

SIMULATION MODEL FOR MULTI-LEVEL DISTRIBUTION SYSTEM PLANNING

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Abstract

The analysis of multi-level inventory/distribution systems has been pursued from both the analytical and simulation viewpoints. The simulation model IDIMS (Inventory and Distribution of Items in a Multi-echelon System) is presented and demonstrated in this paper. IDIMS can handle up to ten separate items at four levels of a distribution system, including a limited-capacity, forecast-driven production entity. The use of continuous or periodic review inventory management systems are allowed and all stochastic events are generated via user-specified probability distributions or empirical distributions input to IDIMS. The use and alteration of system architecture, serving responsibilities, service levels and cost parameters are easily accomplished via input and reasonable default values. IDIMS has been developed for use by industrial management/analysis personnel as well as professors and students in an academic environment to study a wide range of multi-item, multi-level production/inventory/distribution systems.

INTRODUCTION

Virtually all consumer products are delivered to the customer via a distribution system of storage entities which hold the item in inventory until it is ordered or demanded by another storage entity. This type of distribution system is known as a multi-level or multi-echelon inventory/distribution system and may be graphically depicted as a tree structure (Figure 1) in which distribution takes place from the producer through distribution centers, warehouses, and possibly other storage entities, to a local or retail supply (RS) outlet. All entities within each level or echelon perform the same function. The number of entities per level increases rapidly at lower levels of the system due to the one-to-many serving responsibilities. This vertically oriented system is a part of everyday business functions whether the structure

is wholly company-owned or is completely independent in ownership at every level.

In classical inventory theory it is considered correct to analyze inventories and set policies at a distinct stocking point or entity without taking into account many of the possible effects on that entity from other entities with which it interacts. Only recently has the overall multi-level system been investigated and some of the complex interactions and dependencies between inventories been studied.

Questions which management asks about the operation of multi-level systems usually concentrate upon the physical architecture or structure of the system (number of entities, serving responsibilities), required and observed service levels at each entity or level, and the type of inventory management policies used at different levels of the system (continuous review or periodic review). Often general questions concerning cost reduction in inventory are best answered by performing an analysis of a part of the multi-level system.

To analyze this type of inventory/distribution structure with its complex inter- and intra-dependencies, no general closed-form solution exists for optimal inventory policies and structures unless there are many possibly unrealistic constraints placed on the system. Because computer simulation is recognized as a viable analysis tool of complex systems, there have been many simulators developed to study portions and types of multi-echelon systems. This paper describes and applies a new simulator designed to study a wide range of multi-level, multi-item inventory/distribution systems. The simulator, called IDIMS (Inventory and Distribution of Items in a Multi-echelon System), is useful to a wide variety of analysts, ranging from company management of a multi-level system to a student learning the theory and application of inventory and distribution.

Following background material on current work in this area, the design goals and capabilities of IDIMS are discussed. An illustration is included to

