DISCRETE MODELS OF PHYSICAL PHENOMENA

Donald Greenspan
Mathematics Department
University of Texas at Arlington

ABSTRACT: In this paper it is shown how classical physics can be reformulated using only arithmetic. All the usual conservation laws are established in exactly the same form in which they appear in continuous mechanics. New, completely arithmetic models of physical phenomena emerge. These models are more consistent with both the molecular theory of matter and the capabilities of modern digital computers than are classical continuous models. A variety of computer examples will be described and discussed, as will the scientific and educational implications of this new approach.

1. INTRODUCTION

One of the major goals of applied mathematics is the development of modéls of natural phenomena. We do this in order to understand the basic mechanisms of nature, so that we can either control the associated forces or predict events due to forces which are beyond our control.

Good modeling, however, requires a source of exceptional mathematical power. Until recently, the sophisticated concepts of the calculus served such a purpose and resulted in a vast panorama of continuous models. With the development of the high speed digital computer, a new source of power became available, that is, the power to execute arithmetic operations, to store and retrieve numbers from a memory bank, and to make certain logical decisions, all with exceptional speed. Today, we will explore the new type of modeling which has emerged in recent years which utilizes only these computer capabilities.

Since we must replace all the familiar tools of calculus by concepts which utilize only arithmetic in their formulation, it is difficult to know how to begin correctly. For this reason, we will first develop some intuition by considering a simple physical experiment with the very familiar force of gravity.

2. GRAVITY

Consider a metal ball P, situated atop a building 400 feet high, as shown in Figure 2.1, and allow the ball to be dropped vertically from a position of rest. As the ball falls, let us take a picture of its motion every Δt seconds. The value of Δt will depend, of course, on the shutter speed of the particular camera being used. The pictures are taken, then, at the distinct times $t_0, t_1, t_2, t_3, \ldots$, where, $t_0 = 0$, $t_1 = \Delta t$, $t_2 = 2\Delta t$, $t_3 = 3\Delta t, \ldots$, $t_k = k\Delta t, \ldots$. At each time t_k, $k = 0,1,2,\ldots$, let the height of the particle above the ground be given by $x_k = x(t_k)$. From the knowledge that the initial height was $x_0 = 400$ ft., and from the sequence of resulting pictures, one can easily approximate, by ratio and proportion, $x_1, x_2, x_3, \ldots$, and so forth.

Suppose, as a particular example, we have a very slow camera and that $\Delta t = 1$ sec. Then, $t_0 = 0$, $t_1 = 1$, $t_2 = 2$, $t_3 = 3$, $t_4 = 4$. Suppose the heights above ground, at these times, are, to the nearest foot, $x_0 = 400$, $x_1 = 384$, $x_2 = 336$, $x_3 = 256$, $x_4 = 144$, respectively. These data are recorded in column A of Table 1 and will be subjected, next, to mathematical analysis.

By rewriting x_0, x_1, x_2, x_3, x_4 as $x_0 = 400-0$, x_1 - 400-16, $x_2 = 400-64$

$$x_3 = 400-144, \; x_4 = 400-256,$$

which reveal clearly the distance that the object has fallen, one can then factor to yield

$$x_0 = 400 - 16(0)^2, \; x_1 = 400 - 16(1)^2, \; x_2 = 400 - 16(2)^2,$$

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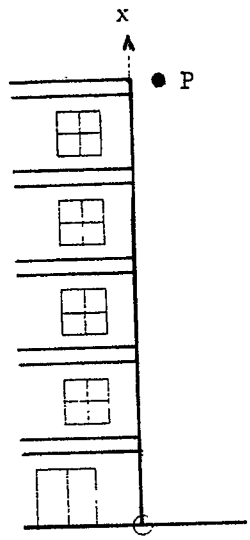

Figure 2.1

TABLE I

Time	A Measured height	B Velocity by calculus	C Acceleration by calculus	D Velocity by arithmetic	E Acceleration by arithmetic
$t_0=0$	$x_0=400$	$v_0=0$	$a_0=-32$	$v_0=0$	$a_0=-32$
$t_1=1$	$x_1=384$	$v_1=-32$	$a_1=-32$	$v_1=-32$	$a_1=-32$
$t_2=2$	$x_2=336$	$v_2=-64$	$a_2=-32$	$v_2=-64$	$a_2=-32$
$t_3=3$	$x_3=256$	$v_3=-96$	$a_3=-32$	$v_3=-96$	$a_3=-32$
$t_4=4$	$x_4=144$	$v_4=-128$	$a_4=-32$	$v_4=-128$	

$$x_3 = 400 - 16(3)^2, \; x_4 = 400 - 16(4)^2$$

which can be written concisely as

$$x_k = 400 - 16(t_k)^2; \; k = 0,1,2,3,4. \tag{2.1}$$

In the traditional manner, one would now interpolate and extrapolate from (2.1) to obtain the continuous formula

$$x = 400 - 16t^2, \tag{2.2}$$

from which, by differentation, one would find

$$v(t) = x'(t) = -32t, \tag{2.3}$$

$$a(t) = v'(t) = -32. \tag{2.4}$$

The particle's velocities $v_0 = v(0)$, $v_1 = v(1)$, $v_2 = v(2)$, $v_3 = v(3)$, $v_4 = v(4)$, at the times of the corresponding heights x_0, x_1, x_2, x_3, x_4, are now determined directly from (2.3), and are recorded in column C of Table 1.

