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AN APPLICATION OF OPTIMIZATION-BY-SIMULATION TO DISCRETE VARIABLE SYSTEMS

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ABSTRACT

An algorithm is being developed for optimization of discrete variable stochastic systems that are modeled by computer simulation. The algorithm is basically a complex type search method that interacts with the simulation model. The capabilities built in the procedure allows it to take into account the stochastic nature of the responses of the simulation model and arrive at a reasonable solution with a certain level of confidence. This paper reports preliminary results obtained from this procedure which is undergoing its final stages of development. An application of this procedure to optimization of operation of a manufacturing cell is also presented.

1. INTRODUCTION

Computer Simulation is often employed to analyze stochastic and complex systems for a given decision policy. Also the outcomes of various policies are usually compared through multiple comparisons or design of experiments employing computer simulation as the means of experimentation. However, if a decision policy is defined as a set of values for several decision variables of the system, simulation can be used as a means of optimization of such systems. In other words, simulation model can be used as an objective function of an optimization model for which the decision variables represent the variables that are to be optimized. To clarify this, consider following examples.

The manufacturing cell shown in Figure 1 consists of three machining centers that perform their corresponding operations on the part that goes through these centers in series [6]. Each machining center consists of a number of similar machines in parallel. In addition, there are a number of robot manipulators that transfer parts from one machine to the next. A buffer stock of a given size is also provided between two machining centers to facilitate the smoothness of the flow of parts through this system. Due to the randomness of the operation times and other complexties involved, this system can best be analyzed through computer simulation. For instance, its production rate can be evaluated for a given number of machines from each type. However, more useful and practical results can be obtained if one could determine the optimal number of machines from each type to employ to maximize the production rate or to minimize the unit production cost under system's constraints.

As another example consider a two way highway one lane of which is blocked in a section of length L for construction purposes [11]. As a result cars from both directions have to take turns for using the open lane. This is accomplished by installing traffic lights at the two ends of the construction zone. This system too can effectively be modeled by computer simulation. For this system one might be interested in minimizing the overall delay per car. The decision variables to be optimized in this system are the lengths of green light in each direction.

In optimizing systems such as above, one is usually interested in finding the optimum values for the control variables such that the total operation of the system is optimized in terms of a given measure of effectiveness. In order to optimize such systems a systematic procedure in the form of an algorithm is needed to be capable of interacting with the simulation model and obtaining the optimal values for the decision variables. One of the main problems in developing such algorithms is the stochastic nature of the responses of simulation models. Since the result of simulation is almost always noise corrupted, comparison of the alternative decision points, which is the building block of almost all search methods, is not always reliable for simulation output analysis.

Several algorithms developed in the literature for optimization by simulation that consider the noisiness of the responses can only be applied to systems whose decision variables can be represented by continuous variables (e.g. SAMOPT by Azadivar [2]). This type of problems include the highway traffic example mentioned earlier, but not the manufacturing system problem, because the decision variables for the latter are discrete. In some cases of discrete variable simulation-optimization problems, rounding up of the continuous optimum values for decision variables have been used [3], [4]. This procedure is not recommended in most cases because it can lead to a point which may or may not represent the real discrete optimum [7].

An algorithm has been proposed for optimization of discrete variable simulation models and has been tested with known stochastic functions [1]. The results from these tests have been very encouraging. In this paper we describe this procedure briefly and apply it to a steady-state simulation model.

2. FORMULATION OF THE PROBLEM

Consider the problem of minimizing a system response f(X) subject to a set of constraints as follows :

Minimize $f(X)$

subject to $a_i \le x_i \le c_i,\ i = 1,2,..,n$ (2.1)

$g_j(X) \le b_j,\ j = 1,2,..,m$ (2.2)

$h_k(X) \le d_k,\ k = 1,2,..,p$ (2.3)

where X is the vector of decision variables, x_i's i = 1,2,....,n and constraints $a_i \le x_i \le c_i$ i = 1,2,...,n in (2.1) are the upper and lower bounds on each variable. These bounds arise from prior knowledge of the system, equipment limitations, and specifications on the product or process. The constraint $g_j(X)$, j = 1,...,m in (2.2) are the explicitly known deterministic functions in terms of the independent variables of X. The constraints $h_k(X)$ in (2.3) are implicitly unknown stochastic functions or other responses from the system that must be evaluated via computer simulation.