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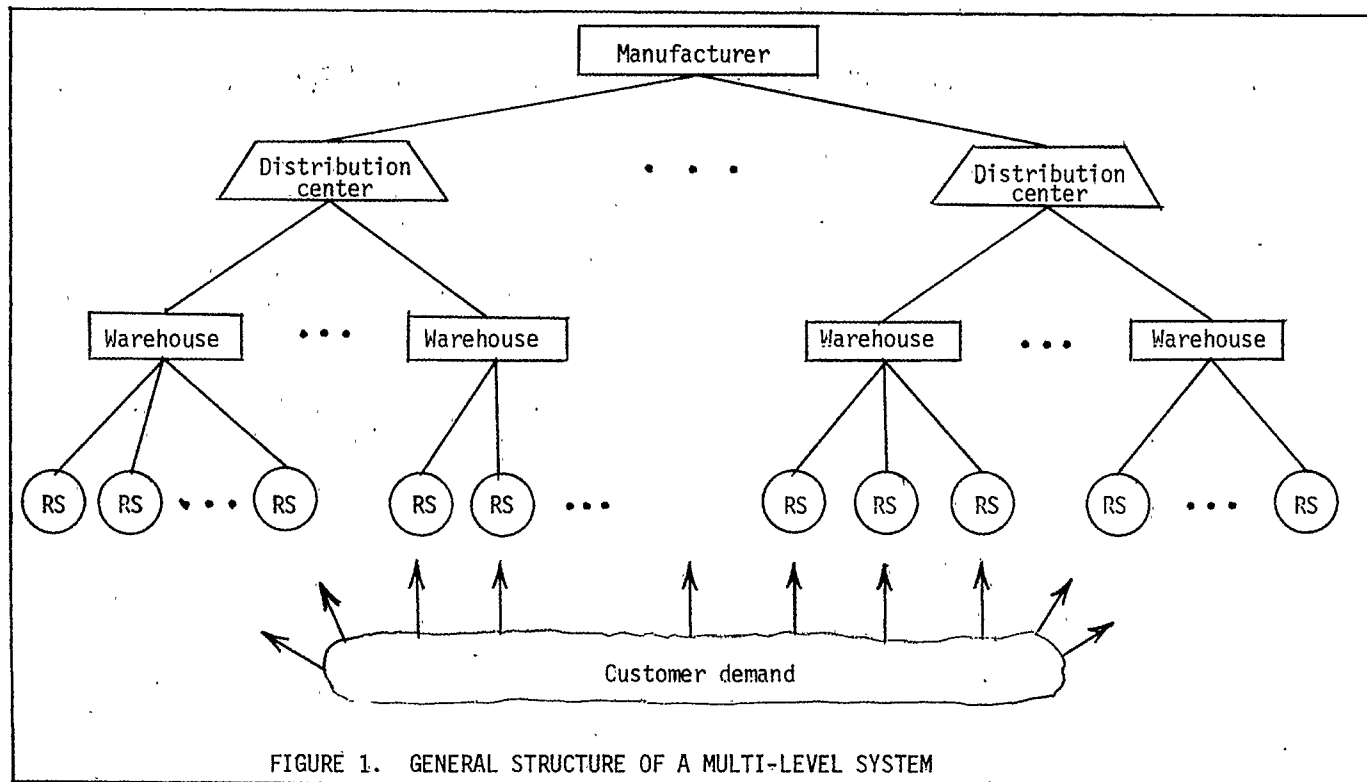


FIGURE 1. GENERAL STRUCTURE OF A MULTI-LEVEL SYSTEM

demonstrate how IDIMS develops and analyzes the structure, policies, stochastics and costs that are inherent to a multi-level system.

BACKGROUND OF MULTI-LEVEL ANALYSIS

There has been considerable work published on multi-level system analysis. One of the first comprehensive surveys was written by Clark (5). Within this article, the evolution of the study of multi-echelon inventory systems was explored. Early work on the understanding and analysis of multi-level systems was outlined and different methods of approaching the solution to problems within the system were detailed. Zangwill (25) created a network model of a deterministic system by linking together several single entities. By applying some restrictions that reduced the computational complexity of the problem, he was able to arrive at optimum production schedules using linear cost functions. Von Lanzener (23) also created a multi-level system using deterministic demands but again the size of the problem was severely limited due to the computational complexity introduced by solving for optimal conditions. The one-warehouse, n-retailer system with deterministic demand was analyzed by Schwarz (19). In this model the average system cost was minimized by the optimum inventory policy. Schwarz and Schrage (20) proposed optimal and near optimal inventory policies for a deterministic multi-echelon production/inventory and assembly system. Geoffrion (9) used cost and sensitivity analysis in a simulator to determine the optimal number of storage locations for a deterministic, multi-level system.

Several stochastic models have been developed. Clark (3) developed a dynamic programming model

for a single product, multi-echelon system with a single entity at each level. The objective of this model is to determine optimal ordering policies under certain cost and policy restrictions. One cost restriction in this model was relaxed by Clark and Scarf (6) to determine bounded solutions at higher levels. Hochstaedter (12) and Hadley and Whitin (10) developed more sophisticated models. Love (13) expanded the stochastic approach with a model of a single item, two entity, two echelon system that calculated the optimum inventory policies. However, there are several restrictions and limitations placed on this model due to the computational complexity of solving for optimality. Deuermyer and Schwarz (8) developed a one-warehouse, n-retailer model that calculates the service level as a function of lot size, leadtimes and known stochastic demand parameters.