Note that formulas (2.3) and (2.4), and the interesting conclusion that the acceleration due to gravity is constant, with the value - 32, have all been deduced from the given distance measurements x_0, x_1, x_2, x_3, x_4.

3. ARITHMETIC MODELING OF GRAVITY

In seeking to develop an arithmetic approach to all of physics, it is reasonable to try to develop, first, an arithmetic model of gravity. Moreover, since gravity is such a simple force, we should try to obtain exactly the same results for velocity and acceleration as those which were obtained in Section 2. After extensive trial and error, this was achieved as follows.

Let us define each particle's velocity $v_k = v(t_k)$, $k = 0,1,2,3,4$ as an average (rather than instantaneous) rate of change of height with respect to time by the arithmetic formula

$$\frac{v_{k+1}+v_k}{2} = \frac{x_{k+1}-x_k}{\Delta t}; \; k = 0,1,2,3. \tag{3.1}$$

Since averaging procedures are both common and useful in the analysis of experimental data, the left-hand side of (3.1) is perfectly reasonable.

Next, for computational convenience, let us rewrite (3.1) in the recursive form

$$v_{k+1} = -v_k+2(x_{k+1}-x_k)/(\Delta t); \; k = 0,1,2,3. \tag{3.2}$$

Assuming that $v_0 = 0$ when a particle is dropped from a position of rest, one finds from (3.2) that

$$v_1 = -v_0+2(x_1-x_0)/(\Delta t) = 0+2(384-400)/1 = -32$$

$$v_2 = -v_1+2(x_2-x_1)/(\Delta t) = 32+2(336-384)/1 = -64$$

$$v_3 = -v_2+2(x_3-x_2)/(\Delta t) = 64+2(256-336)/1 = -96$$

$$v_4 = -v_3+2(x_4-x_3)/(\Delta t) = 96+2(144-256)/1 = -128$$

which are identical with the results of column B in Table 1, and are recorded in column D.

Next, since x_0 and v_0, but not a_0, are known initially, let us define a_k as the average (rather than instantaneous) rate of change of velocity with respect to time by the arithmetic formula

$$a_k = \frac{v_{k+1}-v_k}{\Delta t}; \; k = 0,1,2,3. \tag{3.3}$$

From the values v_k just generated, one finds from (3.3) that $a_0 = a_1 = a_2 = a_3 = -32$, which are identical with entries in column C of Table 1, and are recorded in column E. Formula (3.3) does not allow a determination of a_4 because this would require knowing v_5. Neverthless, the entries do indicate quite clearly that the acceleration due to gravity is constant, with the value -32.

Formulas (3.2) and (3.3) are both recursion formulas. Such formulas are solved numerically with exceptional speed on modern digital computers. Thus, even if the original distance measurements had been exceptionally voluminous, they could still have been recorded and analysed quite easily.

4. ARITHMETIC PHYSICS

Now, just because the arithmetic formulas (3.2) and (3.3) have given the same results as (2.3) and (2.4) does not mean that we have, as yet, a formulation which is of physical significance. Indeed, the physical significance of Newtonian mechanics is characterized by the laws of conservation of energy, linear momentum, and angular momentum, and by symmetry [1]. Surprisingly enough, our approach to gravity will also yield total conservation and symmetry. We will, however, for simplicity only, confine attention here only to the conservation of energy.

For completeness, recall now the fundamental Newtonian dynamical equation:

$$F = ma, \tag{4.1}$$

the classical formula for kinetic energy K:

$$K = \frac{1}{2}mv^2, \tag{4.2}$$

and, for a falling body with $a = -32$, the formula for potential energy V:

$$V = 32mx. \tag{4.3}$$

Recall, also, that the experimental data in column A of Table 1 were obtained from photographs at the distinct times $t_k = k\Delta t$. For this reason, we must concentrate only on these times and hence must rewrite (4.1)-(4.3) as follows:

$$F_k = ma_k;\ k = 0,1,2,\ldots \tag{4.4}$$

$$K_k = \frac{1}{2}m(v_k)^2;\ k = 0,1,2,\ldots \tag{4.5}$$

$$V_k = 32mx_k;\ k = 0,1,2,\ldots \tag{4.6}$$

Let us now define a sum W_n, $n = 1,2,3,\ldots$, which is the discrete analogue of the concept of <u>work</u>, by