3. THE SEARCH PROCEDURE

The simplex method as proposed in [12] and [13] is a search method in which several points (usually n+1 where n is the number of variables) are selected to form a simplex. The objective function is evaluated at all these points and the point with the least desirable value for the objective function is dropped and replaced by a new point according to a particular rule. This process is repeated and as a result the simplex moves steadily towards the region containing the optimum. This search method is applicable to unconstrained optimization problems.

A modification to the simplex method has been proposed by Box [10] to give it the capability of solving constrained problems as well. The modified simplex is called Complex (Constrained-Simplex) method and handles constraints by the use of a flexible simplex of more than n+1 vertices, which can expand or contract in any or all directions. The complex method of Box for continuous variable problems constrained by bound and explicit constraints, can be summarized as follows:

The vertices of initial simplex are randomly generated by the use of pseudo-random numbers and the boundaries of each variable. Each vertex is evaluatd and the worst point is replaced by a point obtained by its reflection through the centroid of the remaining points. If this trial point is also the worst, it is moved halfway towards the centroid of the remaining points. The above procedure is repeated until at least one constraint is violated. If the trial vertex violates a bound constraint on a variable, that variable is re-set to an infinitesimal value inside the appropriate limit. If an explicit constraint is violated, the trial point is moved halfway towards the centroid of the remaining points. This process continues until all points of the simplex collapse into its centroid.

To apply the complex method to the discrete variable simulation models two major points have to be taken into consideration. First, provisions have to be made to assure the points in the original simplex and subsequent vertices consist of integer values for the decision variables. The second major concern is the procedure by which the responses of the simulation at each point are compared. Note that the responses of simulation models are often stochastic. Thus, if a simulation run at one point yields a better response than another point, this does not necessarily mean that the first point is better than the second. It is possible for the variability of the responses at these two points to lead to erroneously preferring one point to another. In the present algorithm [1] these two points and the stopping rules have been treated as explained in following three sections.

3.1 Generation of Discrete Simplex and Vertices

Rather than selecting the initial simplex randomly, exploratory runs are made through a uniform search and a suitable initial point is selected. Using this point and boundaries of each variable, 2n additional points are located. Each pair of these points are selected as almost halfway between the initial point and the boundaries of each variable at each side of the initial point. Using this method of selection of the first simplex helps in making sure that the points are discrete.

After the first simplex has been constructed the subsequent points are found following the rules given by the complex method. However, if a point is not discrete, following the method suggested by Beveridge and Schechter [9] the closest discrete point is selected. Note that this rounding is not the same as solving the problem for the continuous case and then rounding the result. These points are all intermediate points and their function is to move to simplex along a desired direction. In the final stage when a point is to be selected as the optimum, more thorough investigation of the neighborhood of the point is conducted.

3.2 Comparison of Decision Points

The search algorithm for the discrete optimization problem requires that the responses of the system be evaluated at different points and compared. The results of comparison is supposed to reveal the worst point. However, if the response is noise corrupted, the distinction of the worst point is not easy. Since due to the stochastic nature of the problem selecting one point over others is always accompanied with uncertainty, the second best thing to certainty would be to select among two points with a high level of confidence.

Suppose that in addition to the mean response at each point, the confidence intervals on the mean are also evaluated around the mean at a given level. Obviously the range of these intervals will be narrower for longer simulation runs. Let the lower and upper confidence limits on the response of the system at point X at β confidence level be represented by ZL(X) and ZU(X). Now if after comparing the responses at two points, it is found that $ZU(X_1)<ZL(X_2)$. we can almost be sure (with a certain probability) that point X_1 is better than point X_2. If this conclusion is made based on a relatively short simulation run there will not be any need to continue running the model further for the unfavorable point. However, if such a conclusion cannot be reached, running the model for a longer period that results in narrower confidence intervals may provide a better chance for such conclusion.

From the above discussion it is seen that entering confidence intervals into the optimization process results in two major advantages. First, it makes the results of comparisons among the responses of the system at different points more reliable. Secondly, the procedure causes the allowable simulation runs or the computer time be spent more economically.

For constructing confidence intervals, the modified batch means method suggested in [5] and further extended in [1] is employed.

