

## INTRODUCTION TO SYSTEMS ANALYSIS, MODELING AND SIMULATION

J. W. Schmidt

Department of Industrial Engineering and Operations Research  
Virginia Polytechnic Institute and State University  
Blacksburg, Virginia 24060

### INTRODUCTION

Effective execution of managerial responsibility is a perplexing task of ever increasing complexity. This is the result of unpredictable world, national and local economies, the pervasive impact of federal and state regulation, the capricious nature of the consuming public and the relentless and accelerating advance of modern technology. The contribution of each of these elements of business life has led to the perceived need for larger and more complicated organizational structures to effectively carry on business activity. Consequently the manager is forced to recognize and understand the interaction of an increasing number of components within his own organization and its environment. With that recognition and understanding effective management is certainly not assured. But the manager who does not recognize and understand the forces which should affect and are affected by his decisions has little hope of effectively discharging the responsibilities of a managerial position.

Fundamentally a manager is a decision maker. The process of making a decision involves the identification, evaluation and comparison of alternative courses of action. Each action is evaluated on the basis of the decision maker's objectives in light of a variety of conditions which may prevail during the period for which the decision will be in effect. Because of the number of factors which must be considered and the complexity of their interaction, the manager often turns to a system model for quantitative analysis of the impact of each alternative decision under each set of conditions anticipated. Such an analysis can offer significant insight into the propriety of each decision alternative.

Before introducing the subject of models and modeling, a discussion of the context in which the need for models arises is in order. The need for a model usually is the result of identification of a problem which requires a solution. The problem solving process is often referred to as Systems Analysis.

### SYSTEMS ANALYSIS

For the purpose of this discussion the systems analysis process starts with the identification of a problem and includes those activities which lead to the identification and implementation of a solution. While the specifics of the activities included will depend upon the nature and context of the problem and the people who must deal with the problem, several classes of activities or stages of the systems analysis process may be distinguished and include:

1. Problem identification
2. Specification of objectives
3. Definition of the system
4. Model formulation

5. Model verification
6. Model validation
7. Model implementation
8. Model use
9. Solution identification
10. Solution implementation
11. Model revalidation

Of course many problems are appropriately solved without going through each of the steps outlined here. Moreover, even when most or all of these steps are necessary, they may not be carried out as formalized procedures. Finally even when there is a need for formalization of each activity, the solution to the problem may surface prior to the point indicated in this list of activities, step 10. For example, at the outset of a study the analyst may envision the need for a symbolic model to evaluate potential solution alternatives. Before such a model can be formulated the analyst must define the system which is the context for the problem. In the course of studying the behavior of the system the analyst may uncover the solution to the problem, eliminating the need for steps 4 through 8 and step 11.

When all eleven of the activities of systems analysis must be carried out as formal procedures, they do not flow in the simple orderly pattern which the list given above might suggest. A more realistic representation is given in Figure 1. It is not unusual for several activities to proceed in parallel. Moreover, an activity presumed to have been completed at one point in the process may require reevaluation and further study in light of inadequacies revealed at a later stage in the analysis. Such inadequacies often surface during the validation stages of the analysis, and properly so, since the point of validation is to reveal inadequacies in previous steps in the analysis.

A brief examination of the elements of systems analysis and their interrelationships would seem to be in order at this time. Hopefully, this examination will provide a frame of reference for the introduction to modeling which is given in the section which follows.

### PROBLEM IDENTIFICATION

Problems are recognized by the symptoms which they display. Falling revenue, rising cost, poor profit performance, customer complaints, substandard quality, excessive production down time, declining sales, labor dissatisfaction, and labor turnover are just a few examples. Often the remedial action necessary to resolve the problem is evident from the symptoms displayed. The symptoms may also point to a problem which requires no remedial action at all because the problem is minor, because it is viewed as only a temporary disturbance, or because the effort required to resolve the problem cannot be justified

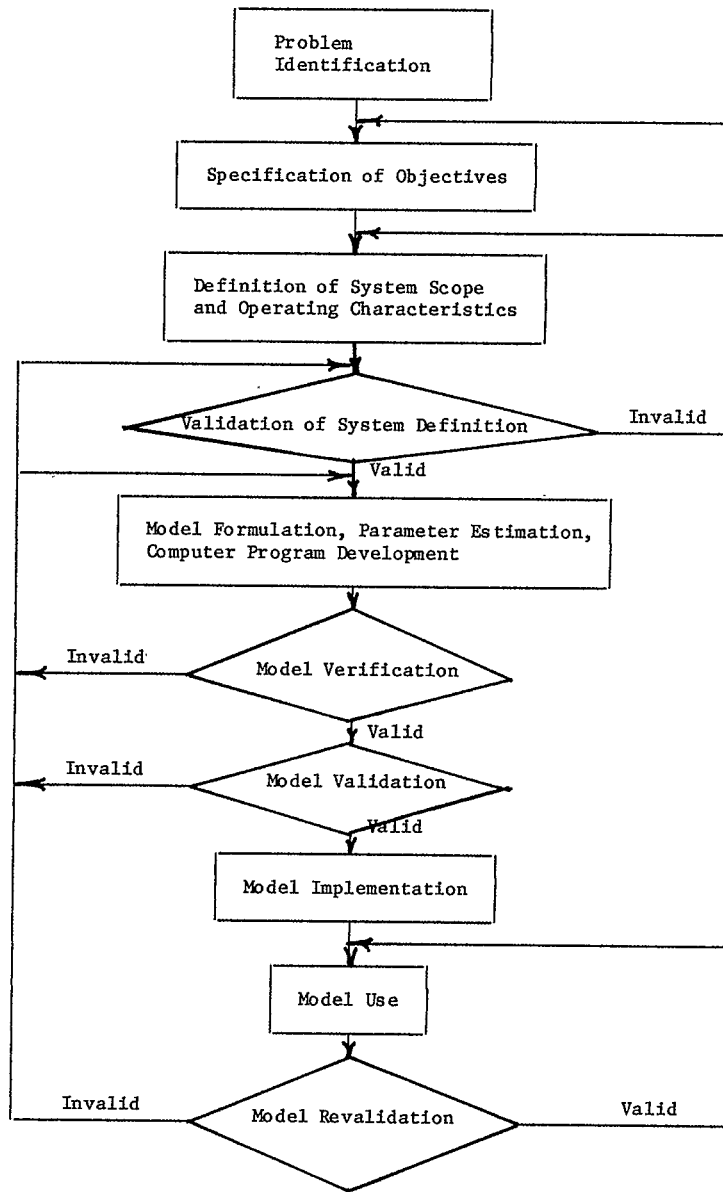


Figure 1: Flowchart for Systems Analysis

by the benefits which would result. Simply stated, some problems are just not worth solving.