$$W_n = \sum_{i=0}^{n-1}(x_{i+1}-x_i)F_i. \tag{4.7}$$

Then, by (3.1), (3.3), (4.4), (4.5) and (4.7),

$$\begin{aligned} W_n &= m\sum_{i=0}^{n-1}(x_{i+1}-x_i)\left(\frac{v_{i+1}-v_i}{\Delta t}\right) \\ &= m\sum_{i=0}^{n-1}\left(\frac{x_{i+1}-x_i}{\Delta t}\right)(v_{i+1}-v_i) \\ &= \frac{m}{2}\sum_{i=0}^{n-1}(v_{i+1}+v_i)(v_{i+1}-v_i) \\ &= \frac{m}{2}\sum_{i=0}^{n-1}(v_{i+1}^2-v_i^2) \\ &= \frac{m}{2}(v_1^2-v_0^2+v_2^2-v_1^2+v_3^2-v_2^2+v_4^2-v_3^2+\ldots+v_n^2-v_{n-1}^2) \\ &= \frac{m}{2}(v_n^2-v_0^2)\ , \end{aligned}$$

so that

$$W_n = K_n-K_0,\ n = 1,2,3,\ldots\ . \tag{4.8}$$

Note in the above derivation that the telescopic sum

$$(v_1^2-v_0^2+v_2^2-v_1^2+v_3^2-v_2^2+v_4^2-v_3^2+\ldots+v_n^2-v_{n-1}^2) = v_n^2-v_0^2$$

is the arithmetic analogue of integration in that it yields a function evaluated at the upper limit, t_n, minus a function evaluated at the lower limit, t_0.

Similarly, since $a_k \equiv -32$, one has from (4.6) and (4.7) that

$$W_n = -32m \sum_{i=0}^{n-1} (x_{i+1}-x_i) = -32mx_n+32mx_0.$$

Thus

$$W_n = -V_n+V_0, \; n = 1,2,3,... \qquad (4.9)$$

Elimination of W_n between (4.8) and (4.9) yields

$$K_n+V_n = K_0+V_0, \; n = 1,2,3,..., \qquad (4.10)$$

which is the classical law of conservation of energy.

It is of primary importance to note that (4.10) is valid independently of Δt, so that our choice of cameras has in fact, no significance at all.

5. LONG AND SHORT RANGE FORCES

Before proceeding to show how to extend the techniques of the previous section to more complex forces, let us examine the nature of those forces which act on all solids, liquids and gases. These fall into two categories, the long range and the short range, both of which act simultaneously and will be included in our new approach to modeling.

The long range forces are those which act on all particles, at all times, over all distances. These forces include gravity, gravitation and coulombic forces.

The short range forces are those which occur only between a molecule and its immediate neighbors. This type of interaction is of the following general nature [1]. If two molecules are pushed together they repel each other, if pulled apart they attract each other. and mutual repulsion is of a greater order of magnitude than is mutual attraction. Mathematically, this behavior is often formulated as follows. The magnitude F of the force $\vec{F}$ between two molecules which are locally r units apart is of the form

$$F = -\frac{G}{r^p} + \frac{H}{r^q}, \qquad (5.1)$$

where, typically,

$$G > 0, H > 0, \; q > p \geq 7. \qquad (5.2)$$

The major problem in any simulation of a physical body is that there are too many component molecules to incorporate into the model. The classical mathematical approach is to replace the large, but finite, number of molecules by an infinite set of points. In so doing, the rich physics of molecular interaction is lost. A viable computer alternative is to replace the large number of molecules by a much smaller number of particles and then readjust the parameters in (5.1) to compensate [2]-[5]. It is this latter approach which we will follow.

6. EXTENSIONS

Guided by the discussion of Section 5, we will now show how to develop an entirely numerical and conservative approach to forces more complex than gravity. Detailed proofs of conservation and symmetry are given in Greenspan [3].

Consider first the planar motion of a single particle under the influence of gravitation. For this purpose, if $\Delta t > 0$ and $t_k = k\Delta t$, $k = 0,1,2,...$, let particle P of mass m be located at $\vec{r}_k = (x_k, y_k)$, have velocity $\vec{v}_k = (v_{k,x}, v_{k,y})$, and have acceleration $\vec{a}_k = (a_{k,x}, a_{k,y})$ at time t_k. In analogy with (3.1) and (3.3), let

$$\frac{\vec{v}_{k+1}+\vec{v}_k}{2} = \frac{\vec{r}_{k+1}-\vec{r}_k}{\Delta t}, \; k = 0,1,2,... \qquad (6.1)$$

$$\vec{a}_k = \frac{\vec{v}_{k+1}-\vec{v}_k}{\Delta t}, \ k = 0,1,2,\ldots \ . \tag{6.2}$$

