not contribute to miss distance.

Proportional Lead Guidance (PLG) has been proposed by Kuhn (7). This law is similar to the PNG and is defined by:

$$T_s \dot{\psi} = \theta - \psi$$

where, as before, ψ is the angle of the LOS relative to a fixed coordinate frame, θ is the missile attitude angle and T_s is the seeker time constant. The PLG law develops a lead angle which is proportional to the LOS rate. Kuhn pointed out that PLG is not as effective as PNG against a maneuvering target, but yields a better performance when wind shear disturbances are present.

One needs to note that in the realm of miss distance and LOS angle angular rate, a direct hit is uncommon due to the many sources of errors. The main sources of errors according to Stallard (5) are: receiver noise, range-independent noise due to roughness in tracking servo, scintillation noise, initial heading error, acceleration bias within the autopilot, LOS-rate bias and gravitational pull. However, if the miss distance is sufficiently small, the interceptor can still produce a kill.

Next, the attitude pursuit and velocity pursuit will be briefly considered (8). In attitude pursuit, the guidance objective is to keep the centerline of the missile pointed at the target. If a missile flies an angle-of-attack while maneuvering, the velocity vector will be lagging the vehicle pointing direction. The attitude pursuit guidance mechanization is designed to decouple the angle-of-attack from the target seeker and thus miss distance performance is improved by an amount proportional to the vehicle angle of attack. On the other hand, velocity pursuit guidance attempts to keep the velocity vector of the missile pointed at the target. This is usually achieved by mounting a target sensor on an air vane which indicates relative wind direction. The main error in this scheme will be the difference between this velocity vector and a true velocity vector. A study comparing the qualities and sensitivities of LOS, pursuit, and PNG for air-to-ground and air-to-air missiles is reported in Reference 9. References 8 and 10 present thorough documentation on the many reported modifications to the above mentioned classical techniques.

In Reference 11, Bishop et al. analyzed the performance of PNG in a non-homing ballistic application. They have compared four guidance laws: (i) PNG, (ii) Integral guidance, (iii) Perturbation guidance, and (iv) Cross-product guidance. The integral guidance is designed to use the LOS angle as well as the LOS rate to bring the missile to an intercept trajectory with the target. Thus, the PNG law is modified to take this form:

$$\dot{\gamma}_m = N_1 \dot{\psi} + N_2 \int_{t_0}^{t} \dot{\psi}\, d\tau$$

One obvious advantage to the inclusion of the integral term is that it results in a reduction of the error to zero in the presence of a constant disturbance (similar to integral control action in feedback systems). Stated differently, the missile tends to return to the nominal trajectory after a disturbance.

The perturbation guidance uses perturbations of the position and velocity from the standard trajectory to calculate the acceleration required to return the missile to the standard trajectory. It has been shown that the guidance gain in this case is proportional to R^2, and that although the guidance gain may be very large at large distances from the target, the dispersions from the nominal trajectory are expected to be small. On the other hand, the cross-product guidance is based on the prinicple that if a missile velocity vector is aligned with the target, the cross-product of the range and velocity is zero. The guidance law is thus chosen such that if this cross-product is non-zero, the missile is commanded to reduce its magnitude. It has been shown, however, that cross-product guidance, when used by itself, is not accurate enough. However, a combination of cross-product guidance with PNG or integral guidance may provide a system that has low cross range dispersion. For the application presented in Reference (11), integral guidance performed best, based on miss distance, actuator duty cycle, peak fin deflection, peak angle of attack and peak cross range dispersion.