Organizations are controlled by decision makers who "decide" what courses of action should be taken and see to the implementation of their decisions. Decision makers determine which problems should be

solved, which objectives should be achieved and which solutions should be implemented. The decision maker may not be involved in the day to day activities associated with the development of a solution to the problem. Nonetheless, he must be convinced that the solution proposed will solve the problem. Otherwise the solution will not be implemented. Thus the role

of the decision maker is of fundamental importance to the successful solution of a problem. In general, the more involved the decision maker is in the process of seeking a solution, the greater will be his understanding of and confidence in the solution finally achieved, which in turn enhances the likelihood that the solution will be implemented.

#### SPECIFICATION OF OBJECTIVES

The individual who assigns the task of finding a solution to the analyst is not necessarily the decision maker or may be only one of the decision makers concerned with the problem. The analyst's first problem is then identification of who he is working for; that is, the decision maker with the problem. In a large organization this may not be a simple exercise. Intergally related to identification of all relevant decision makers is the task of defining the characteristics which a solution to the problem should possess. Otherwise the analyst has no way of knowing when a solution has been achieved.

Since a problem solution will be implemented only with the agreement of the decision maker, the analyst must understand the objectives of the decision maker. An objective in this context is an outcome of a course of action which the decision maker desires. Thus, from the analyst's point of view a course of action is a solution to the problem if it results in outcomes which lead to achievement of the decision maker's objectives. In this sense the objectives define the characteristics of a solution. This is not intended to suggest that such a solution is possible. The objectives defined may be in conflict with one another. Moreover, individual objectives may be unachievable under any circumstances. Hence, in accepting the decision maker's objectives as characteristics which a solution to the problem should possess, the analyst defines aspiration levels to be sought and the decision maker's objectives become those of the analyst.

#### DEFINITION OF THE SYSTEM

Whether or not a decision maker's objectives are achieved in implementing a problem solution depends, in part, upon the response of factors outside his/her control. These factors include elements of the decision maker's organization and elements of the organization's environment. Since these factors are not under the control of the decision maker, the analyst must infer how each will respond to a specific problem solution and how the inferred response will affect the achievement of the objectives. It follows that the inferences drawn will be appropriate only if the analyst has identified all pertinent factors and understands their behavior and interaction. The set of all factors within the organization and its environment which affect the achievement of objectives defines the scope of the system with which the analyst must deal. The operating characteristics of the system define the collective, interactive behavior of the elements within the system.

Identifying the decision maker's objectives and defining the system are not simple, single stage processes. The analyst's initial perception of the objectives will almost certainly change at some point during the problem solving exercise. More realistically, this perception will probably go through a series of changes as the analyst's understanding of the decision maker and the system improves. In a similar manner

the analyst's perception of the scope and operating characteristics of the system can be expected to change also.

Proper specification of objectives and definition of the system are fundamental to successful solution of the problem. Because it is easy to misinterpret both, the analyst should maintain a continuing skepticism on both counts. To reinforce, clarify or correct his interpretation of objectives and system definition requires a close working relationship between the analyst, the decision maker and others familiar with the system. These individuals are invaluable in assisting the analyst not only in formulating an interpretation of objectives and system behavior but also in validating that interpretation. Basically, the analyst formulates an interpretation of objectives and system definition through information provided by others, observation of the system and the analysis of data. The analyst validates that interpretation by communicating it back to those who understand the objectives and the system, by verification through observation of the system and by comparing his interpretation with appropriate historical data.

#### MODEL FORMULATION

The discussion thus far leads to the conclusion that a course of action is a solution to the problem if and only if its implementation leads to a system response which achieves the decision maker's objectives. Consequently, the analyst needs a vehicle for testing potential solutions. The vehicle used for this purpose must provide a means of determining or predicting how the system will respond to a given course of action in a manner that will indicate whether or not the response leads to achievement of the objectives. One such vehicle is the system itself. However, while experimentation with the system itself is sometimes feasible, more often it is impractical. In the latter case the analyst requires a surrogate for the physical system which provides an efficient, economical means for testing alternative solution proposals.

The surrogate used as a testing device is called a model and may be defined as a representation of some aspect of reality without the presence of that reality. Models and system modeling will be discussed in more detail later. For the present a model may be anything from an intuitive concept which exists only in the analyst's mind to a physical reproduction such as a pilot manufacturing plant. Further the analyst's model may be a collection of separate models each intended to deal with different aspects of the problem. While the model is only an approximation of reality it permits the analyst to deduce how the system will respond to a solution proposal and whether that response will lead to achievement of the decision maker's objectives.

#### MODEL VERIFICATION AND VALIDATION

Since the results obtained from the analyst's model will form the basis for evaluating the effectiveness of a solution proposal in achieving the objectives sought, an erroneous model may lead to wholly inappropriate solutions. It follows that the analyst should seek as much assurance as possible that the model performs in a manner which adequately describes the behavior of the physical in response to solutions proposed and yields measures of system performance

which provide an adequate basis for determining objective achievement. The process of determining whether or not the model is an acceptable descriptor of reality is called validation.

At this point in the problem solution process, model validation is a two stage process. The first stage involves the determination of whether or not the model performs in the manner intended by the analyst. Some authors refer to this process as model verification. If the model does not describe system response in the manner intended by the analyst, there is little reason to believe that it will describe that response as it will occur in the physical system. Even when the model does function in the manner intended, there is no guarantee that the response portrayed will be indicative of that of the physical system. The analyst's model is based upon his interpretation of the system and its operation. If it faithfully reflects that interpretation and that interpretation is in error, then the system behavior described by the model will also be in error. Nonetheless, as a first step toward model validation the analyst should determine whether or not the model functions in a manner which reflects his interpretation of the behavior of the system.