To relate force and acceleration at each time t_k, we assume a discrete Newtonian dynamical equation

$$\vec{F}_k = m\vec{a}_k \tag{6.3}$$

where

$$\vec{F}_k = (F_{k,x},F_{k,y}). \tag{6.4}$$

Suppose now that a massive object, like the sun, whose mass is M, is positioned at the origin of the XY coordinate system and is assumed to have no motion. Then, in analogy with the continuous, conservative gravitational force on P, which has components

$$F_x = -\frac{GMm}{r^2}\frac{x}{r}, \ F_y = -\frac{GMm}{r^2}\frac{y}{r},$$

where G is a constant, the arithmetic and conservative gravitational force on P is taken to have components [3]:

$$F_{k,x} = -\frac{GMm}{r_k r_{k+1}} \cdot \frac{(x_{k+1}+x_k)/2}{(r_{k+1}+r_k)/2} = -\frac{GMm(x_{k+1}+x_k)}{r_k r_{k+1}(r_k+r_{k+1})}, \tag{6.5}$$

$$F_{k,y} = -\frac{GMm(y_{k+1}+y_k)}{r_k r_{k+1}(r_k+r_{k+1})} \tag{6.6}$$

where

$$r_k^2 = x_k^2+y_k^2, \ k = 0,1,2,\ldots \ . \tag{6.7}$$

Now, gravitation is a $1/r^2$ law. Suppose, as in classical molecular mechanics, one would desire an arithmetic and conservative formulation of a $1/r^p$, $p \geq 2$, law of attraction. Then, in this case, (6.5) and (6.6) need be modified only as follows [6]:

$$F_{k,x} = -\frac{GMm\left[\sum_{j=0}^{p-2}(r_k^j r_{k+1}^{p-j-2})\right](x_{k+1}+x_k)}{r_k^{p-1} r_{k+1}^{p-1}(r_{k+1}+r_k)}, \ G \geq 0 \tag{6.8}$$

while $F_{k,y}$ is the same as $F_{k,x}$ except that x and y are exchanged. In the particular case where $p = 2$, (6.8) reduces to (6.5).

In classical molecular mechanics, however, particles attract like $1/r^p$ only when they are relatively far apart. When the are close, they repel like $1/r^q$, $q > p$. To simulate both these effects, simultaneously, it follows directly from (6.8) that the conservative formulas are

$$F_{k,x} = -\frac{GMm\left[\sum_{j=0}^{p-2}(r_k^j r_{k+1}^{p-j-2})\right](x_{k+1}+x_k)}{r_k^{p-1} r_{k+1}^{p-1}(r_{k+1}+r_k)} + \frac{HMm\left[\sum_{j=0}^{q-2}(r_k^j r_{k+1}^{q-j-2})\right](x_{k+1}+x_k)}{r_k^{q-1} r_{k+1}^{q-1}(r_{k+1}+r_k)}, \ G \geq 0, \ H \geq 0, \tag{6.9}$$

while $F_{k,y}$ is the same as $F_{k,x}$ except that x and y are exchanged.

Finally, with regard to the motion of a single particle, it is of interest to note that all the arithmetic conservative formulas developed thus far are special cases of the following general formula [7]. For any Newtonian potential $\phi(r)$, let

$$\vec{F}_k = -\frac{\phi(r_{k+1})-\phi(r_k)}{r_{k+1}-r_k} \cdot \frac{\vec{r}_{k+1}+\vec{r}_k}{r_{k+1}+r_k}. \tag{6.10}$$

Arithmetic formula (6.10) conserves exactly the same energy, linear momentum and angular momentum as does its continuous, limiting counterpart

$$\vec{F} = -\left(\frac{\partial\phi}{\partial r}\right)\frac{\vec{r}}{r}. \qquad (6.11)$$

Moreover, the singularity which results if $r_k = r_{k+1}$ is removable.

Since we have now explored rather completely the motion of a single particle, the next extension is to a system of particles $P_1, P_2, \ldots, P_n$. To do this, let particle P_i of mass m_i be at $\vec{r}_{i,k} = (x_{i,k}, y_{i,k})$, have velocity $\vec{v}_{i,k} = (v_{i,k,x}, v_{i,k,y})$ and have acceleration $\vec{a}_{i,k} = (a_{i,k,x}, a_{i,k,y})$ at time t_k. Position, velocity and acceleration are assumed to be related by

$$\frac{\vec{v}_{i,k+1}+\vec{v}_{i,k}}{2} = \frac{\vec{r}_{i,k+1}-\vec{r}_{i,k}}{\Delta t} \qquad (6.12)$$

$$\vec{a}_{i,k} = \frac{\vec{v}_{i,k+1}-\vec{v}_{i,k}}{\Delta t}. \qquad (6.13)$$

If $\vec{F}_{i,k} = (F_{i,k,x}, F_{i,k,y})$ is the force acting on P_i at time t_k, then force and acceleration are assumed to be related by

$$\vec{F}_{i,k} = m_i\vec{a}_{i,k}. \qquad (6.14)$$

If, in particular, we assume that all particles interact with all other particles with attraction like $1/r^p$ and repulsion like $1/r^q$, then the arithmetic, conservative force on each P_i, $i = 1,2,\ldots,n$, is given, in analogy with (6.9), by

$$\vec{F}_{i,k} = m_i \sum_{\substack{j=1\\ j\neq i}}^{n} \left\{ m_j \left(-\frac{G\left[\sum_{\xi=0}^{p-2}\left(r_{ij,k}^{\xi}\, r_{ij,k+1}^{p-\xi-2}\right)\right]}{r_{ij,k}^{p-1} r_{ij,k+1}^{p-1}(r_{ij,k}+r_{ij,k+1})} + \frac{H\left[\sum_{\xi=0}^{q-2}\left(r_{ij,k}^{\xi}\, r_{ij,k+1}^{q-\xi-2}\right)\right]}{r_{ij,k}^{q-1} r_{ij,k+1}^{q-1}(r_{ij,k}+r_{ij,k+1})} \right) \times \left(\vec{r}_{i,k+1}+\vec{r}_{i,k}-\vec{r}_{j,k+1}-\vec{r}_{j,k}\right) \right\}, \qquad (6.15)$$

where $G \geq 0$, $H \geq 0$, $q > p \geq 2$, and $r_{ij,k}$ is the distance between P_i and P_j at t_k.