There are problems with the stochastic as well as the deterministic approach to multi-echelon inventory analysis--most of which deal with the computational complexity of the problem. Models that derive optimal policies are very restrictive in either the size of the system modelled or in the parameters within the system. The use of computer simulation has been considered an alternative to analytical approaches to multi-echelon inventory problems as early as 1960 when Clark (4) simulated a system that was similar to his dynamic programming model described previously. Simulation proved to be a powerful tool in analyzing multi-echelon inventory systems and has been applied to many aspects of the system.

Sherbrooke (21) developed METRIC and Muchstadt (16) updated this system with MOD-METRIC to examine the repair of aircraft engines within the repair/inventory/distribution system of the Air Force.

Heier and Jones (11) simulated a physical distribution system to improve the number, placement and capacity of warehouses for a national food corporation. Thornley (22) also used simulation for modelling different distribution strategies within a large department store chain. Blank (1) applied simulation to provide a practical solution to the distribution and management of inventory in a multi-level, multi-item environment within a communications corporation. Meyer and Groover (14) applied simulation to analyze the effects of a unit step input on demand in a single-item, multi-level system. A 'false order' effect that was modelled and suppressed by Meyer and Groover (14) and Clark (5) has been addressed by Burns and Sivazlian (2) who offer a smoothing concept to suppress this adverse effect.

The multi-echelon inventory/distribution system remains a fertile area for continued research and applications. Wagner (24) provided a comprehensive review of the status of inventory management systems with enumeration of practical problems that need further study. One of the problem areas is the multi-item environment.

With all the work presented it is still not very easy for the inventory analyst or manager to perform multi-level system investigation at a relatively high (global) level. IDIMS is specifically designed for this type of investigation in which the effects of different inventory management policies, demand patterns, physical configurations (architecture) and service-level requirements are simulated and analyzed. All these characteristics are easily changed at each level of the system and the effects are observed on costs, order quantities, observed service levels, etc. at each level of the system for each item or product line simulated.

The remainder of this paper presents an overview of the capabilities of IDIMS. Updating of inventory parameters via a special, user-imposed, feature called PAM (Parameter Adjustment Module) is discussed and illustrated. An example run of IDIMS is presented and possible alternatives to improve the operation of the modeled system are discussed.

SYSTEM DESIGN AND CAPABILITIES OF IDIMS

DESIGN GOALS

In the design and development of IDIMS there were three important goals that helped form the structure of the model from the initial design to the final output. These are:

1. The simulator should be user-oriented.

This requires the preparation of good documentation and user's guides, as well as applicability by many different types of users under different circumstances. This development goal is accomplished by providing a large amount of flexibility for the user in designing the structure or architecture of the simulated system and the type of inventory and distribution system management to be modelled within the architecture. This large flexibility makes it mandatory that the user provide a large amount of data to accurately model the system. However, to make the model more applicable to users in education and industry, a liberal amount of default values (on system

parameters) are included. These default values are practical in nature and, whenever possible, they are a function of other user-input data so that the simulator is realistically adaptable to the system being modelled. The listing of default values is provided in the user's guide.

2. The decision rules should be based on applications and theory. Building a practical yet theoretically correct model was another goal during the design and development stages. Classical inventory theory is integrated into the decision rules within the model. However, the user has the opportunity to bypass the theory, for example in the evaluation of management-specified techniques for implementing inventory policies for periodic and continuous review systems. The user has the versatility to eliminate or include some features. For example, the forecaster may be eliminated, it may take the form of a simple horizontal forecaster, or it may be a complex forecaster including trend and seasonality with adaptive smoothing. There are allowances within the production entity allowing the user to set limits on production changes, and smooth the forecast value via the previous month's production.

3. The model should be portable.

The goal of portability required that IDIMS be transferrable to different makes and sizes of machines. Portability was attained by using FORTRAN, a very common computer language and one that a great majority of computers can compile. Machine dependencies in the coding of the model were avoided to ensure easy transfer from machine to machine.

