

FINDING EQUILIBRIUM BIDDING STRATEGIES FOR A SIMULATED TIMBER AUCTION*

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Abstract

This paper reports on a Monte Carlo simulation model of estimating and bidding in an unusual form of auction used by the U.S. Forest Service to sell logging rights. The model results must be precise enough to find Nash equilibrium bidding strategies and to distinguish how various measures of auction performance differ under variation in current auction rules. This paper briefly describes the auction, several different computational approaches, and their relative effectiveness. We state several conclusions we were able to reach using the simulator. These include identifying a situation in which changed auction rules could significantly increase government revenue.

Introduction

We report here on the development of a simulation model to analyze a particular form of auction used by the U.S. Forest Service to sell logging rights in national forests of the western United States. Currently, this auction is a progressive oral auction in which bids are vectors of unit-prices for each marketable species of tree on a tract. The total amount of a bid is the inner product of the bid vector with a vector of Forest Service estimates of the quantities of each marketable species. The whole tract is awarded to the high bidder figured on this basis, but the payment by the winner is made at the time the trees are cut and is based upon the actual, not estimated, quantity harvested. Bidders have discovered that they can profit by "unbalancing" or "skewing" their bids to take advantage of perceived Forest Service estimating errors -- especially an overestimate of a minor species. The Forest Service has been criticized [1] for sales in which this occurred, and is considering rule changes to limit skewed bidding. [2,3] The bidding literature [4,5] contains insightful analysis of how bidders can best bid in such unit-price auctions, but only a little analysis of how bid takers can best manage them. [6]

The Simulator and Bidder Strategies

Our simulator models the quantity and value estimating processes of the seller and of each bidder, and each bidder's strategy choice. It performs a Monte Carlo simulation of the auction to evaluate a variety of measures of auction performance, including the probability that at least one acceptable bid is received, the expected revenue received by the seller, and the probability that each bidder (including each less efficient one) wins. These simulations can be repeated for different sets of auction rules in order to estimate the effects of rule changes.

Several kinds of bidding strategies are treated by the model. The model handles bidders who skew their bids in the manner suggested by Stark [4] and those who do not skew their bids. The model characterizes a bidder's strategy in terms of two factors: the maximum fraction of his or total value estimate the bidder is willing to bid (if competing bids make this necessary) and, for skewing bidders, the additional fraction of the estimated additional profit from skewing that the bidder would "give back" in the form of further bid increases (again, if competing bids make this necessary). Bidders' strategies characterized in terms of these two fractions may be classified either as "prespecified" or "optimal." For bidders with "optimal" strategies, the model adjusts the two fractions that compose the strategy so that (within tolerances) no further adjustments could increase that bidder's expected profit. If there are two or more bidders with "optimal" strategies, then the strategies of each must be adjusted to find (to within tolerances) a Nash equilibrium set of strategies in which no optimizing bidder can unilaterally improve his or her own expected profit.

Stark's original work on skewed bidding [4] showed that the optimal pattern in skewing bids is the solution of a linear programming problem and, furthermore, that in certain situations the optimal pattern of skewing could be determined without resorting to iterative calculations (such as the simplex method). We have used this result and extended our ability to obtain essentially noniterative solutions to two additional kinds of constraints, including those proposed by the U.S. Forest Service. [2,3]

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Computational Approaches

Finding optimal or equilibrium strategies is computationally difficult because each evaluation of the objective functions of the bidders for a given set of strategies requires a Monte Carlo simulation. Such evaluations involve a substantial computational effort, one that increases as the square of the required precision.

We have used three different methods for equilibrium calculations. The first method involves running a set of Monte Carlo simulations in a pattern suggested by experimental design considerations, fitting quadratic surfaces to the results, finding the equilibrium on the quadratic surfaces, and iterating with a new pattern centered around this point. This method is related to an approach discussed by Glad and Goldstein. [7] The second approach is specialized only to situations in which all optimizing bidders are identical for strategic purposes. It looks for symmetric sets of equilibrium strategies by varying one bidder's strategy and using the results to make a damped adjustment of all bidders' strategies. The third method is more general, applying to both symmetric and asymmetric auctions. It determines each bidder's best strategy response for several fixed values of the opponents' strategies, and fits functional forms to these values. We find the equilibrium strategies by locating the intersection of these functions.

Of the three methods, the third is generally superior, and was used to produce most of our results. The first approach is the most direct, but our problem seems to require strong curvature of the fitted surfaces or unreasonably good initial guesses for convergence. The second method (when it can be used) converges quite quickly for reasonably good initial guesses, but provides no direct way to estimate the uncertainty of the equilibrium except replicating the entire algorithm. This replication, if required, would make it more costly than the third method. Versions of all three methods have been implemented on the CDC 7600 and/or LBL's MIDAS parallel processor computer.

The second and third approaches both require finding a bidder's "best response strategy" to opponent's fixed strategies. We have tried two different approaches to this subproblem: an iterative application of simplex search [8] and a fairly straightforward application of classical response surface methodology. Of these, the second method has been more useful.

Conclusions

Among the preliminary results using the simulator are the following:

- (1) Equilibrium sets of strategies exist and can be found even when optimizing bidders have moderately different relative values for the timber.
- (2) For some auction conditions, near-optimal strategies for a bidder (given the strategy of his competitors) cover a wide range of "givebacks" of anticipated profits from skewing. Hence, little is lost by assuming that "giveback" is arbitrarily fixed rather than optimized. This significantly eases the computational burden of finding equilibrium strategies.
- (3) To the extent that equilibrium strategies can be relied upon, it appears that in auctions that have two or more symmetric optimizing bidders who are also skewing bidders, much of the apparent revenue gain by winning bidders who skew is competed away.
- (4) In these auctions, equilibrium bidding strategies are more aggressive with fewer bidders. Unlike sealed-bid, first-price auctions, this is true even when comparing two-bidder auctions with three-bidder auctions.
- (5) At equilibrium, expected government revenue increases and the expected profit of the winning bidder decreases with the number of symmetric bidders.
- (6) In auctions with two otherwise symmetric bidders, only one of whom skews, government revenue at equilibrium can be significantly increased by auction rules that limit the amount of unbalancing.

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