It should be noted, in particular, that only for simple forces, like gravity, do the continuous and discrete approaches yield exactly the same dynamical behavior. In general, the two approaches yield results which differ by terms of order $(\Delta t)^3$, even though they both conserve exactly the same physical invariants.

7. DISCRETE MODELS

The arithmetic approach developed thus far lends itself naturally and consistently to discrete, or particle-type, models of complex physical phenomena [2], [3]. These have been developed in both the conservative, implicit fashion and the less expensive, nonconservative, explicit fashion. Viable discrete models have been developed for vibrating strings; heat conduction and convection; free surface, laminar and turbulent fluid flows; shock wave generation; interface problems; and elastic vibration. The mechanisms in these models are always consistent with classical molecular mechanics and the modeling applies with equal ease to both linear and non-linear phenomena.

For illustrative purposes, we will summarize next a variety of computer simulations using discrete models. Whenever possible, the derived physical insights and advantages will be described.

Figures 7.1(a) and (b) show a bar and its particle simulation. Particles P_5, P_6 and P_7 were heated, or, in kinetic terms, their velocities were increased, and Figures 7.1(c)-(g) show the subsequent conductive heat transfer through the bar. The model is both conservative and nonlinear. It is also satisfying, physically, that the bar need not be infinite, as is usually required in continuous modeling.

Figure 7.2 shows the conservative, elastic vibration of a flexible bar from a position of tension. What

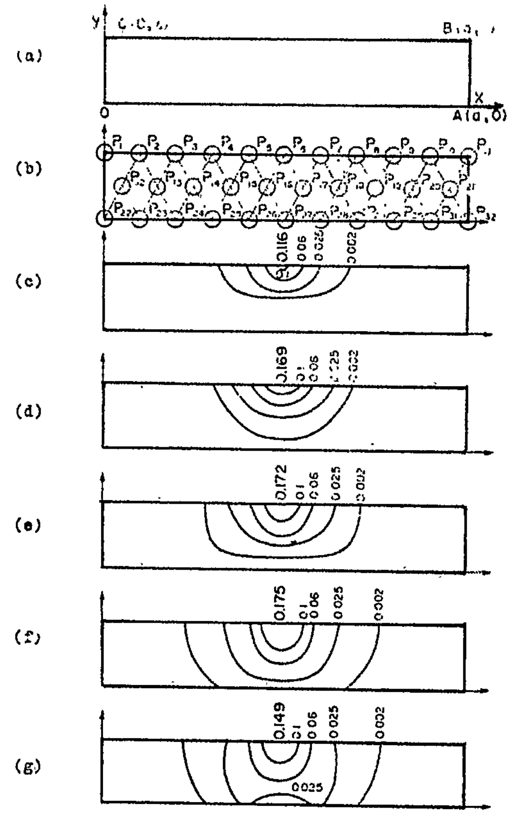

FIGURE 7.1

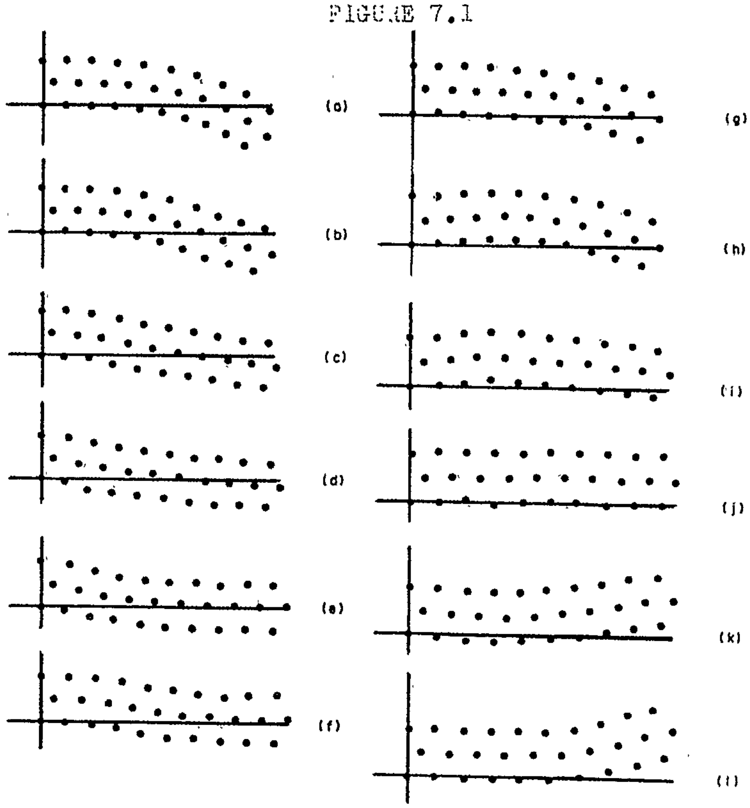

FIGURE 7.2

emerges clearly is that the bar does not swing smoothly, but flutters up, due to waves which travel through the bar as part of its gross upward motion. Engineers have been aware, for some time, of such waves in vibrating materials.