ARCHITECTURES ALLOWED IN IDIMS

The modeler has the ability to include up to ten distinctly different items and four levels in the distribution system, including the production level and three storage levels (distribution center DC, warehouse WHSE, and retail supply RS). Figure 2 shows a maximum balanced architecture of the 14 entities that is allowed. A balanced structure indicates that each entity within a certain echelon has an equal number of entities reporting to it. Figure 3 illustrates an unbalanced architecture that is regular, that is, each entity within a certain level reports to an entity within the next higher level.

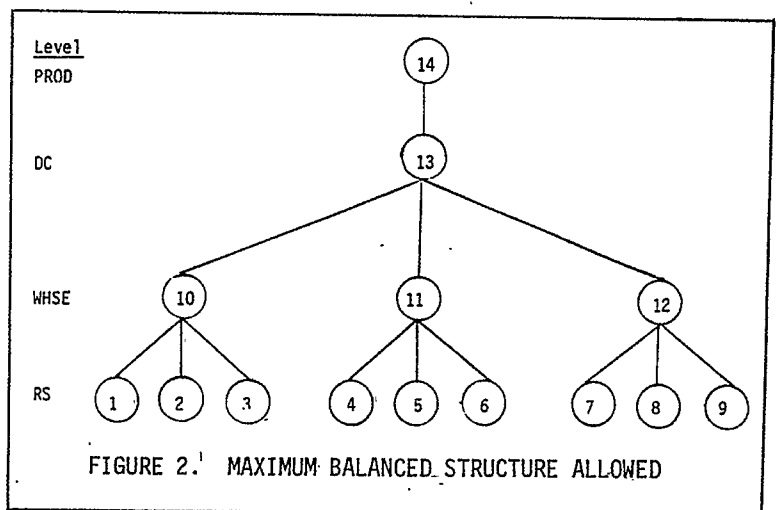


FIGURE 2. MAXIMUM BALANCED STRUCTURE ALLOWED

In a non-regular structure an entity in a level may obtain service that is more than one level higher, such as the two RS entities reporting directly to the DC instead of a WHSE in Figure 4.

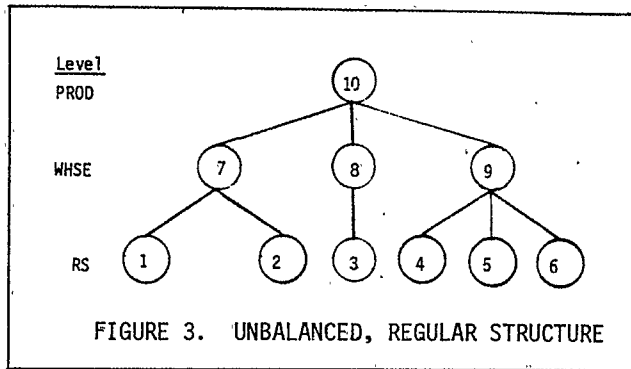


FIGURE 3. UNBALANCED, REGULAR STRUCTURE

The following limits are imposed on the number of entities at each level:

Level	Number
Retail supply	1 to 9 (one required)
Warehouse	1 to 3 (one required)
Distribution center	0 to 1 (can be eliminated)
Production	1 (required)

The RS level in IDIMS is the only one that deals directly with the customer to fill demand for different items from the stock on hand. Although it is required that each RS (and for that matter each entity in the system) carry all of the items specified by the user, the user may enter the demand distribution parameters such that no demand is generated for some items at certain RS entities. This capability will allow the user to simulate different stock items at different RS entities. The RS level is the only level that is charged a stock-out cost if the demand is not filled. Stock-outs at higher levels are accumulated in the monthly reports, but there is no cost penalty for not filling a demand.

WHSE is the next higher level and although it is common for the RS level to be restocked from the WHSE level, user input specifies from where entities are restocked, as in the unbalanced, non-regular structure shown in Figure 4. The distribution center (DC) level is above the WHSE level and functions in much the same way. The highest level, production (PROD), is the one at which all of the items are 'made' and introduced into the distribution system. The user again has a large amount of flexibility in structuring the production storage limits and a limit on the production percentage change from month to month.

The system modeled by IDIMS is imposed to be a 'pure' arborescent system for each item, that is, all the items that are stored and distributed from a single entity are supplied by another single entity at least one level higher. There are no alternate suppliers or other sources for stock when stockouts or heavy demands are experienced.