If the model adequately captures the analyst's understanding of the system and that understanding is free of error then the model should provide an adequate description of the behavior of the physical system. The second stage of model validation is intended to determine whether or not the model describes the behavior of the system, given that it describes the analyst's interpretation of that behavior. Hence, the analyst is, in effect, validating his interpretation of the behavior of the system. While the first stage of validation is based upon a comparison of model results with those expected by the analyst, the second stage is based upon a comparison of model and physical system results to the extent possible.

At this stage of the validation process, the results of the model may sometimes be compared with those of the system where data describing system results are available. It is important to note that such a comparison is appropriate only if the environmental conditions governing the operation of the physical system and those governing the model are the same. In addition, the analyst should exercise care in selecting data from the physical system to assure, if possible, that the system described by that data is the same as that which is being modeled. The system modeled may be one which is planned for the future or may be the system in current operation. In the former case, comparisons between model and system results may be impossible simply because no such physical system exists.

Under the most ideal conditions perfect correspondence between the model's portrayal of system behavior and the actual behavior of the system cannot be achieved. A model is only an approximation of reality. Bearing this in mind, a difference between model and system results must be anticipated. This difference is usually referred to as model error.

The validation process deals with the evaluation of the magnitude of model error rather than its existence. The magnitude of model error evident as a result of the validation effort may render the model acceptable or unacceptable for the analyst's purposes. This determination calls for the definition of levels of model error which allow the analyst to classify the model as acceptable or unacceptable. These levels

define the criteria for model validity and are, more often than not, multidimensional.

For the purpose of solving the problem at hand, the model is intended to be used as a vehicle for evaluating the effectiveness of various solution proposals. An effective solution is one which leads to achievement of the decision maker's objectives. An evaluation by a model that represents the behavior of the system poorly offers little credible information about the effectiveness of the solution proposal tested. This argument leads to the necessity for the validation process just discussed. If the validation effort leads to the conclusion that the model can be accepted as an adequate descriptor of system behavior, the analyst then has the vehicle necessary to test the effectiveness of solution proposals.

#### MODEL IMPLEMENTATION

Having an acceptable model at hand, the analyst might proceed to evaluate a variety of solution proposals until one or several are identified by the model as leading to achievement of the objectives sought. In some cases the model itself may identify the solution proposals to be tested in addition to testing their effectiveness. Although the process of defining potential solutions and testing their effectiveness may be time consuming, the process of solving the problem once the analyst has an adequate model seems straightforward. Unfortunately this is the case only in the unlikely event that the decision maker is willing to place blind faith in the analyst's recommendations. In essence then, before the decision maker accepts solution recommendations he must be convinced of the credibility of the model.

Gass and Joel (22) address the subject of user confidence in model results in detail. They define seven criteria for user confidence and suggest a scale for measuring the degree to which each criterion is met. Shannon (46) identifies seven properties which a model should possess to maximize the chance that the model will be found convincing by the decision maker. Shannon also lists ten points which should be considered in presenting model results to the decision maker.

The importance of model implementation is often overlooked. It is not surprising then that inadequate implementation is cited as a major reason for the failure of many operations research studies. Even when the analyst realizes its importance, implementation is often left largely to others who do not possess a comprehensive understanding of the model. As a result model output is often misinterpreted, input data are not properly maintained and updated and the model may be applied to the analysis of problems for which it is ill-suited. The inevitable outcome of incomplete implementation is loss of confidence in the model and finally its rejection as useless.

#### MODEL USE

A model is a means to synthetic experimentation with the physical system. The design of the experiment to be conducted includes specification of alternative solution proposals to be examined and the operating and environmental conditions assumed to prevail for the period of time considered in the experiment. The model describes how the system will behave under the scenario defined by the experimental design. The

resulting output of the model should include measures of system performance which can be used to determine whether or not the objectives sought are achieved by the solution examined. The hopeful result of repeating this process for a variety of potential solutions is the identification of at least one which satisfies the decision maker's objectives.

#### MODEL REVALIDATION

As it has been described here, a model is an approximation to reality which represents the analyst's perception of system behavior. The analyst's perception of the system is based upon his understanding of the system at the time of model development. Given that the model is adequate, the problem solution adopted should yield the results indicated by the model, at least for a short period of time. That is, model prediction should prove reliable provided that the system components do not change and that they continue to operate under the same rules of behavior and environmental conditions as those upon which the model is based. Given sufficient change in the components of the system, the behavior of those components or environmental conditions, the initial solution will become unsatisfactory, requiring a revised solution to the problem.

Realistically every specific solution to a problem must be viewed as a temporary measure. As a result the decision maker may face the same problem repeatedly. Having used the model to assist in the selection of a solution to the problem when it was initially posed, it seems reasonable to reapply the model each time a solution revision is required. However, this immediately leads to the question of continued adequacy of the model. That is, if the current solution to the problem is or may be obsolete because of change in the real world, that same change may mean that the model is no longer an adequate descriptor of the real world. Hence before applying the model to evaluation of revised solutions, the analyst should first reevaluate its validity.

Since the model should be used to evaluate potential problem solutions only if it is an adequate descriptor of system behavior, model validation should be viewed as a recurrent process throughout the life of the model. Each time the revalidation effort identifies inadequacies in the current model, the model may require structural modification. Hence model development occurs in an evolutionary fashion throughout the life of the model.

Since the need for continued revalidation can be anticipated at the outset of development of the initial model, the analyst should identify the data necessary for revalidation in the course of the model development effort. As a part of model implementation the analyst should specify how and when the data should be collected and the manner in which they should be summarized. Because the data collection effort is planned in advance, the data available for revalidation should provide the basis for continuing comprehensive validation of the model after it is implemented.

#### SYSTEM MODELING

As already mentioned, a model may be described as the representation of some aspect of reality without the presence of that reality. In this sense models have been used by man throughout recorded history. A

photograph, painting or drawing is a two-dimensional representation of the visual aspects of the reality portrayed, a sound recording is an auditory representation and a scale model is a spacial representation. While such models may be of use, the manager is usually concerned with models incorporating a higher level of abstraction. These include financial models such as a balance sheet or profit and loss statement, simulation models and mathematical models. Mathematical models represent the highest level of abstraction and simulation models the next highest level.