Figure 7.3 shows how particles of a liquid emerge from a nozzle at relatively low speeds. The flow is what is usually called laminar. As the particle velocities are increased moderately, the rows of particles maintain their relative positions, as shown in Figure 7.4, but the flow is becoming relatively chaotic. The disturbance arises from the increase in velocities, since faster moving particles can come closer to each other than can more slowly moving ones, and greater repulsive forces thereby result. Finally, in Figure 7.5, the velocities have been increased to the point where the rows no longer maintain their relative positions, and the motion is called turbulent, since it simulates the rapid appearance and disappearance of many vortices [3]. In continuum mechanics, there is as yet no viable model of turbulence, although it is known that most fluid flow is of this nature.

If one does not have sufficient resources for the implicit, conservative modeling described thus far, or if one wishes to simulate nonconservative phenomena, then one can still formulate and study discrete models by using nonconservative explicit formulas [3]. Some of the models which have been developed in this fashion will be described next.

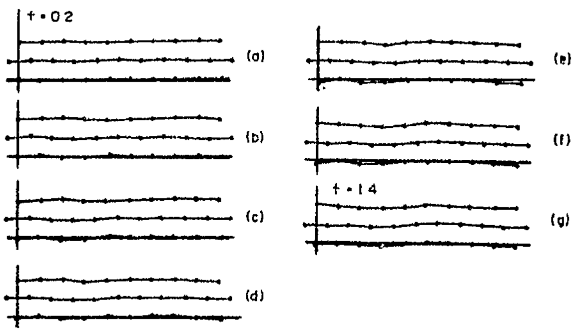

FIGURE 7.3

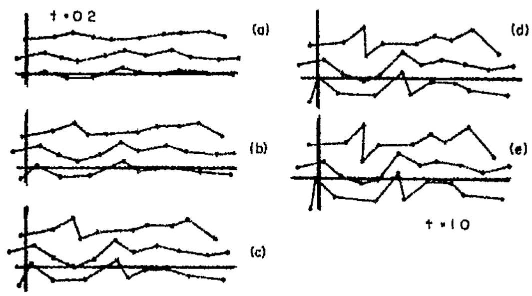

FIGURE 7.4

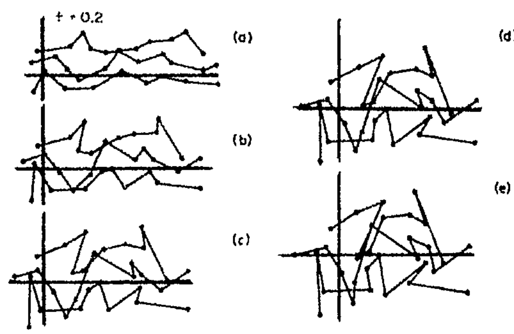

FIGURE 7.5

Figures 7.6(a)-(d) show how shock waves can be generated. In (a) is shown a gas in a long tube. The gas particles are distributed relatively uniformly. In (b), a plunger has been inserted into the tube. When the plunger is moved slowly down the tube, as shown in (c), the gas simply reorganizes itself into a new, but relatively uniform, distribution. However, when the plunger is moved down the tube at a very high speed, then gas particles do not have the time to reorganize, and they pack up on the face of the plunger, as shown in (d). The gas now separates into two portions, one that has, approximately, the initial density, and one that is highly dense and is impacted on the face of the plunger. The boundary between these two portions is called a shock wave and the computer generation of 'realistic' shock wave formation is shown in Figure 7.7. We have used the term 'realistic' because this model also includes the heating of the walls of the tube, a phenomenon of fundamental importance which is usually too difficult to incorporate into continuous models.

Figure 7.8 shows a computer generated heavy gas. Figure 7.9 shows the path of a relatively light particle which is ejected after insertion into the gas, thus establishing the property of buoyancy. Figure 7.10 shows expansion, convective motion, and vortex development in the gas due to heating at the lower right hand corner of the container.

Figures 7.11-7.14 show the entry and dispersion of a heavy liquid drop into a liquid well. The wave generation is shown readily by reexamining the same figures, but following the motion of various liquid columns, as shown in Figures 7.15-7.18.

Figures 7.19-7.23 show a circularization process of the biological species called volvox. This circularization is needed to simulate an inversion process inherent in the maturation of volvox.

8. REMARKS

In this final section, let us discuss some of the nuances and implications of the approach we have pursued.