INVENTORY MANAGEMENT POLICIES AND STOCHASTICS

All of the echelons in IDIMS have several characteristics in common. They each accumulate carrying costs, item costs and ordering costs and each storage entity has inventory policies and service

levels that are structured by user inputs. The RS and WHSE levels can manage items using either a (s,Q) continuous review system where a fixed order quantity Q and reorder point s are the parameters,

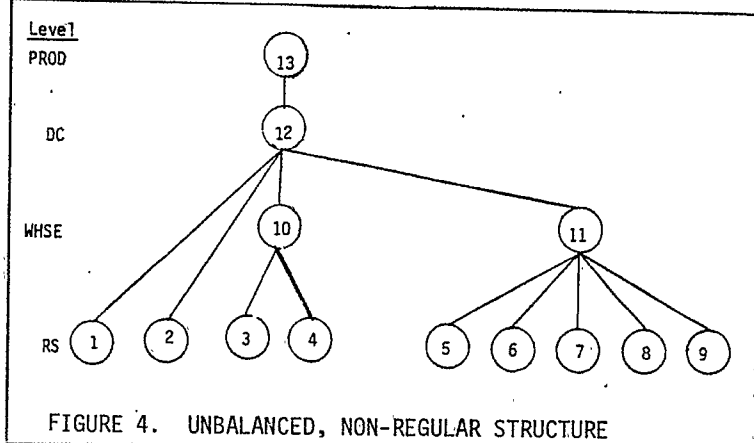


FIGURE 4. UNBALANCED, NON-REGULAR STRUCTURE

or a (R,S) periodic review system in which the reorder interval R is fixed and the maximum inventory level S are the parameters. At the DC level only the (R,S) policy is allowed because most large storage entities actually employ a reorder interval (ROI) policy. The items are distributed with a user-selected, probabilistic lead-time from the production entity through the storage and distribution 'pipeline' formed by DC and WHSE to the final RS level.

Simulations are generally employed to model the actions and reactions of a physical system over a period of time. The physical variations in the system being simulated are generated using stochastic properties. The probability distributions for inventory and production properties used in IDIMS, such as demand, time between demands, leadtimes and production times are estimated and approximated to one of the following probability distributions:

Uniform	$U(a,b)$
Normal	$N(\bar{X},s^2)$
Log-normal	$LN(\bar{X},s^2)$
Weibull	$W(a,b)$
Exponential	$\exp(\lambda)$
Poisson	$P(\lambda)$

Values from the selected distribution are computed via the proper inverse function using a U(0,1) pseudo-random number generated by the multiplicative congruential method.

If no common distribution is acceptable, the user may enter empirical data in the form of a cumulative density function (cdf) for up to six different distributions for special or non-standard forms for any stochastic event in IDIMS. To make IDIMS more applicable to actual situations the cdf may be entered using up to 10 variable values. The user inputs both axis values and the Monte-Carlo sampling of the cumulative probability scale imposes a value on the variable.

Deterministic simulation can be accomplished for any stochastic event in IDIMS by inputting the appropriate parameters into one of the distributions.

The most popular are to specify the same parameter values in the uniform, $U(a,a)$, or the normal in the form $N(\bar{X},0)$. In both cases the inverse function is a constant equal to the input value.

DEMAND GENERATION

IDIMS can best be described as a 'pull' system. That is, the demand is generated at the bottom of the system and the units are 'pulled' through the distribution system. This type of system is opposite to that of a 'push' system where the demand is forced through the system from the production level. The demand is the most important part of the simulator as practically all other events depend upon this event. To generate demand that is stochastic or deterministic the user specifies the distributions and parameters that describe the demand and the time between demands. The actual demand D_{it} for item i in time period t may be represented as a function of the average \bar{d}_{it} of the selected distribution.

$$D_{it} = f(\bar{d}_{it})$$

The simulator will, at user request, include trend as a constant monthly multiplier of the mean of the demand distribution.

$$D_{it} = \begin{cases} \min\left\{(\bar{d}_{i,t-1} \times T_i), T_U\right\} & T_i > 1.0, T_U > 0 \\ \max\left\{(\bar{d}_{i,t-1} \times T_i), T_L\right\} & T_i < 1.0, T_L > 0 \end{cases}$$