The purpose of any model is to describe the essential characteristics of the system portrayed. A simulation model is only one of many types of models which might be used to this end. Before discussing specific types of models and their characteristics an examination of the pros and cons of system models in general would seem in order.

The analysis of any system is generally the result of the need to better understand the behavior of the system. The manager may wish to know how the system will function under a variety of conditions, whether the system should be modified to more efficiently achieve its intended function, or simply to better understand the operational characteristics of the system. Given sufficient time and financial resources, goals such as these could be achieved through experimentation with the physical system itself. For example, one could implement successive modifications of the system in an attempt to achieve more efficient performance. To determine how the system will behave under conditions which might arise in the future, one might simply wait for those conditions to arise and observe the resulting behavior of the system. However, experimentation with the physical system is usually not economically feasible since it may seriously disrupt the overall operation of the organization of which it is a part. Simply waiting for those conditions under investigation to arise may be self-defeating in that failure to predict the impact of those conditions may lead to serious consequences with respect to the performance of the system. Thus, experimentation with the physical system is usually to be discouraged, although it is occasionally employed with satisfactory results. Consequently the analyst seeks a surrogate for the physical system which can be manipulated easily and economically.

The logical alternative to experimentation with the physical system is experimentation with a model of the system which is intended to approximate the behavior of the system. The accuracy of the approximation will usually diminish as the complexity of the real system increases. However, models which are only gross approximations to reality can yield important information about the system and often lead to system modifications which result in more efficient and effective performance of the system.

#### MODEL CLASSIFICATION

Models may be classified in a variety of ways. In this discussion the following five dimensions of model classification will be considered:

1. the manner in which the model describes the system,
2. the purpose of the model,
3. the description of the time dependent behavior of the system,

4. the description of the random behavior of elements of the system,
5. the description of system change as a discrete or continuous phenomenon,

Considering the manner in which a model represents a system, a model may be iconic, analogue or symbolic. The common property of iconic models is reproduction of a physical characteristic of the entity modeled, although the scale of the model may differ from that of the physical entity. That is, an iconic model looks like the entity modeled. A three dimensional template representing the layout of an office and a pilot manufacturing plant are two examples of iconic models.

The common feature of analogue models is replacement of a property of the physical system by a substitute property in the model. For example, a graph of the price of a stock over time is an analogue model where stock price and time are the properties modeled. In the graph time is replaced by distance along the x-axis and stock price is measured by vertical distance on the y-axis. A thermometer is another analogue model where temperature is replaced by the height of the mercury in the thermometer.

Mathematical and simulation models are classes of symbolic models. The common characteristic of symbolic models is the replacement of properties of the physical system by symbols. For example, if C is the unit cost of an item, x is the number of units purchased and y is the total cost of x units, then  $y = Cx$  is a mathematical or symbolic model relating total cost to the number of units purchased.

A model may be classified by the purpose for which it is developed. In this context a model may be descriptive or normative. A descriptive model is one which simply describes the behavior of properties of the system modeled. The output of such a model is not intended to recommend a course of action but rather to describe what happens. A model which is intended to recommend or prescribe a course of action is called a normative model. Such models are also referred to as prescriptive. More often than not a normative model results from the manipulation of or operation on a descriptive model.

Elmaghraby (14) identifies five important uses of models. First models are an aid to understanding. The very process of attempting to develop a model requires a thorough understanding of the entity modeled. Further, once developed, experimentation with the model often provides insight into relationships which govern the behavior of interacting system components. Second, models are of assistance in communication particularly in explaining interactive relationships. Third, models are frequently used for the purpose of instruction and training. Business games are a prime example of this use of models. Fourth, models play an important role in prediction. Among the most important applications of mathematical and simulation models is that of predicting the response of the system under study to a variety of conditions which may arise and decision alternatives which may be applied in the future. Finally, models provide a vehicle for synthetic experimentation with and control of the system. By varying input parameters to the model the analyst may study the behavior of the physical system, as represented by the model, under a variety of operating conditions and decision alternatives. The result of such experimentation might be the selection of the decision alternative which leads to optimum control of the system.

Models may be further categorized according to whether or not they portray the behavior of the physical system over time. A model which describes the behavior of a system throughout a given time interval is called a dynamic model. A model which portrays the behavior of a system at a single point in time is called a static model. As an illustration consider a system model which describes the mean cost of production per unit manufactured. If the model portrays the fluctuation in the mean throughout the period of production then the model is dynamic. If the model yields only the mean for the entire production period then the model is static. Quite often static models result from operation on or manipulation of dynamic models. For example, if one is concerned with the steady state behavior of a system, the steady state model may be obtained by examining the limiting behavior of the analogous dynamic model.

The fourth dimension of model classification deals with whether or not the model explicitly recognizes the presence of random variation in the system modeled. Very few real world systems, if any, are free of the influence of unpredictable or random behavior of the elements of the system and its environment. A deterministic model is one which does not recognize the random component of such behavior. While a system may be influenced by random behavior, the impact of that behavior may be sufficiently slight that the random component may be ignored for practical purposes. In such cases a deterministic model is entirely appropriate. A model which explicitly captures the random components of system behavior is called a probabilistic or stochastic model.

The final dimension for model classification deals with the manner in which the model represents change within the system. If a model describes change in the status of the system as occurring only at isolated points in time, the model is called discrete. On the other hand, if the model treats change as a continually occurring phenomenon then the model is called continuous. Simple queueing systems are representative of discrete change systems in that the status of the system, measured by the number of units in the system or the number waiting for service, may change only at those points in time at which either an arrival or a service takes place. If, however, the measure of system status is the percentage of time the system is busy, then the system must be considered one of continuous change. While the process modeled may be continuous, a discrete model may provide an adequate approximation to the behavior of the system.

By their nature mathematical and simulation models are symbolic. While both types of models may be either descriptive or normative, more often than not simulation models are descriptive. A review of the literature on modeling would indicate that static mathematical models are more prevalent than their dynamic counterpart. Conversely dynamic simulation models are reported more frequently than static simulation models. A wide variety of deterministic and stochastic mathematical models are reported in the literature, the type of model being dependent upon the nature system modeled. However, simulation modeling is applied to the analysis of stochastic systems more frequently than it is to deterministic systems. Finally, change is treated as a discrete phenomenon more often than a continuous phenomenon in the case of both mathematical and simulation modeling. While continuous mathematical and simulation

models are certainly not uncommon, continuous change systems are frequently approximated by discrete models.