III. MODERN GUIDANCE TECHNIQUES

In his recent survey of guidance techniques, Kelly (10) has grouped the modern techniques into four categories based on linear-quadratic theory, linear quadratic Gaussian theory, linear exponential Gaussian theory, and differential game theory. Kreindler has shown (12) that PNG is the optimal control for the restricted case when target maneuvers and interceptor dynamics are ignored and the linear-quadratic (LQ) problem is thus solved to minimize

$$J = c\, x_1^2 (t_f) + \int_0^{t_f} u^2(t)dt$$

where c is a positive weighting factor, $x_1(t_f)$ is the miss distance measured at the final time and u(t) is the control. As early as 1965 Bryson (13) had presented a solution for the LQ interceptor problem for the case of a target in free fall and

Modern Guidance Techniques (continued)

showed that if all the weighting is on miss distance then:

$$u = 3 V_c \dot{\psi}$$

where u is the optimal lateral acceleration and V_c is the closing velocity (approximately constant); this form corresponds to a PNG with a navigation ratio of 3.

The optimal control to intercept a deterministic maneuvering target using a quadratic performance index has been presented (14) and expanded to the case when the estimated final miss is computed based only on displacement and velocity (15). Williams (16) has solved the LQ problem for the special case of a missile represented by a second-order model and a target maneuver described by a decaying exponential. Cottrell (17) has solved the LQ problem and obtained the optimal control as a function of missile time constant, time-to-go, closing velocity, and the missile target acceleration. Other contributions have been reported by Kim and Grider (18) and York and Pastrick (19) where minimization was performed with constraints at final time. An interesting study that showed how to use performance index terms to limit terminal errors has been reported in (23). Recently, Stallard (31) reported an optimal missile guidance for low miss and perpendicular impact minimizing a performance index which is the sum of five terms.

In 1965, Kishi and Bettwy (20) addressed a spectrum of design possibilities in applying the Linear-Quadratic Gaussian (LQG) theory to homing guidance. Ignoring the missile dynamics they obtained optimal and suboptimal systems for the cases of: initial offset only, target maneuver, and initial offset coupled with measurement noise. In a similar sense, References (2) and (5) have addressed the LQG problem of interceptor guidance with random target acceleartion and noisy measurement. Assuming small LOS rotation, Deyst and Price (22) have developed and evaluated four different guidance laws for tactical missiles taking into consideration a number of factors and constraints including target maneuvering capability, homing sensor measurement errors, missile autopilot dynamics, bounded control variables and launch initial conditions. York, St. Clair and Pastrick introduced an algorithm to help choose weighting parameters in implementing a missile optimal law (24). Nesline and Zarchan (32) have recently compared a modern guidance system to a classical PN homing missile guidance system in terms of performance, robustness and ease of implementation. This study reported interesting results on how quality of component tolerances or measurement errors relate with missile miss distances. The interested reader should consult references (8) and (10) for more extensive bibliographies.

The problem of target motion analysis (TMA) is of similar interest to the naval combat researchers. In many situations, bearing is the only information available and bearings-only target motion analysis is the foundation of the TMA process. The problem of performing TMA using noisy bearing-measurement derived from multiple observation platforms or from a single moving observer has been recently discussed (25). The extended Kalman filter has been applied to bearings-only target tracking (26) where it was demonstrated how bearing and range estimation errors may interact to cause filter instability.

IV. INTERRUPTED GUIDANCE TECHNIQUES

A typical missile trajectory might include the following phases: boost, midcourse, and terminal pahses. Target acquisition normally occurs prior to or during midcourse guidance, and the seeker head control circuits use the error outputs of the signal processor to track the target (that is to say, maintain seeker antenna/target alignment). Rate gyro signals may be stored in the guidance computer. If target acquisition is lost, the pointing error signals may be disconnected from the head control circuits and the stored head rates used to control the seeker head. This mode thus keeps the seeker head turning at a rate proportional to the stored turning rate. One primary purpose of midcourse guidance may be to increase missile range and facilitate a steeper dive angle during terminal guidance so as to improve warhead patterns. The terminal guidance phase may consist of a proportional navigation course until target intercept unless target acquisition is lost. If acquisition is lost, the stored head rates drive the seeker head at a rate proportional to the turning rate that existed at acquisition loss. Blind mode operation results in seeker head tracking and missile steering which optimize the probability of target reacquisition and/or target intercept.