3.3 Stopping Rules

In this algorithm there are two stopping rules.

a) The algorithm stops when all points in the simplex collapse into one of the vertices. This point is then identified as the optimum. There is an option available to the user to request the evaluation of the neighborhood of this point for a possible better optimum.

b) It is possible that at the final stages of the process all points of the simplex be tried for the reflection without success. In that case the best point in the current simplex is identified as the optimum. The option of the searching the neighborhood of this point is also available.

4. APPLICATION TO A ROBOTIC MANUFACTURING SYSTEM

To demonstrate the capabilities of the algorithm it is applied to a robotic manufacturing cell consisting of several robot manipulators and three work stations (roughing lathes, finishing lathes and grinders) in series. Each work station consists of several similar machines each of them capable of performing the required operation independently (see Figure 1).

The workpieces are introduced into the system by the conveyor which can be adjusted to any speed. Each part is first picked up by a robot and is loaded onto a lathe for rough sizing. During the time the workpiece is on the lathe, it does not require the robot's attention. When this operation is completed, the robot returns to the lathe and unloads the workpiece , and loads it onto a second lathe for finishing. Because the roughing operation is faster, intermediate storage is provided between the two lathes. This means that if none of the finishing lathes are available, the robot will store the workpiece in the buffer area. As before, the robot's attention is not required during the finishing operation. Upon completing the finishing operation, the robot unloads the workpiece from the lathe and moves it to a grinder and returns its attention to other machines. Upon completing the grinding operation, the workpiece is automatically unloaded onto a conveyor which removes the completed part from the manufacturing cell.

The robot manipulators are employed for five different duties: to load the first lathe; to unload this machine and load the second lathe; to unload the first lathe and put the wrokpiece in the buffer; to remove parts from the buffer storage and load the finishing lathe; to remove the parts from the finishing lathe and load the grinder. Under normal conditions, several workpieces are in the system at the same time. The robots respond to requests for services based on a priority list for the operations. For example, if more than one workpiece is waiting to be moved, the priority will be assigned in the order of finishing lathe, roughing lathe, buffer storage and input conveyor respectively. Priorities are assigned in a way to make the flow of workpieces more efficient. For instance if the robot unloads the roughing lathe prior to the finishing lathe, the part will become a burden on the limited capacity of the buffer storage.

The objective for this optimization problem is to select the optimum values of controllable variables to minimize the unit production cost. There are four discrete valued decision variables ; the number of robot manipulators, and the number of machines in each work station. The size of buffer and the speed of robots and machines are assumed to be constant. The variables and parameters of the system are defined as follows:

* x_1 = number of roughing lathes
* x_2 = number of finishing lathes
* x_3 = number of grinders
* x_4 = number of robot manipulators
* Buffer size : 30
* Service time of roughing lathe: exponentially distributed with a mean of 3 min
* Service time of finishing lathe: exponentially distributed with a mean of 4 min

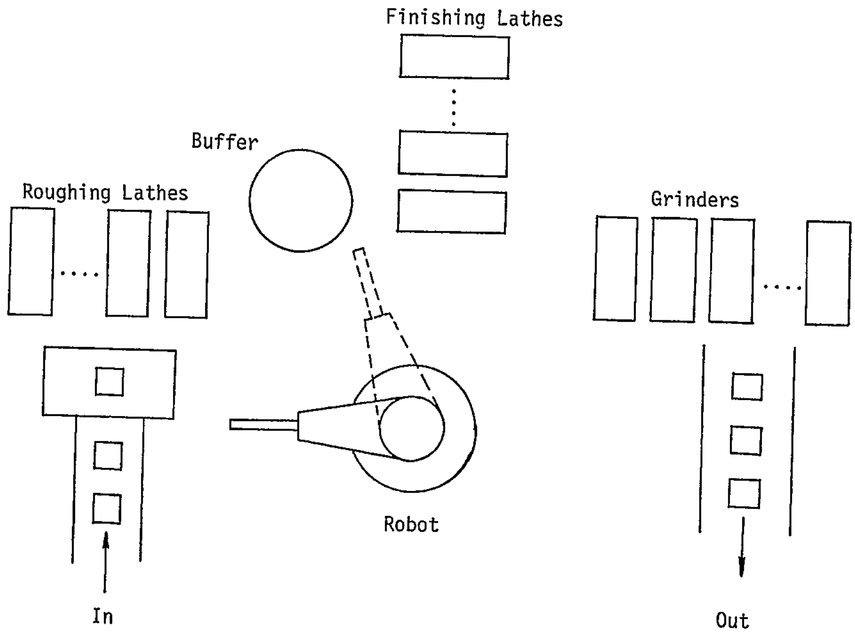

Figure 1: An Example of A Robotic Manufacturing Cell

* Service time of grinder: exponentially distributed with a mean of 2 min
* Transport time by robot: exponentially distributed with a mean of 0.5 min