#### SIMULATION AND/VS. MATHEMATICAL MODELS

Mathematical and simulation models are used to describe the interactive behavior of a system and its environment under prescribed conditions of operation. The input to either type of model usually defines the operating conditions assumed and the decision alternatives considered while the output of the model describes the resulting response of the organization and its environment. Model output usually includes measures of system or organization performance such as profit, cost, level of service, sales volume, product quality, etc. Through analysis of the measures of performance associated with a given decision alternative one can often determine whether or not that decision alternative will lead to achievement of the decision maker's objectives. Whatever the measure of performance, an important feature of mathematical and simulation models is their ability to provide quantitative information, providing a basis for assessment and comparison of alternative decision strategies.

Mathematical Models. Mathematical models are characterized by one or a series of equations relating the measure(s) of system performance to the variables which affect system performance, and equations or inequalities which define constraints on the range and character of values which the variables of the system may assume. The variables of the system may be classified as decision variables, variables under direct control from within the system, and variables which cannot be directly controlled. Uncontrollable variables may be further classified as those which are not influenced by other variables, independent variables, and dependent variables whose values are determined by the values of the decision variables, the independent variables and other dependent variables.

The equations defining a mathematical model usually attempt to describe system behavior in aggregate form. To illustrate, consider a facility which serves customers in the order in which they arrive. Suppose that the time between successive customer arrivals is an independent exponential random variable with mean arrival rate  $\lambda$  and that the time required to provide the service is also an independent exponential random variable with mean service rate  $\mu$ . Under appropriate assumptions the steady state mean total time a customer spends in the system is given by  $W_t$  where

$$W_t = 1/(\mu - \lambda) \quad (1)$$

The equation for  $W_t$  is a mathematical model for the steady state mean total time a customer spends in the system as a function of  $\lambda$  and  $\mu$  for  $\mu > \lambda$ . This model describes steady state mean total time in the system in aggregate form in that it makes no attempt to deal with the behavior of individual customers.

Mathematical models have long been a basic tool of the physical sciences and engineering. During World War II such models were used as a basis for the analysis of military operational problems. With the successful experience gained during World War II, similar modeling efforts were applied to the analysis of organizational systems after the war, leading to the emergence of the discipline of operations research. This discipline is also referred to as management science, systems analysis and systems engineering, although the

latter two terms are also applied to other disciplines as well. More recently mathematical modeling has been successfully applied to problems in agriculture, forestry, sociology, psychology and education.

Simulation Models. The distinction between a mathematical and a simulation model is not one which may be definitively drawn. In fact, there are many who use the terms synonymously. However, the distinction recognized here may be illustrated by a simple example. Consider again the service facility example cited in the preceding section. A simulation model of this system would attempt to track the behavior of the system on a customer by customer basis, in much the same manner as would an observer of the physical system who attempts to estimate the mean total time a customer spends in the system. The difference lies in the manner in which the times between successive customer arrivals and customer service times are generated. In observing the physical system, customers define the points in time at which they arrive, and thereby customer interarrival times, and the time to complete the service of a customer is generated by the service facility. The observer merely records when each customer enters and exits the system, the difference between the exit and entry times and computes the average of these differences after observing a long series of customers. The simulation model also "observes" the system and carries out the same computations as the observer of the physical system. However, in addition the simulation model defines or "generates" the times of arrival of customers and the times required for the service of each. The method of generation of these random variables involves synthetic sampling from the appropriate probability distribution. The synthetic sampling technique used is called the Monte Carlo Method.

In the waiting line example the simulation model was described as attempting to mimic the event by event, activity by activity, customer by customer behavior of the system. This approach to modeling the behavior of systems is a common denominator of what is generally regarded as discrete event simulation.

The distinction just drawn between a mathematical model and a simulation model is somewhat idealized. In a simulation model one often employs a mathematical model to describe phenomena which influence the behavior of entities which are in turn "tracked" by the model on an individual basis. In the service facility example one of the activities performed by the simulation model is generation of exponential times between customer arrivals. In applying the Monte Carlo method to do so, the simulation model employs a mathematical model for the exponential probability distribution.

The fundamental operation of a time dependent simulation model is shown graphically in Figure 2. As Figure 2 illustrates each event initiates a reaction by the system. The system reaction may include the initiation or termination of activities. Activity in the system leads to changes in the status of system. The model then moves forward in time to the next event and the process repeats until the simulation experiment is completed.

Some authors trace the origin of simulation modeling to the early sampling experiments of W.S. Gosset, who published under the name Student (48). However, the foundations of modern simulation methodology are usually attributed to the works of Ulam (50) and von Neumann (51). Their work, conducted in the late

1940's, involved the analysis of nuclear-shielding problems through a technique which they termed "Monte Carlo Analysis." However, it was not until the early 1950's, with the arrival of high-speed computing equipment, that the horizons for application of simulation were broadened to the point where it became available and practical for the analysis of engineering, business, and behavioral problems. Since that time simulation has been applied in such diverse areas as:

- The Analysis of Air Traffic Control Systems
- The Analysis of Large-Scale Military Operations
- Communication Systems Analysis
- Job-Shop Scheduling
- Analysis of the U.S Economy
- Production Planning and Inventory Control
- Determination of Manpower Requirements
- Instructional Modeling for Higher Education
- Energy Supply and Demand Analysis
- Competitive Market Analysis
- Housing Market Analysis
- Transportation Planning
- Financial Investment Analysis
- Man-Machine Interface
- Corporate Planning