It is obvious from the preceding discussion that a selective turn-off, or interrupted scheme could be an effective tool in denying guidance data to a missile for a critical time before impact. Some aspects of selected schemes that may be used under interrupted guidance conditions are presented next.

A. Main Beam

The main beam concept is one in which the guidance scheme must be designed to be compatible with infrequent homing data (27) (28). Data may be transmitted on an infrequent basis (low-rate illumination). This concept allows distinguishing between energy received from the main lobe of the target radar and sidelobe level. The autopilot/guidance system concepts may use (i) a full inertial system with target location algorithm, (ii) attitude inertial with target homing, or (iii) terminal homing with PN. Some key guidance issues are the impact accuracy of the guidance system and its sensitivity to update rates, missile time constant, missile closing velocity and RF receiver characteristics. It is to be noted that the miss distance accuracy may, in general, be a strong function of the time interval between the last illumination of the target and impact.

B. Memory Mode Guidance

Memory Mode Guidance is a scheme that utilizes modern inertial guidance techniques along with sophisticated processing of seeker data to locate a target and hopefully results in hitting atarget even if the target shuts down during the missile's flight. The system uses seeker angles and angular rates to locate the target in an inertial reference frame. An inertial guidance system calculates guidance commands required for homing on the target. The system is designed to update calculations of target position based on received data; however, in memory-mode operations the system guides the missile to its best estimate of target position; see Figure (3).

One study (29) explored the previously discussed integral guidance scheme to cause lofted trajectories and to control the final approach angle. Integral guidance is an off-shoot of PNG and may have advantages over PNG in trajectory shaping. In integral guidance, the position of the missile is constrained to a particular trajectory with respect to the target.

C. Bearings-Only Tracking Techniques

The problem of estimating the position and velocity of an object traveling on a straight line course from a history of bearing measurements has been considered (25), (26). The most familiar example of such a problem is the estimation of aircraft heading, altitude and speed based on noisy azimuth, elevation and range measurements obtained by radar. Another interesting application exists in the ocean environment. In such two-dimensional bearings-only target motion analysis (TMA), a moving observer (own ship) monitors noisy sonar bearings to an acoustic source (target); measurements are obtained and processed to obtain estimates of source position and velocity. It is in this sense that similarity exists between this technique and the ones discussed earlier. TMA with multiple observation platforms has also been reported (25).

In modeling bearings-only motion analysis, one may use the form of a linear state estimation when the measurements are generated from the bearing observations. This requires the knowledge of the emitter's bearing, location of an observer, and time of observation. In a typical situation, observer motion is often restricted to straight line segments and each segment is termed a "leg" or "phase". It has been found, that in the case of a single moving observer, the dynamic process in unobservable on the first leg.

Recursive least squares, Kalman filtering, and the instrumental variable approach have been used in this estimation problem. Convergence to the complete solution was achieved following observer maneuvers and that resulted in the dynamic process becoming observable. However, difficulties have been reported in the application of the extended Kalman filter technique to the TMA problem where premature collapse of the covariance matrix and solution divergence resulted (30).

V. SIMULATION RESULTS

In this section, a digital computerized model written in the ACSL (33) language has been modified. Proportional navigation is used as the terminal phase guidance law. A single radar is assumed to be the only source of energy available in the simulation environment. Two scenarios of signal interruptions have been assumed: in one set-up the victim radar will be allowed to shutdown and continue to do so until impact (blind-mode); in the second scenario, it is assumed that the source will not be acquired for a fixed period of time (few seconds, say) and then reacquired again. The missile performance has been judged by the simple criterion of missile missdistance from the target at impact. In this simulation, memory mode guidance (see section IV. B) will be used under these interrupted conditions. This relatively sophisticated model also includes a noise source to the boresight error.

Figure (4) depicts variation of missdistance versus time representing the length of the blind-mode (before impact) phase; larger missdistances occur as the length of the shutdown period increases. Figure (5) shows simulation results based on a fixed period of shutdown (of the victim radar) occuring at specified instant after missile launch, the signal is assumed to be reacquired again. The ability of the missile to hit the target under these conditions is obvious once the signal has been reacquired again and a critical time

Simulation Results (continued)

has elapsed allowing the missile to readjust its position towards the target.

