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INITIAL TRANSIENT EFFECTS IN THE FREQUENCY DOMAIN

Douglas J. Morrice

Department of Management Science
and Information Systems
University of Texas
Austin, Texas 78712

Lee W. Schruben

School of Operations Research
and Industrial Engineering
Cornell University
Ithaca, New York 14853-7501

Sheldon H. Jacobson

Department of Operations Research
Weatherhead School of Management
Case Western Reserve University
Cleveland, Ohio 44106

ABSTRACT

In this paper we discuss the effects of initial transients on the harmonic estimates used in frequency domain simulation factor screening experiments. For certain additive initial transients, we show that frequency domain methodology (FDM) remains asymptotically valid though such transients change the distribution of the harmonic component estimates for finite sample sizes.

1. INTRODUCTION

Traditional time domain factor screening experiments attempt to determine the importance of input factors by changing the inputs after each complete run of the simulation. By looking at the outputs from many simulation experiments, each with different inputs, inferences can be drawn concerning the significance of various factors in the input/output model relationship. This basic strategy is the foundation of response surface methodology (Myers 1971).

As an alternative to traditional factor screening approaches, Schruben and Cogliano (1987) introduced *Frequency Domain Methodology* (FDM). The motivation for FDM is to determine the same information (i.e., the importance of factors in the input/output model relationship) using only two runs of the simulation model. The key difference between frequency domain and time domain experiments is that for frequency domain experiments, the inputs are *varied* during the simulation run, while in time domain experiments, they are kept *fixed* throughout the entire run. By varying the inputs during the run, one is able to shift the anaylsis from the time domain into the frequency domain.

A common problem in steady state simulation output analysis is accounting for initial transient effects for finite simulation run lengths. In this paper, we discuss how initial transients affect the harmonic estimators in frequency domain experiments. In Section 2, background on frequency domain experiments is given. Section 3 contains an examination of the effects of an additive transient term in the metamodel relating the input and the output of the simulation. A summary of the results and some concluding remarks are contained in Section 4.

2. BACKGROUND

Consider a simulation model with K continuous input factors, $\{X_k, k = 1, ..., K\}$ and output, Y. In FDM, the assumption is made that the relationship between the inputs and the output is adequately represented by the metamodel relationship,

$$Y(t) = \sum_{k=1}^{K} \sum_{r=0}^{q_k} h_k(r) X_k(t-r) + \varepsilon(t), \tag{2.1}$$

where,

- $t = 0, ..., N-1$, is an index which counts the observations generated within a run of the simulation model,
- $h_k(r)$ is the rth unknown coefficient in the memory filter associated with input factor k, and
- $q_k < N$ is the lag length of the memory filter for the kth input factor,
- $\varepsilon(t)$ is a zero-mean covariance stationary error process.

The purpose of FDM is to detect the importance of an input factor in relation to the output (i.e., to determine the significance of unknown coefficients). This goal is achieved by making two runs of the simulation experiment. On the first run, the input factors are held fixed at nominal levels described by,

$$X_k(t) = X_k(0),$$

for $k = 1, ..., K$. The purpose of this run, called the *noise run*, is to estimate the variance of the error process, $\{\varepsilon(t)\}$, in the frequency domain. The second run, called the *signal run,* involves varying each input factor according to the expression,

$$X_k(t) = X_k(0) + a_k \cos(2\pi\omega_k t),$$

for $k = 1, ..., K$, during this simulation run. The $\{a_k\}$ are the *oscillation amplitudes* and the $\{\omega_k = p_k/N, p_k \in \{1, ..., [N/2]-1\}\}$ are the *driving frequencies* (note that here '[.]' denotes the integer part of the quotient). The purpose of the signal run is to estimate the strength of signals in the output affected by the input factors.

On each run, a frequency domain statistic called the *periodogram* is constructed. It is defined as,

$$I(\omega_j) = \left(\frac{2}{N}\right) \left| \sum_{t=0}^{N-1} Y(t)\, e^{-i\omega_j t} \right|^2. \tag{2.2}$$

For the noise run, the periodogram provides a frequency domain estimate, at frequency ω_j, of the variance of the error process; on the signal run, it provides a frequency domain estimate of the strength of the output signal associated with each input factor. Suppose the $\{\varepsilon(t)\}$ have the following additional properties:

i) $\varepsilon(t) = \sum_{s=-\infty}^{\infty} w(s)\, e(t-s)$, where $\sum_{s=-\infty}^{\infty} |w(s)| < \infty$,

ii) $\gamma(h) = \mathrm{Cov}(\varepsilon(t), \varepsilon(t+h))$ is real, absolutely summable, and symmetric,

iii) $\{e(t)\}$ are independent with distribution function $\{F_t(e)\}$ where $E(e(t)) = 0$, $V(e(t)) = \sigma^2$, and $\sup_{t=0,1,\ldots} \int_{|e|>d} e^2\, dF_t(e) \to 0$, as $d \to \infty$.