Advantages of Mathematical and Simulation Models.  
 The advantages of mathematical and simulation modeling are viewed here in relative terms since each is the most probable alternative to the other. The principle advantages of mathematical modeling lie in the interpretation of model output and the efficiency of execution on a digital computer. Given that the pertinent behavior of a physical system can be

captured with equal validity either through a mathematical model or a simulation model, the mathematical model will usually execute more quickly on a digital computer. This advantage accrues to a mathematical model since models of this type deal directly with the aggregate behavior of the system while a simulation model tends to focus on the behavior of individual entities; computing aggregate measures of system performance, for example, by averaging individual component behavior. The second advantage, clarity of output interpretation, arises when the system modeled is subject to the influence of unpredictable or random variation and when that component of system behavior is to be captured by the model. Since a mathematical model deals directly with aggregate measures of system behavior such as means, medians, variances or quantiles, the output of the model for a given input data set is one or more constants. That is, if the scenario modeled is repeated using the same input data set the output for the scenario will not change. However, in dealing with a stochastic system a simulation model generates specific values for the random variables which influence system behavior, the aggregate measures of performance being computed at the end of the simulation experiment or scenario. Output measures of performance for a simulation experiment are then functions of the values of the random variables generated in the course of the experiment. If the same scenario is again analyzed through simulation a new set of values of the random variables will be generated which, although possessing the same probabilistic characteristics, will have different numerical values than the sequence generated in the first simulation experiment. Since the measures of

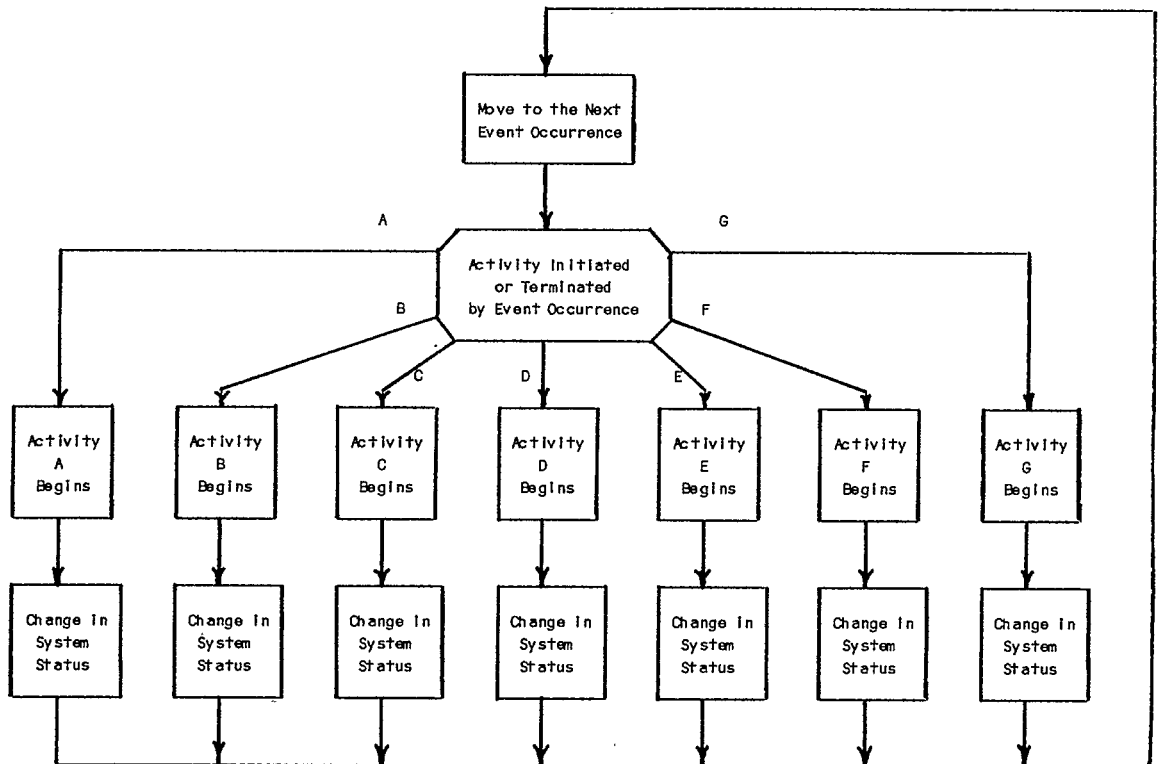


Figure 2: Fundamental Operation of a Time Dependent System Simulation Model



system performance from the two simulation runs are functions of different sequences of values of the random variables, differences between the numerical values of the output variables for the two simulation runs are inevitable. This difference in results occurs because the output of the simulation run is composed, at least in part, of random variables, clouding the interpretive value of the output. Proper interpretation of such output usually requires the application of statistical techniques while such is not the case when a mathematical model is used.

One often hears or reads phrases such as "the only game in town," "the court of last appeal" and "when all else fails, simulate," applied in reference to simulation modeling. Such references relate to one of the principle advantages of simulation modeling - the versatility of its application. At least in theory, any physical system which can be understood can be simulated. Many systems which have not been successfully modeled mathematically may be simulated with little difficulty. In general the level of mathematical sophistication necessary to model a system mathematically exceeds that required for development of the corresponding simulation model; frequently the difference is substantial. Thus a system which is intractable from a mathematical point of view, at least as far as the analyst is concerned, may well be within his grasp when approached through simulation.

The importance of the decision maker's confidence in the problem solution proposed was discussed in describing the stages of the systems analysis process. Since the model is the vehicle used to evaluate solution alternatives, the confidence the decision maker has in the solution proposed will usually depend, at least in part, upon his confidence in the model which in turn may depend upon his ability to understand how the model works. Typically managers have a limited knowledge of mathematics. As a result the task of explaining how the model works and convincing a manager that the equations included in a mathematical model do in fact describe the behavior of the physical system may be a challenging one indeed. This task is not usually as difficult in the case of a simulation model. A simulation model traces the behavior of the system on an event by event basis, appropriately modifying the status of the system as each event occurs. This can be demonstrated for the manager by displaying each event as it occurs and the status changes which take place coincident with each event. While the manager may not understand "how" the model works, the display of event by event behavior is often convincing evidence that it "does" work.

In summary, the advantages of mathematical modeling are efficiency of execution on a digital computer and simplicity in interpreting model output. However, efficiency of computer execution may not be a significant consideration if computer time is a resource in abundant supply or if the run time of the simulation experiment is not significant. The principle advantages of simulation modeling lie in the variety of systems which may be successfully modeled, the availability of the technique to professionals who do not possess a strong background in mathematics or probability and the relative ease in demonstrating what the model does. However, a reasonably strong background in statistical analysis may be required for proper interpretation of the output of a simulation model.