For the specific missile under consideration, the effect of length of the phase-before-terminal on missdistance has also been addressed, Fig. (6). Under PNG, if an estimate of the downrange distance to the target is available at missile launch, one might consider the possibility of adjusting the length of the phase-before-terminal to achieve best results. To put it differently, if the downrange is estimated before launch, a missile with a fixed length of phase-before-terminal should be launched only when the distance to target is expected to yield minimum missdistance under these conditions.

REFERENCES

1. F. P. Adler, "Missile Guidance by Three-Dimensional Proportional Navigation," Journal of Applied Physics, Vol. 27, No. 5, May 1956, pp. 500-507.
2. S. A. Murtaugh and H. E. Criel, "Fundamental of Proportional Navigation," IEEE Spectrum, December 1966, 75-85.
3. J. P. Janus, "Homing Guidance - A tutorial report," Report No. TOR 469(9990)-1, Aeorspace Corporation, El Segundo, California, December 10, 1964.
4. L. D. Paarmann et al, "Guidance Law Handbook for Classical Proportional Navigation," Tactical Weapon Guidance and Control Information Analysis Center, GACIAC HB 78-01, June 1978.
5. D. V. Stallard, "Classical and Modern Guidance of Homing Interceptor Missiles," Raytheon Co., Report, April 1968.
6. G. M. Siouris, "Comparison Between Proportional and Augmented Proportional Navigation," Nachrichtentechnische Zeitschrift, pp. 278-280, July 1974.
7. H. L. Kuhn, "Proportional Lead Guidance in a Stochastic Environment," Ph.D. Dissertation, University of Florida, 1968.
8. H. L. Pastrick, S.M. Seltzer and M. E. Warren, "Guidance Laws for Short Range Tactical Missiles," Proceedings of the AIAA Guidance and Control Conference, pp. 1-11, January 1969, New Orleans.
9. R. Goodstein, "Development of Control System Requirements," AGARD Report: Guidance and Control of Tactical Missiles, May 1972.
10. W. C. Kelly, "Homing Missile Guidance - A Survey of Classical and Modern Techniques," U. S. Army Missile Command (Unpublished report), 1978.
11. C. M. Bishop, O. P. Cerny and J. D. Alexander, "Inertial Guidance Law Study for the Simplified Inertial Guidance Demonstration (SIG-D) Missile," Computer Sciences Corporation, Report No. CSC/TR-76/5420, April 1976.
12. E. Kreindler, "Optimality of Proportional Navigation," AIAA Journal, Vol. 11, No. 6, June 1973.
13. A. E. Bryson, "Linear Feedback Solution for Minimum Effort Interception, Rendezvous, and Soft Landing," AIAA Journal, Vol. 3, No. 8, August 1965.
14. V. Garber, W. Flory and R. Dickson, "Optimum Intercept Laws," U. S. Army Missile Command, Report No., RD-TR-67-10, December 1967.
15. R. Dickson, "Optimum Intercept Law: Part II," U. S. Army Missile Command, Report No., RD-TR-68-2, September 1968.
16. G. D. Willems, "Optimal Controllers for Homing Missiles with Two Time Constants," U. S. Army Missile Command, Report No. RE-TR-69-20, October 1969.
17. R. G. Cottrell, "Optimal Intercept Guidance for Short Range Tactical Missiles," AIAA Journal, Vol. 9, No. 7, July 1971.
18. M. Kim and K. Grider, "Terminal Guidance for Impact Attitude Angle Constrained Flight Trajectories," IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-9, No. 6, November 1973.
19. R. J. York and H. L. Pastrick, "Optimal Terminal Guidance with Constaints at Final Time," Journal of Spacecraft and Rockets, Vol. 14, No. 6, June 1977, pp. 381-383.
20. F. H. Kishi and T. S. Bettwy, "Optimal and Sub-Optimal Designs of Proportional Navigation Systems," Proc. of the Symposium on Recent Advances in Optimization Techniques, John Wiley & Sons, New York, 1965.
21. A. E. Bryson and Y. C. Ho, "Applied Optimal Control," Blaisdell Publishing Co., Waltham, Mass., 1969.