- where T_i = user-specified trend factor for item i
- T_U = user-specified upper limit for demand distribution average (a positive trend)
- T_L = user-specified lower limit for demand distribution average (a negative trend)
- $\bar{d}_{i,t-1}$ = demand mean from the previous period

The average demand may be written

$$\bar{d}_{it} = \bar{d}_{i,t-1} \times T_i = \bar{d}_{i,t-2} \times T_i^2 = \dots = \bar{d}_{i1} \times T_i^{t-1}$$

Figure 5 shows the effect of a negative trend ($T_i < 1.0$) on D_{it} , the actual demand generated. The application of a trend factor to the demand of an item may be used to show the increasing demand of a newly-introduced item or the decline of demand on an item that is being phased out. There is also an intermediate trend factor that the user may apply to the demand average during the simulation. This new trend factor takes the place of the previously described trend factor to model an in-simulation change in trend. Together these two trend factors may model the rise and fall of the demand of an item or item interaction for the eventual decline of an old product after the demand for a new product rises.

In addition to trend, the simulator is equipped to add a seasonal effect to the demand of an item. These monthly, user-input seasonal multipliers change the demand distribution average each month (Figure 6).

$$D_{it} = \bar{d}_{it} \times S_{ik}$$

where

- k = index locating period t within the season ($k = 1,2,\dots,12$)
- S_{ik} = annual seasonal adjustment for item i and month k

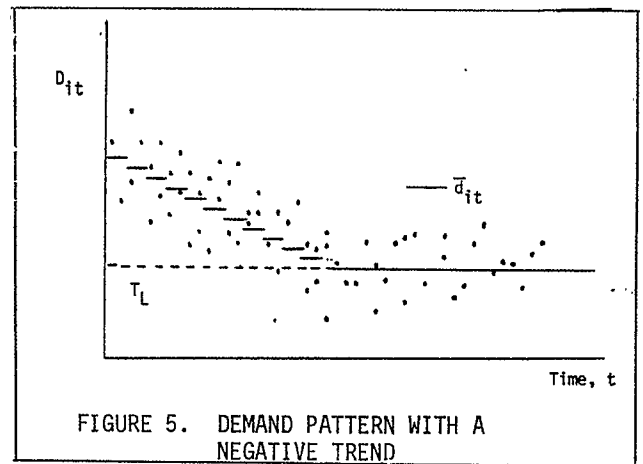


FIGURE 5. DEMAND PATTERN WITH A NEGATIVE TREND

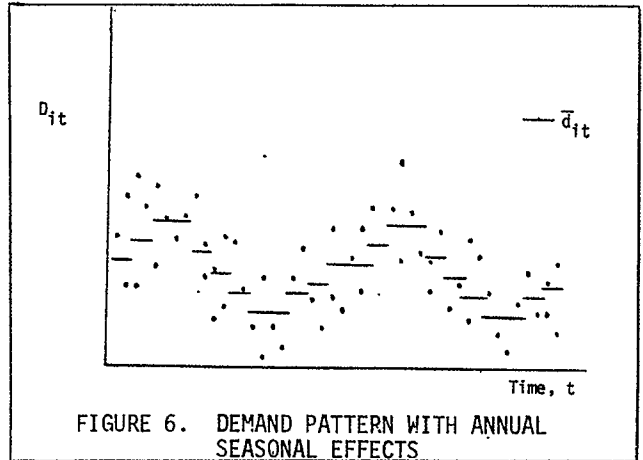


FIGURE 6. DEMAND PATTERN WITH ANNUAL SEASONAL EFFECTS

PRODUCTION ENTITY AND THE FORECASTER

Unlike many other production/inventory/distribution simulators, the supply of the items from the production entity in IDIMS is not infinite. There may be definite restrictions on the quantity to be produced via the amount of percentage change allowed on the quantity of production from month to month and the storage limit placed on each item at the production entity (PROD).

Simulation of production runs by item i occur each month t with the quantity produced P_{it} based on the forecasted quantity (described below), the previous month's production run, the production change limit and the maximum storage limit for each item. Figure 7 indicates how the actual production rate can be affected by the forecast value and the storage limit.