First, note that a completely arithmetic approach has been developed elsewhere [9] for Special Relativity. Thus, the two major disciplines of deterministic physics have been given arithmetic formulations.

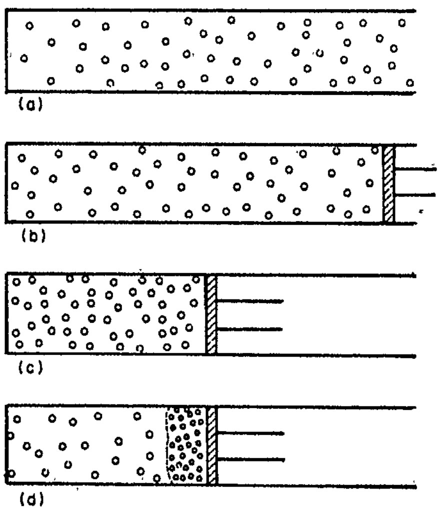

FIGURE 7.6

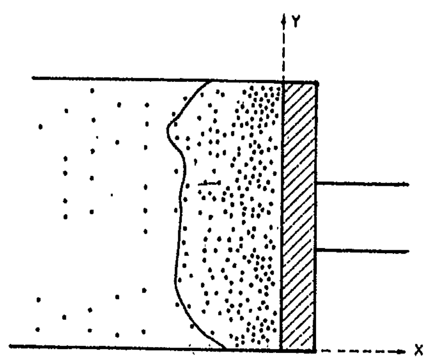

FIGURE 7.7

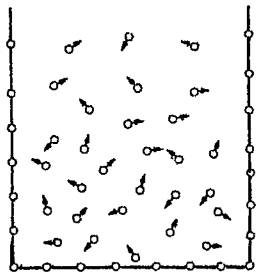

FIGURE 7.8

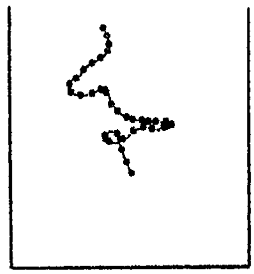

FIGURE 7.9

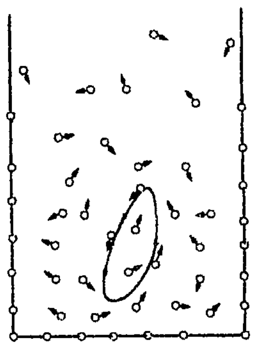

FIGURE 7.10

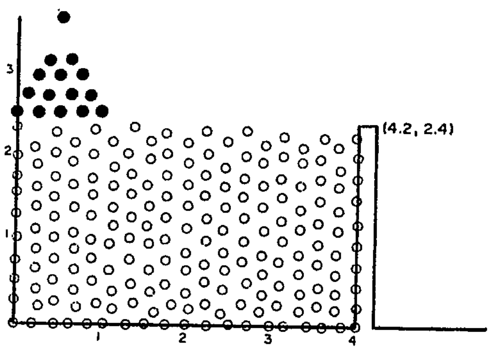

FIGURE 7.11

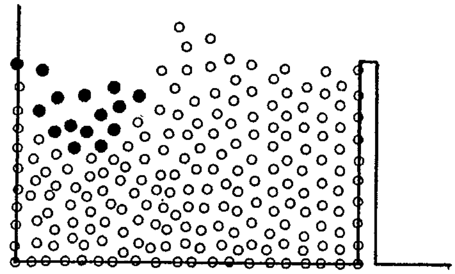

FIGURE 7.12

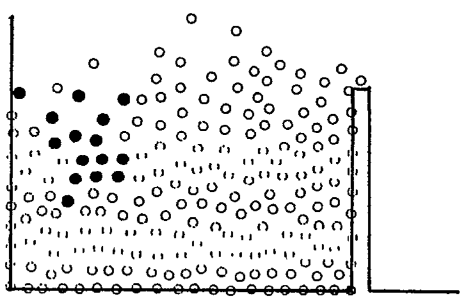

FIGURE 7.13

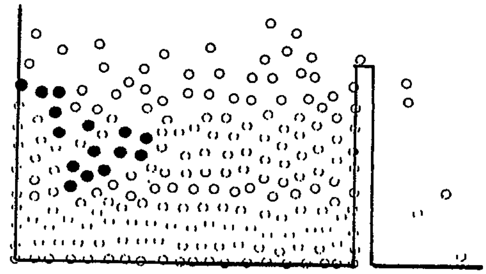

FIGURE 7.14

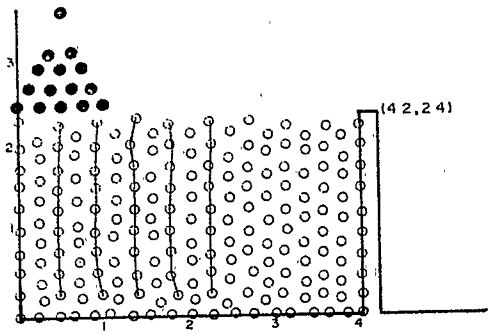

FIGURE 7.15

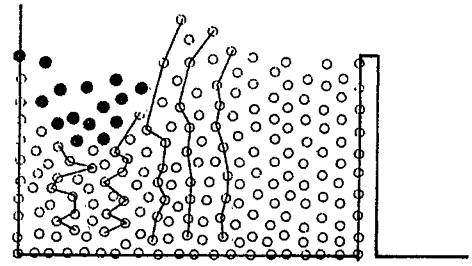

FIGURE 7.16

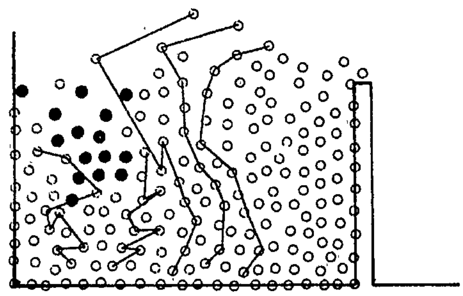

FIGURE 7.17

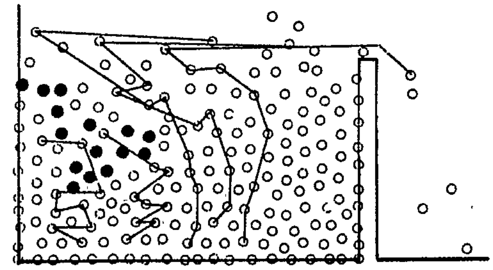

FIGURE 7.18

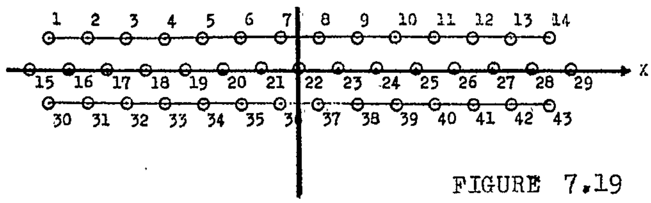

FIGURE 7.19

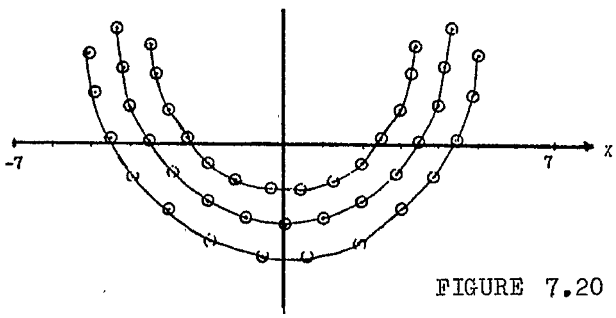

FIGURE 7.20

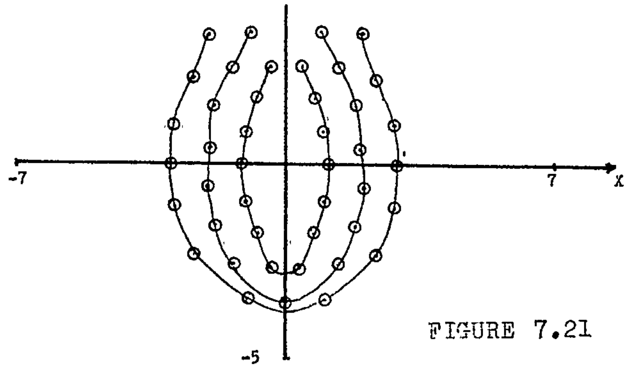

FIGURE 7.21

Second, note that a generalized form of Newton's iteration method [10], which does not use matrix inversion, applies easily to all nonlinear systems associated with the conservative approach of Section 6. In this connection, it is worth emphasizing that all models in this paper have been nonlinear. Note also that the explicit models are natural candidates for the application of parallel computers.