22. J. Deyst and D. F. Price, "Optimal Stochastic Guidance Laws for Tactical Missiles," Journal of Spacecraft and Rockets, Vol. 10, No., 5, May 1973.

23. E. C. Dalbernie et al., "Merging Conventional and Optimal Control Techniques for Practical Missile Terminal Guidance," Proc. of AIAA Conf., 1975.

24. R. J. York, D. C. St. Clair and H. L. Pastrick, "Choosing Weighting Parameters in Simulating a Missile Optimal Control Law," Proceedings of the 10th Annual Pittsburg Conference on Modeling & Simulation, April 1979.

25. A. G. Lindgren and K. F. Gong, "Position and Velocity Estimation Via Bearing Observations," IEEE Trans., Vol. AES-14, No. 4, July 1978, pp. 654.

26. V. J. Aidala, "Kalman Filter Behavior in Bearings-Only Tracking Applications," IEEE Trans., Vol. AES-15, No. 1, Jan 1979, pp. 29.

27. Memory Main Beam ARM: Feasibility Analysis, IBM Study, (Secret), Document AS148, 1978.

28. Mainlobe ARM Study, Raytheon Company (Secret), Document TDW-79-15, February 1976.

29. Computer Sciences Corporation Document: "Memory Mode Guidance for ARM Missiles," CSC/TR-77/5505, (Confidential), December 1977.

30. D. J. Murphy, "Noisy Bearings-Only Target Motion Analysis," Ph.D. diss., Department of E. E., Northeastern University, 1970.

31. D. V. Stallard, "Optimal Missile Guidance for Low Miss and Perpendicular Impact," (paper # 79-1734), AIAA Conference on Guidance and Control, Boulder, Colorado, pp. 294-305, Aug. 1979.

32. F. W. Nesline and P. Zarchan, "A New Look at Classical versus Modern Homing Missile Guidance," (Paper #79-1727), AIAA Conference on Guidance and Control, Boulder, Colorado, pp. 230-242, August 1979.

33. E. L. Mitchell and J. S. Gauthier, "Advanced Continuous Simulation Language (ACSL)," SIMULATION, March 1976, pp. 72-78.

TABLE 1 MISSILE CATEGORIZATION BY GUIDANCE TYPE AND FUNCTION

GUIDANCE TYPE	CURRENT SYSTEMS			DEVELOPMENTAL SYSTEMS		
	AIR DEFENSE	GENERAL SUPPORT	CLOSE SUPPORT	AIR DEFENSE	GENERAL SUPPORT	CLOSE SUPPORT
NONE			FFAR		GSRS	VIPER
INERTIAL		PERSHING I LANCE				
COMMAND	HERCULES		TOW DRAGON SHILLELAGH	ROLAND		
ACTIVE					PERSHING II	
SEMI-ACTIVE	HAWK			PATRIOT		HELLFIRE COPPERHEAD
PASSIVE	CHAPARRAL REDEYE			STINGER		