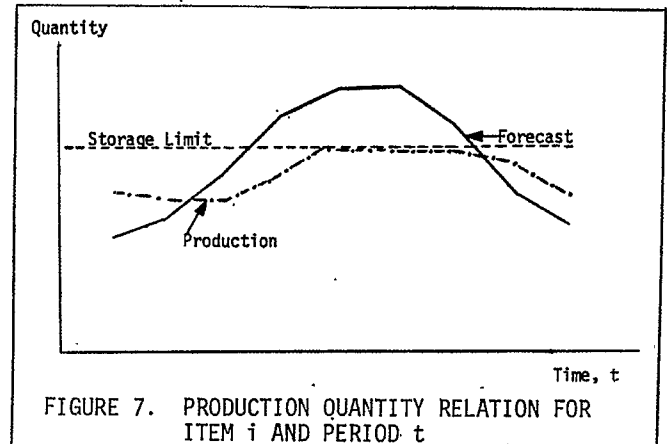


FIGURE 7. PRODUCTION QUANTITY RELATION FOR ITEM i AND PERIOD t

Simulation of Multi-Level Distribution Systems (continued)

Since in application most production/inventory/distribution systems have some sort of forecasting ability to anticipate future demand, IDIMS is equipped with this same capability. The forecaster (FC) is dependent upon user inputs. The user may leave FC in a simple, horizontal state which applies single exponential smoothing; if there is some trend apparent, an initial smoothed trend value will change FC to a double exponential smoothing model. The basic equation used in IDIMS is:

$$F_{i,t+1} = E_{t+1} + (1/\alpha)(\hat{T}_t) \quad 0 < \alpha < 1$$

where

$$F_{i,t+1} = \text{forecasted value for item } i \text{ for next month}$$

$$E_{t+1} = \text{single exponentially smoothed forecast}$$

$$= \alpha D_{it} + (1 - \alpha)F_{it}$$

$$\alpha = \text{smoothing constant, } 0 \leq \alpha \leq 1$$

$$\hat{T}_t = \text{current value of the smoothed trend (automatically equal to zero for single exponential smoothing)}$$

The same seasonal factors that are used to change the demand average are also used to give FC the ability to include seasonality in the forecast. The addition of seasonal influences S_{ik} changes the forecast to:

$$F_{i,t+1} = [E_{t+1} + (1/\alpha)(\hat{T}_t)] S_{ik}$$

An important capability of FC is the use of adaptive smoothing via the Trigg/Leach method (18). Adaptive smoothing allows the smoothing constant α to no longer remain constant but to react to the difference (error) between the last forecast and the actual demand. There is a ceiling of $\alpha = 0.6$ on the adaptive smoothing constant in IDIMS.

The amount of each item that is actually produced each month is not exactly the forecasted value. Smoothing of the amount forecasted with the last production rate P_{it} dampens the changes in production thus counteracting any 'false order effect' or over-reaction to changes in forecast values.

$$\hat{F}_{i,t+1} = \alpha_{PR}(F_{i,t+1}) + (1 - \alpha_{PR})P_{it}$$

where:

$$\hat{F}_{i,t+1} = \text{actual amount to be produced in period } t + 1$$

$$\alpha_{PR} = \text{smoothing constant input by the user}$$

This smoothing of production occurs in addition to the percentage change limit previously discussed.

PARAMETER ADJUSTMENT MODULE (PAM)

The updating of inventory management parameters such as the EOQ, s and safety factor k used in safety stock computations is of prime importance to the operation of multi-level distribution systems. IDIMS allows the user to select periodic updates using the observed (simulated) demand and input service level requirements or using the

specialty-designed updating functions in PAM. The well-known service function $G(k)$ is written

$$G(k) = \frac{Q}{\sigma_L} (1 - SL)$$

where Q = order quantity

σ_L = standard deviation of lead time demand

SL = management-specified service level

k = safety factor from standard normal distribution

Parr (17) devised an approximating relation for $G(k)$ in the form

$$G(k) = \exp -[0.92 + 1.19k + 0.37k^2] \quad (2)$$

which Der Tatevasion (7) used to derive k as a function of observed variables only.