#### MODEL TRANSLATION

With the exception of very simple models, model

execution is carried out on a computer. A computer is used for one of two reasons. First, a computer possesses the capability to carry out a series of complex computations, either logical or mathematical, at speeds far beyond human capability. Second, a computer possesses the capability to store and accurately retrieve information. While the limits of the capacity of the human mind to store and retrieve information have not been precisely defined, it is well known that conscious human recall is subject to significant error particularly where large volumes of data are concerned.

Translation of the model into a medium which can be interpreted by the computer is the purpose of a programming language or program package. For the purpose of this discussion a distinction needs to be drawn between a programming language and a program package as the terms will be used henceforth. A programming language is a well defined set of commands or statements which the analyst may use to define the logic and calculations necessary to execute operation of the model. Once the appropriate commands are read by the computer, the analyst need only define the data set necessary to describe the scenario to be modeled, feed that data set to the machine and execution of the model takes place. A programming language offers the analyst the ability to construct and translate a wide variety of models for execution on a computer.

A program package includes the code or sequence of commands necessary for model execution allowing the analyst to bypass the programming effort otherwise necessary. The program package requires definition of input data only for model execution. Because the code is already incorporated in a program package, the user may possess no knowledge of programming nor is he required to acquire such knowledge. However, the fact that the code is predefined implies that the model is limited to description of the behavior of those systems which fall in the class for which the package was developed. For example, specific packages have been developed for quality control, inventory, facilities planning, materials handling and network systems to name but a few. While an inventory control program package may describe the behavior of a large variety of inventory systems it would be of little use to the analyst concerned with quality control or a communication network. On the other hand, a programming language might be used to develop a model for any of these applications and many others as well.

#### LANGUAGE CATEGORIZATION

Programming languages are divided into two categories for the purposes of this discussion, general purpose languages and special purpose languages. General purpose languages are designed to provide a translation mechanism for a broad range of problems and their solution, extending well beyond the generic applications which the analyst has in mind. Examples of general purpose languages are FORTRAN, PL/I and BASIC. A special purpose language is designed to provide a translation mechanism for a broad range of problems also, but in this case the range is limited to a specific generic class of problems. Of particular interest here are simulation languages. A simulation language may be used to translate a virtually limitless number of simulation models for execution on a computer but in general would be of little value in mechanizing an accounting system, maintaining and retrieving personnel information or

solving a linear programming problem.

The principle advantages offered by general purpose languages are flexibility in the design of the computer program used to execute the model, flexibility in the presentation of results from the model analysis, more efficient utilization of memory, and reduced running time. Simulation languages usually lead to reduced programming and debugging time and because of their structure actually assist the analyst in designing the model. Perhaps the most significant advantage of general purpose languages lies in the fact that if the analyst knows anything about computer programming he is probably more familiar with one of the general purpose languages than with a simulation language. Thus the advantages of reduction in programming time offered by a simulation language may be offset by the additional expenditure of time required to learn the language. On the other hand, if the cost of the learning experience is spread over a variety of anticipated future applications the payback on the investment of time may be handsome indeed. Shannon (46) presents an excellent discussion of general purpose languages and simulation languages including an elaboration of the relative advantages of each. Sargent (44) compares the characteristics of five widely used simulation languages.

Just as a simulation language is appropriate for only a small portion of the applications for which a general purpose language may be used, a simulation or model package is designed to deal with a relatively small portion of the applications for which a simulation language might be used. For example, consider the problem 1) of simulating the movement of air carriers in and out of a major air terminal throughout a one year period and 2) of simulating the behavior of demand for a particular product for a one year period. In either case one might choose to translate the model through a general purpose language or through a simulation language. Further the analyst may find that models for both systems have already been developed and coded (a special purpose package), eliminating the need for extensive programming. However, it is unlikely that he would find a single package which could be used to successfully describe the behavior of both systems. Hence a special purpose package is developed to model the behavior of a specific class of systems only. A special purpose package may be coded in one of a variety of general purpose or simulation languages.

The principle advantage of a special purpose package is reduced programming time. In many cases the analyst need only define the input data required by the package to define the system modeled. Different system configurations may be modeled simply by altering the input data. Despite their convenience, the decision to use a special purpose package should be taken with caution. All such packages include assumptions about the manner in which the systems modeled operate. All too frequently critical assumptions concerning the operation of the system are not expressed in the package documentation. Failure to recognize the assumptions underlying the package often leads to results which bear little relationship to the system under study or, worse still, to serious error in interpreting the behavior of the system which the package purports to describe.

#### REFERENCES

1. Ackoff, R. L., "Unsuccessful Case Studies and Why", *Oper. Res.*, Vol. 9, 1960, pp 259-263.

2. Ackoff, R. L., "Scientific Method: Optimizing Applied Research Decisions", New York: Wiley, 1962.
3. Banks, J. and J. S. Carson, "Discrete-Event System Simulation", Englewood Cliffs, NJ, Prentice-Hall, 1984.
4. Bonini, C. P., "Simulation of Information and Decision Systems in the Firm", Chicago: Markham, 1967.
5. Boulding, K., "General Systems Theory", *Mgt. Sci.*, Vol. 2, 1956, pp. 197-208.
6. Bratley, P., B. L. Fox and L. E. Schrage, "A Guide to Simulation", New York: Springer-Verlag, 1984.
7. Bulgren, W. G., "Discrete System Simulation", Englewood Cliffs, NJ: Prentice-Hall, 1982.
8. Carter, G. and E. J. Ignall, "A Simulation Model of Fire Department Operations: Design and Preliminary Results", The New York City Rand Inst., New York, NY, Tech Report No. R-632-NYC, 1970.
9. Churchman, C. W., "An Analysis of the Concept of Simulation", Symposium on Simulation Models (A. C. Hoggatt & F. R. Balderston, Eds.), Cincinnati: South-Western, 1963.
10. Churchman, C. W., R. L. Ackoff and E. L. Arnoff, "Introduction to Operations Research", New York: Wiley, 1959.
11. Churchman, C. W., "Prediction and Optimal Decision", Englewood Cliffs, NJ: Prentice-Hall, 1961.
12. Coplin, W. D. (Ed.), "Simulation in the Study of Politics", Chicago: Markham, 1968.
13. Dutton, J. M. and W. H. Starbuck, "Computer Simulation of Human Behavior", New York: Wiley, 1971.
14. Elmaghraby, S. E., "The Role of Modeling in IE Design", *Industrial Engineering*, Vol. 19, No. 6, June, 1968, pp. 11-15.
15. Emshoff, J. R. and R. L. Sisson, "Design and Use of Computer Simulation Models", New York: MacMillan, 1970.
16. Fishman, G. S., "Principles of Discrete Event Simulation", New York: Wiley, 1978.
17. Forrester, J. W., "Industrial Dynamics", Cambridge, Mass.: M.I.T. Press, 1961.
18. Forrester, J. W., "Urban Dynamics", M.I.T. Press, 1969.
19. Franta, W. R., "The Process View of Simulation", New York: Elsevier North Holland, 1977.
20. Gass, S. I., "Documenting a Computer-Based Model", *Interfaces*, Vol. 14, No. 3, May-June, 1984, pp. 84-93.
21. Gass, S. I., "Evaluation of Complex Models", *Comput. Op. Res.*, Vol. 4, No. 1, 1977, pp. 27-35.