Then for large sample sizes, on the control run and on the signal run, under the null hypothesis, (2.2) is approximately distributed as $\alpha_j \chi^2(2)$, where

$$\alpha_j = \begin{cases} \sum_{|h| \le \left[\frac{N}{2}\right]} \gamma(h) e^{-i\omega_j h}, & N \text{ odd} \\ \sum_{|h| < \left[\frac{N}{2}\right]} \gamma(h) e^{-i\omega_j h} + \gamma\left(\frac{N}{2}\right) e^{-i\omega_j \left(\frac{N}{2}\right)}, & N \text{ even} \end{cases}$$

On the signal run, under the alternative hypothesis, (2.2) has an approximate $\alpha_j \chi^2(2;\delta_j)$ distribution, where,

$$\delta_j^2 = \frac{N \left| \sum_{r=0}^{q_j} h_j(r)\, e^{-i\omega_j r} \right|^2}{2\, \alpha_j}.$$

(see Morrice [1990]).

Denoting the signal and noise periodograms as I^s and I^n, respectively, a ratio statistic called the Signal-to-Noise Ratio (SNR) is defined as,

$$\mathrm{SNR}(\omega_j) = \frac{I^s(\omega_j)}{I^n(\omega_j)}. \tag{2.3}$$

If ω_j is the term indicator frequency for X_j, then $\mathrm{SNR}(\omega_j)$ can be used to test the significance of the FDM hypothesis,

$$H_{0j}: \sum_{r=0}^{q_j} h_j(r)\, e^{-i\omega_j r} = 0,$$

$$H_{Aj}: \sum_{r=0}^{q_j} h_j(r)\, e^{-i\omega_j r} \ne 0.$$

For large sample sizes, (2.3) has an approximate $F_{2,2}$ distribution under the null hypothesis and an approximate $F_{2,2;\delta_j}$ distribution under the alternative hypothesis.

3. AN ADDITIVE INITIAL TRANSIENT

The metamodel relationship in (2.1) is based on the assumption that all initial transient effects have been removed from the simulation output process. In practice, for finite simulation run lengths, some initial transient will be present. To reflect this , we rewrite (2.1) as,

$$Y(t) = \sum_{k=1}^{K} \sum_{r=0}^{q_k} h_k(r) X_k(t-r) + f(t) + \varepsilon(t), \tag{3.1}$$

where f(t) is a deterministic function of the index, t, and typically depends on the initial state of the system through some nuisance parameters. For example, $f(t) = \beta\, e^{-\alpha t}$, where the unknown parameters $\alpha > 0$ and β depend on the initial state of the system.

Since f(t) is not related to the input factors, its periodogram components appear in both the signal and noise runs. Also, since f(t) is a deterministic component in (3.1), it only changes the distribution of the periodogram ordinates through a non-centrality parameter. If on the signal run the frequency ω_j is the term indicator frequency for an important term in the model then,

$$\begin{aligned} E[I^s(\omega_j)] &= E\left[\left(\frac{2}{N}\right) \left| \sum_{t=0}^{N-1} Y(t)\, e^{-i\omega_j t} \right|^2\right] \\ &= \left(\frac{N}{2}\right) \left| \sum_{r=0}^{q_j} h_j(r)\, e^{-i\omega_j r} \right|^2 + \left(\frac{2}{N}\right) \left| \sum_{t=0}^{N-1} f(t)\, e^{-i\omega_j t} \right|^2 \\ &\quad + 2\left(\sum_{r=0}^{q_j} h_j(r) \cos(\omega_j r)\right)\left(\sum_{t=0}^{N-1} f(t) \cos(\omega_j t)\right) \\ &\quad + 2\left(\sum_{r=0}^{q_j} h_j(r) \sin(\omega_j r)\right)\left(\sum_{t=0}^{N-1} f(t) \sin(\omega_j t)\right) \\ &\quad + \left(\frac{2}{N}\right) E\left[\left| \sum_{t=0}^{N-1} \varepsilon(t)\, e^{-i\omega_j t} \right|^2\right] \end{aligned} \tag{3.2}$$