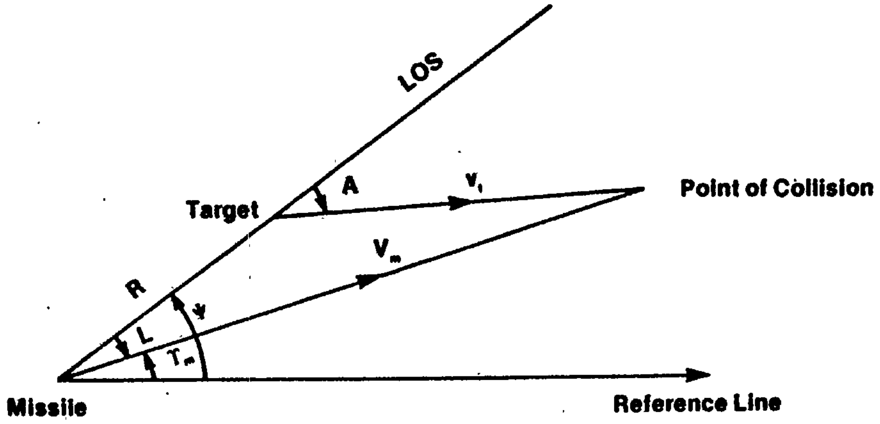
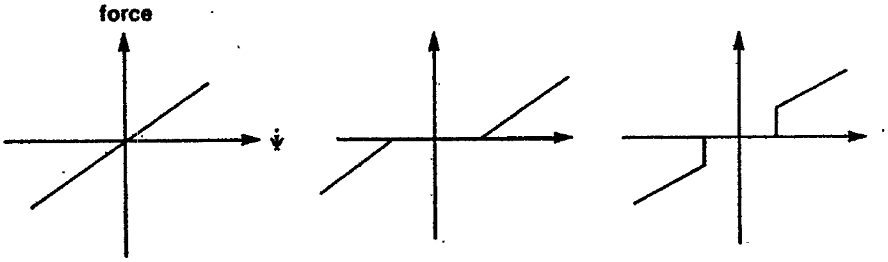