22. Gass, S. I. and L. S. Joel, "Concepts of Model Confidence", Oper. Res. Div., Cent. for Applied Math., Nat. Bur. of STDS., Washington, DC, Report NBSIR 80-2053, June, 1980.
23. Geisler, M. A., "The Use of Man-Machine Simulation for Support Planning", Naval Res. Logistics Quart., Vol. 7, 1960, pp. 421-428.
24. Guetzkow, H., "Simulation in International Relations, Developments for Research and Teaching", Englewood Cliffs, NJ, Prentice-Hall, 1963.
25. Guetzkow, H., P. Kotler and R. L. Schultz, "Simulation in Social and Administrative Science", Englewood Cliffs, NJ: Prentice-Hall, 1972.
26. Hermann, C. F., "Crisis in Foreign Policy: A Simulation Analysis", New York, Bobbs-Merrill, 1969.
27. King, G. W., "The Monte Carlo Method as a Natural Mode of Expression in Operations Research", J. Oper. Res. Soc. Amer., Vol. 1, 1963, pp. 46-51.
28. Kiviat, P. J., R. Villanueva and H. M. Markowitz, "Simscrip II.5 Programming Language", E. C. Russell (Ed.), Los Angeles: CACI, Inc., 1973.
29. Kleijnen, J. P. C., "Statistical Techniques in Simulation, Part I", New York: Marcel Dekker, 1974.
30. Kleijnen, J. P. C., "Statistical Techniques in Simulation, Part II", New York: Marcel Dekker, 1974.
31. Kresge, D. T. and P. O. Roberts, "Techniques of Transportation Planning: Systems Analysis and Simulation Models", Washington, DC: Brookings Inst., 1971.
32. Law, A. M. and W. D. Kelton, "Simulation Modeling and Analysis", New York: McGraw-Hill, 1982.
33. Mihram, A. G., "Simulation: Statistical Foundations and Methodology", New York: Academic, 1972.
34. Morris, W. T., "On the Art of Modeling", Mgt., Sci., Vol. 13, No. 12, 1967.
35. Mullin, J. K., "Simulation vs. Analysis", Commun. of the ACM, Vol. 26, No. 8, Aug., 1983, pp. 570-571.
36. Orcutt, G. H., M. Greenberger, J. Korbel, and A. M. Rivlin, "Microanalysis of Socioeconomic Systems: A Simulation Study", New York: Harper & Row, 1961.
37. Pegden, C. D., "Introduction to Siman", Systems Modeling Corp., State College, PA, 1982.
38. Pomerantz, A. G., "Predict Your System's Fortune Use Simulation's Crystal Ball", Computer Decisions, Vol. 2, 1970, pp. 16-19.
39. Pritsker, A. A. B., "Models Yield Keys to Productivity Problems, Solutions", Ind. Engr., Vol. 15, No. 10, Oct., 1983, pp. 83-87.
40. Pritsker, A. A. B., "Introduction to Simulation and Slam", New York and West Lafayette, IN: Halsted, Wiley & Systems Publishing Co., 1984.
41. Ratcliffe, L. L., B. Vinod, and F. T. Sparrow, "Optimal Prepositioning of Empty Freight Cars", Simulation, Vol. 42, No. 6, June, 1984, pp. 269-275.
42. Reitman, J., "Computer Simulation Applications", New York: Wiley, 1971.
43. Richardson, J., "Models of Reality: Shaping Thought and Action", Paris, France: Lomond, 1984.
44. Sargent, R. G., "An Introduction to the Selection and Use of Simulation Languages", Proceedings No. 268, Modeling and Simulation of Arlonics Systems and Command, Control and Communications Systems, Paris, France, 1977.
45. Schriber, T. J., "Simulation Using CPSS", New York: Wiley, 1974.
46. Shannon, R. E., "Systems Simulation: The Art and The Science", Englewood Cliffs, NJ: Prentice-Hall, 1975.
47. Shannon, R. E. and W. E. Biles, "The Utility of Certain Curriculum Topics to Operations Research Practitioners", Oper. Res., Vol. 18, 1970, pp. 741-745.
48. Student, "On the Probable Error of a Mean", Biometrika, Vol. 6, No. 1, 1908.
49. Taha, H. A. and F. M. Stephen, "Modeling with Imperfect Data: A Case Study Simulating a Biological System", Simulation, Vol. 42, No. 3, Mar., 1984, pp. 109-115.
50. Ulam, S., "On the Monte Carlo Method", Proc. of the Second Symposium on Large-Scale Digital Calculating Machinery, 207, 1951.
51. Von Neumann, John, "Various Techniques Used in Connection with Random Digits", Nat. Bur. Std. Math. Ser., Vol. 12, 1951, pp 36-38.
52. Wagner, H. M., "Principles of Operations Research", Englewood Cliffs, NJ: Prentice-Hall, 1969.
53. Zeigler, B. P., "Theory of Modeling and Simulation", New York: Wiley, 1976.