The first term in the above expression follows from the trigonometric relationships found on page 392 of Priestley (1981). Note that all cross product terms containing $\{\varepsilon(t)\}$ evaluate to zero since $E[\varepsilon(t)] = 0$. Assuming $\{\varepsilon(t)\}$ has the properties stated in Section 1, then,

$$E\left[\left| \sum_{t=0}^{N-1} \varepsilon(t)\, e^{-i\omega_j t} \right|^2\right] \approx N\alpha_j \tag{3.3}$$

(see Proposition 4.5.2 on page 130-131 of Brockwell and Davis (1987)). Substituting (3.3) into (3.2) and comparing the result with the expectation of an non-central chi-square distribution with two degrees of freedom (see Scheffe (1959), page 414), yields the non-centrality parameter,

$$\delta_j'^2 = \frac{N \left| \sum_{r=0}^{q_j} h_j(r)\, e^{-i\omega_j r} \right|^2}{2\, \alpha_j} +$$

$$\frac{2\left|\sum_{t=0}^{N-1} f(t)\, e^{-i\omega_j t}\right|^2}{N\,\alpha_j} + \left(\frac{2}{\alpha_j}\right)\left(\sum_{r=0}^{q_j} h_j(r)\cos(\omega_j r)\right)\left(\sum_{t=0}^{N-1} f(t)\cos(\omega_j t)\right) + \left(\frac{2}{\alpha_j}\right)\left(\sum_{r=0}^{q_j} h_j(r)\sin(\omega_j r)\right)\left(\sum_{t=0}^{N-1} f(t)\sin(\omega_j t)\right) \quad (3.4)$$

Since the first term in (3.4) is O(N), then as long as,

$$\sum_{t=0}^{N-1} f(t)\cos(\omega_j t) \text{ and } \sum_{t=0}^{N-1} f(t)\sin(\omega_j t) \quad (3.5)$$

are o(N), the influence of all other terms will diminish as N approaches infinity. A sufficient condition for this to occur is,

$$\sum_{t=0}^{N-1} |f(t)| < \infty$$

(see Priestley (1981), Section 4.5.1). As an illustration, consider $f(t) = c_1$ (a constant) for $t = 0$ and $(\beta / t^2)\sin(\omega t)$ for $t > 0$, where f(t) is absolutely summable. Then both summations in (3.5) are bounded above, in magnitude, by a function that is O(ln(N)).

On the noise run or equivalently under the null hypothesis, the distribution is no longer $\alpha_j\chi^2(2)$ but rather $\alpha_j\chi^2(2;\delta_j'')$. To determine δ_j'', we can use the expectation in (3.2) which for the noise run becomes,

$$2\,\alpha_j + \frac{2\left|\sum_{t=0}^{N-1} f(t)\, e^{-i\omega_j t}\right|^2}{N}$$

From the expectation of a non-central chi-square distribution, $(\delta_j'')^2$ is given by,

$$\frac{2\left|\sum_{t=0}^{N-1} f(t)\, e^{-i\omega_j t}\right|^2}{N\,\alpha_j}$$

If the summation in (3.5) are o(N), then δ_j'' diminishes to zero as N approaches infinity.

A consequence of the transient term for finite sample sizes is that the distribution of the SNR under both the null and alternative hypotheses becomes a doubly non-central F. Under the null hypothesis, the distribution of (2.3) is a $F_{2,2;\delta_j'',\delta_j''}$; under the alternative hypothesis, (2.3) has an $F_{2,2;\delta_j',\delta_j''}$ distribution. In the limit, as N approaches infinity, these distributions converge to $F_{2,2}$ and $F_{2,2;\delta_j}$, respectively, the case with no initial transient effect.

4. CONCLUDING REMARKS

In this paper we have discussed the effect of an additive initial transient on the harmonic estimates used in frequency domain simulation factor screening experiments. We showed that under reasonable conditions, the harmonic effect is asympotically negligible. This implies that the initial transient effect is no worse, asymptotically, in the frequency domain than in the time domain.

For finite sample sizes, we showed how the additive initial transient effect changes the distribution of the SNR, which in turn may affect the power of the statistic for rejecting the null hypothesis. In order to reduce an additive initial transient effect, the difference between corresponding observations from the signal and noise runs can be considered. If the transient is identical for both runs, then it is, eliminated in this difference between the signal and noise runs (see Jacobson (1990) for some related work).

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