The operating cost for each machine or robot is selected as follows;

* Operating cost of roughing lathe = $ 40.00/hour
* Operating cost of finishing lathe = $ 50.00/hour
* Operating cost of grinder = $ 20.00/hour
* Operating cost of robot = $ 90.00/hour

The unit production cost is calculated as follows;

* Unit production cost = Total operating cost/Number of units produced

4.1 Unconstrained Optimization

The unconstrained optimization problem for the above manufacturing system is represented as

$$\text{minimize} \quad y = f(X)$$

$$\text{subject to} \quad 1 \le x_1 \le 10, \quad 1 \le x_2 \le 10, \quad 1 \le x_3 \le 10, \quad 1 \le x_4 \le 3$$

Here $f(X)$ is the noise corrupted response of the simulation model for a set of values for decision variables represented by the vector X. To optimize the problem, the algorithm was run with the simulation program which is written in SLAM simulation language. The algorithm stopped after evaluating 35 points. All points in the last simplex were used for reflection, but no better point was found. Then according to the second stopping rule given in 3.3.b the optimal solution was obtained as

x_1 = 5 roughing lathes
x_2 = 5 finishing lathes
x_3 = 5 grinders
x_4 = 3 robots
Optimal unit production cost = $ 11.65
Confidence interval on the optimum value = [11.57, 11.72]

4.2 Constrained Optimization

The general constrained optimization problem may consist of bound, explicit and implicit constraints. For this problem, in addition to the bound constraints an explicit constraint (g_1) and an implicit constraint (h_1), are considered. The explicit constraint is an analytical function of the decision variables and represents the limit on the available budget to spend on all machines. The implicit constraint is imposed on the acceptable minimum production rate which is a response of the simulation model.

$$g_1 = 2000x_1 + 3000x_2 + 1000x_3 + 5000x_4 \le 45000$$
$$h_1 = \text{Expected production rate per hour} \ge 50 \text{ units} \quad (4.1)$$

Due to the stochastic nature of h_1, the above representation of the constraint for h_1 is not mathematically correct, because the left hand side is a random variable while the right hand side is a deterministic value. This constraint can better be stated in conjunction with a probability assigned to the possibility of violation of the constraint. This can be accomplished by expressing it as

$$P[\, h_1 \ge 50 \,] = 1 - \beta \quad (4.2)$$

where β $(0 < \beta < 1)$ is the risk that the decision maker is willing to accept for violating the constraint. The constraint in (4.2) is now in a form that can be handled by the optimization process. We write this constraint as:

$$HL_1 \ge 50 \quad (4.3)$$

where HL_1 is the lower limit on the response h_1 with β=0.1 confidence level.

After 29 experimental search points the algorithm stopped when all vertices of the last simplex coincided. This satisfied the first stopping rule given in 3.3.a. The optimal solution obtained is

x_1 = 5 roughing lathes
x_2 = 5 finishing lathes
x_3 = 4 grinders
x_4 = 2 robots
Optimal unit production cost = $ 11.99
Confidence interval on the optimum value = [11.80, 12.17]

5. EVALUATION OF THE RESULTS

The results from this application show that the proposed method in [1] is an effective tool for optimizing discrete variable stochastic systems through simulation. In unconstrained problem only 1.17% of possible points were evaluated (Number of evaluated points/All possible points in the feasible region = 35/3000 = 1.17%). In addition, by using the sequential procedure to compare alternative systems the computer time was utilized very economically. More specifically, since for many points the conclusion was reached before running the simulation for the full length of the simulated period, a considerable amount of the computer time was saved. For instance, for the unconstrained problem, the average simulation time per run was 36.57% less than the time specified for a full length run. The saving was 34.48% for the constrained problem.

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