Next, let us try to delineate clearly the fundamental differences between discrete and continuous modeling. For simplicity, consider, for example, water in a glass. There are, approximately, 10^{28} molecules of water, which are far too many to deal with directly. In continuous modeling, the 10^{28} molecules are replaced by an infinity of points. In the discrete modeling the 10^{28} molecules are replaced by say, 10^3 particles, with a simultaneous compensating adjustment of molecular parameters. Both approaches are only approximations. The discrete approach was not realizable before the availability of modern computers. In this connection, it is worth observing also that many modelers believe firmly that 10^{28} is so large that it can be approximated well by infinity. This can be dispelled, but only by the precise mathematical statement that 10^{28} points form a set of measure zero in any infinite set of points. Loosely stated this means that when one creates an infinite set of points, then 10^{28} points are completely lost within these.

From the educational point of view, the current availability of inexpensive computers means that exciting mathematics and physics, considered previously to be 'advanced', can be presented now at a most elementary level. Indeed, there is not one model described in Section 7 which cannot be presented after a first course in trigonometry.

Finally, from the scientific point of view, the availability of discrete models provides researchers with additional tools in their study of natural phenomena, and the importance of these new tools cannot be underestimated at a time when both sub-atomic and cosmic physics are revealing that Nature is far from the simplistic entity it was once thought to be.

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