Fig. 1



(a) True PNG

(b) PNG with Bias

(c) PNG plus Dead Space

Fig. 2

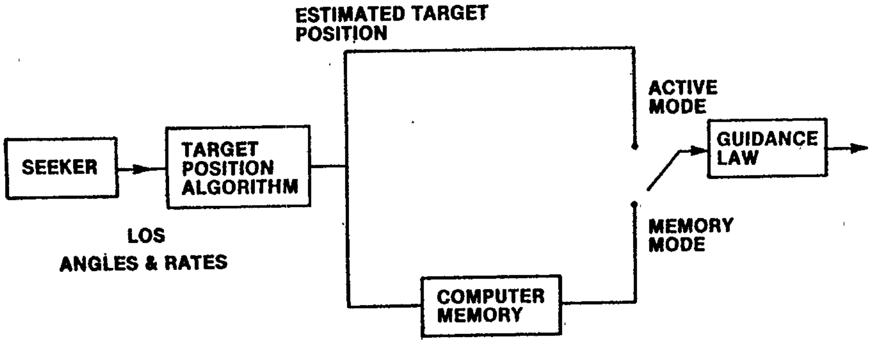


Fig. 3

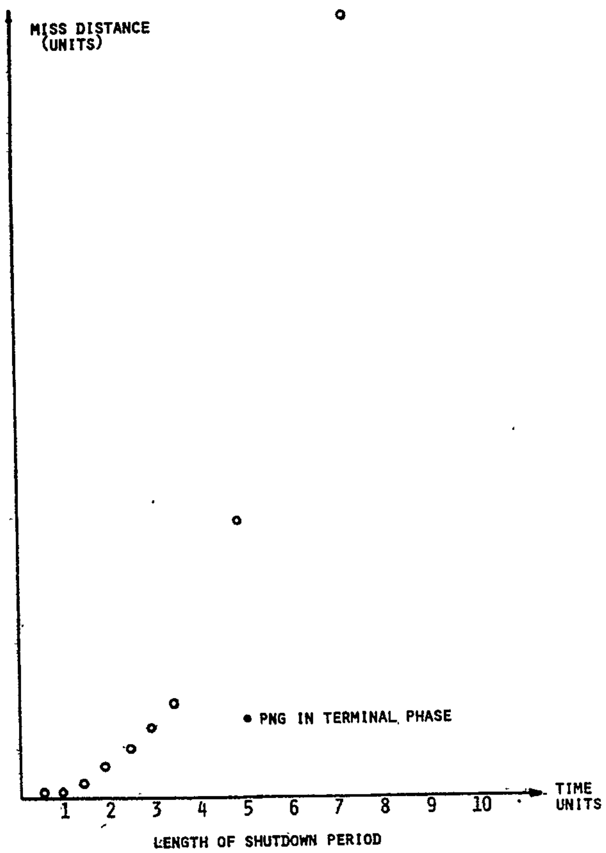


Fig. 4

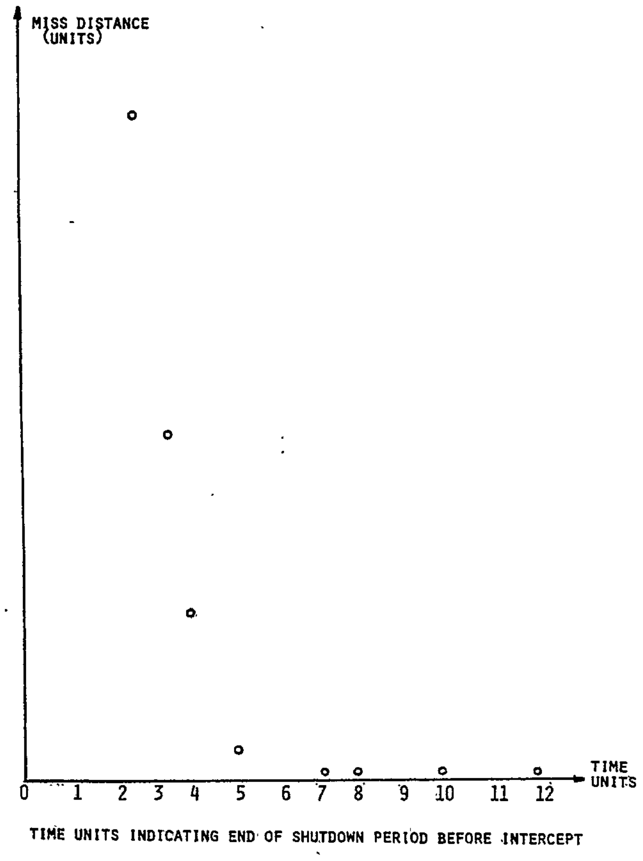


Fig. 5

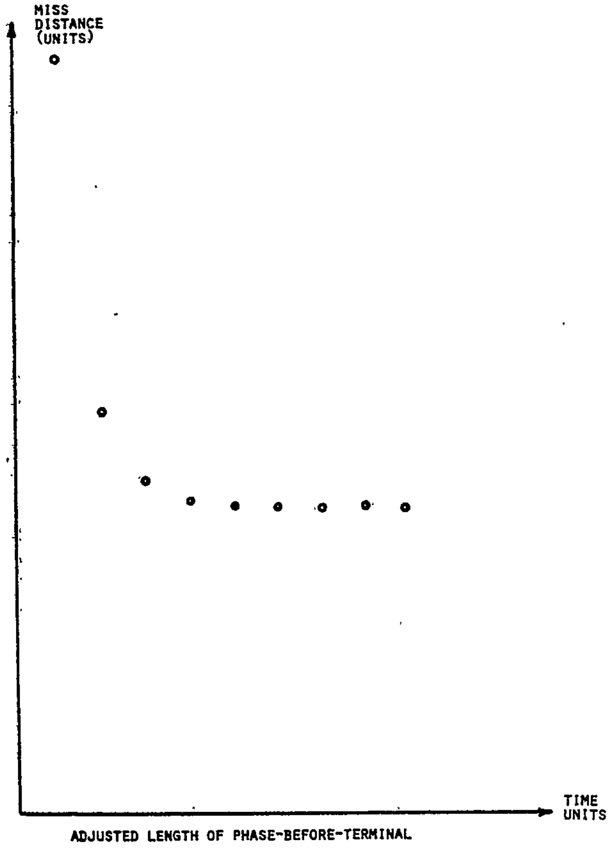


Fig. 6