$$k \exp[k(1.19 + 0.37k)] = \left[\frac{1}{2 - 2} \right] \left(\frac{AI}{SS} - 1 \right) (2.5093) \quad (3)$$

where AI = observed average inventory level
 SS = observed safety stock level

Substitution of Eq. (2) into Eq. (1) and solution for Q results in

$$Q = \{ \exp[-(0.92 + 1.19k + 0.37k^2)] \sigma_L SL \} / (1 - SL) \quad (4)$$

Equations (3) and (4) are used in PAM to compute updated values of k and Q for the continuous review (Q,s) system. Similar analysis is performed for the periodic review (R,s) system.

Additional derivation for cost analysis of the total variable cost (TVC) allows the user to bound the observed TVC using a percentage of the optimal TVC. System cost values outside the limit cause the EOQ value to be adjusted accordingly and a new safety factor k to be computed. The user may eliminate PAM from IDIMS or cause it to 'adjust' the inventory management parameters at the RS level of the architecture every six months of simulated time.

ILLUSTRATION

All input to IDIMS is summarized on a form, one of which is developed for each item to be simulated. Output is given for each month and item at each entity in the distribution structure (typical print-out in Table 1). The data in Table 2 represents the input necessary for one of the two items within the illustrated structure. The structure has six RS entities, three reporting to each of the two WHSE entities. RS operates under a ROP system while WHSE uses the ROI policy. Table 2 indicates that Item 1 has a normally distributed demand distribution with various means and standard deviations. This item is a dynamic, high usage item with an exponential distribution for time between demands and both positive trend and seasonality factors applied to the demand mean. Item 2 (input not shown) is higher priced than Item 1, has a uniform demand distribution and exponential time between demands. There is no seasonality accompanying item 2 and a small positive trend factor is offset by the late, declining trend which is initiated at time 400. These two items are different in many of their structuring parameters, however, one fact that the items have in

Simulation of Multi-Level Distribution Systems (continued)

Item 1		Month						6-mo. Avg.
		12	13	14	15	16	17	
Service Level (%)	RS1,2,3	75.4	80.5	60.8	85.7	75.1	100.0	79.6
	RS4,5,6	92.8	79.4	64.3	80.3	74.9	56.6	74.3
	WHSE1,2	17.3	10.5	26.2	47.7	7.3	13.3	16.6
Order Fill Rate (%)	RS1,2,3	9.8	9.2	27.7	41.7	9.5	16.7	15.6
	RS4,5,6	39.0	11.4	24.9	54.0	5.3	10.9	17.5
	WHSE1,2	100.0	100.0	100.0	94.7	67.3	89.4	91.0
Qty. Lost to Storage Limits (unit)	RS1,2,3	64	0	45	596	0	490	199.2
	RS4,5,6	33	0	42	88	23	0	31.0
	WHSE1,2	644	472	472	843	376	388	532.5
Item 2								
Service Level (%)	RS1,2,3	71.4	77.0	87.0	59.8	23.8	83.2	68.2
	RS4,5,6	53.8	58.9	84.3	56.7	95.9	72.7	70.8
	WHSE1,2	13.1	32.1	31.9	22.7	9.0	41.8	23.3
Order Fill Rate (%)	RS1,2,3	8.1	29.7	22.3	31.2	6.9	32.1	12.9
	RS4,5,6	9.7	24.7	42.3	24.0	11.1	54.2	24.4
	WHSE1,2	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Qty. Lost to Storage Limits (unit)	RS1,2,3	0	0	0	0	0	3	0.5
	RS4,5,6	0	21	45	18	12	48	24.0
	WHSE1,2	153	156	306	190	343	343	248.5

TABLE 3. CONSOLIDATED RESULTS FOR INITIAL SIMULATION RUN

CONCLUSION

The IDIMS simulator has been developed to analyze both the "as-is" and "to-be" environments of a multi-level distribution system. It is well-adapted for both a management and a cost/stochastic analysis viewpoint. The versatility in distribution system architecture, inventory management policies and adaptations to real-world management and control make it a potentially important automated tool in the support of the design, analysis implementation and maintenance of many multi-item, multi-level production/inventory/distribution systems.

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