

A REPLICATION APPROACH TO INTERVAL ESTIMATION IN SIMULATION

M. Murat Köksalan

Industrial Engineering Department
Middle East Technical University
06531 Ankara, Turkey and
Krannert School of Management
Purdue University
West Lafayette, Indiana 47907

Nail Basöz

Industrial Engineering Department
Middle East Technical University
06531 Ankara, Turkey

ABSTRACT

In this study we modify an earlier approach developed for reducing the bias of the estimator for the mean response in simulation caused by the initial conditions. We try to balance the bias of the estimator in a simulation run by imposing a bias in the opposite direction in a companion run by suitably setting its initial conditions. We present analytical results for the bias of our estimator for AR(1) and M/M/s processes. We suggest making independent replications of the pairs of runs to construct a confidence interval for the mean response. We present some empirical results about the coverages and precisions of the confidence intervals. The results suggest that the idea of balancing a bias with a bias in the opposite direction is promising.

1 INTRODUCTION

The statistical estimation of the output response in steady state simulations has been widely studied. Commonly, the problem of constructing a confidence interval (c.i.) for the mean response is addressed. In classical statistical analysis the sample mean and the variance of the sample mean estimated from independent and identically distributed (i.i.d.) random variables are used to construct a c.i. for the mean. The estimation problem in simulation is nontrivial because the output process obtained from a simulation run is usually highly correlated. Many approaches in the literature transform the simulation output in order to obtain a new process that is approximately uncorrelated. For a review of different approaches, the reader is referred to Bratley, Fox, and Schrage (1987), Fishman (1978), Law and Kelton (1991), and Schmeiser (1990). Many of the developed approaches have desirable asymptotic properties. Sargent, Kang, and Goldsman (1988)

demonstrate that small sample properties of an approach may be quite different from its asymptotic properties and approaches that have better asymptotic properties do not necessarily have better small sample properties.

Another important problem in steady state simulation output analysis is the effect of initial conditions. In order to estimate the steady state performance of a system we need to observe the system in steady state. However, we have to initialize a simulation run from some state and it usually takes the run a while to reach steady state. A common approach is to allow the simulation to run for a while before starting to collect statistics. Many approaches have been developed to detect when the effect of initial conditions is eliminated. Wilson and Pritsker (1978a, b) review some of these approaches and they suggest a procedure for evaluating the performance of the approaches. Gafarian, Ancker, and Morisaku (1978) conducted an empirical study and concluded that none of the rules they tried for detecting steady state was satisfactory. Schruben (1982) and Schruben, Singh, and Tierney (1983) develop tests to detect the presence of initial transient effects in simulation output.

If the effect of initial conditions could be eliminated quickly, then constructing a c.i. for the mean response would be trivial. One could make independent replications of the simulation run and construct a c.i. using the i.i.d. observations obtained from each replication. The problem with this approach is that one has to incur the effect of initial conditions in each replication. Kelton and Law (1983) develop an approach where they try to detect the start of steady state. They make independent replications and construct a c.i. for the mean response after eliminating some initial observations from each replication. Kelton (1989) suggests that stochastic initialization of different replications may reduce the effect of initial conditions

and lead to the elimination of fewer initial observations. Based on a first-order autoregressive (AR(1)) process, Kelton and Law (1984) also present results in favor of an independent replications approach that deletes some initial observations.

Deligonul (1987) develops an approach to reduce the initial bias. The approach is called antithetic bias reduction (ABR) where the bias in a simulation run is tried to be balanced with a counter bias in a companion run. Our approach is a variation of the ABR approach. We review the ABR approach, develop our approach and present some analytical results in the next section. In section 3 we report some empirical results and we present our conclusions in section 4.

2 DEVELOPMENT OF THE APPROACH

In this section we first briefly review the ABR approach and then discuss our approach and present some analytical results.

2.1 The ABR Approach

Let X_0 denote the initial state of the system in a simulation run and X_1, X_2, \dots, X_m be the output stochastic process obtained from the simulation run. Let $\bar{X}_1 = \sum_{j=1}^m X_j/m$ given $X_0 = x^-$. Assuming that the steady state mean of the process is μ , the bias of X_0 for

μ is $x^- - \mu$ and an estimate for this bias is $x^- - \bar{X}_1$. A bias in the opposite direction can be created by initializing a simulation run at $x^+ = \bar{X}_1 + (\bar{X}_1 - x^-) = 2\bar{X}_1 - x^-$ and obtaining m new observations. Then $\bar{X}_2 = \sum_{j=1}^m X_j/m$

given $X_0 = x^+$ is found and $X^* = (\bar{X}_1 + \bar{X}_2)/2$ is used as the estimate for μ . Deligonul shows that for an AR(1) process the ABR approach reduces the bias quadratically with the number of observations whereas the bias reduction is linear with the number of observations in a single run of the simulation. Deligonul also presents favorable results for a machine-repair type closed queuing system.

2.2 A Variation of the ABR Approach

The ABR approach would work well when x^+ and x^- are equally biased on opposite sides of the mean and the convergence toward the mean is monotonic and at an

approximately equal rate from x^+ and x^- . The monotonic convergence is also assumed by others and seem to hold for a wide range of systems, especially queueing systems (see Kelton and Law 1983, 1985). In order to have x^+ and x^- equally biased on opposite sides of μ , however, we need to have

$$x^- - \mu = \mu - x^+ = \mu - (2\bar{X}_1 - x^-)$$

or taking the expectations

$$x^- - \mu = \mu - 2E(\bar{X}_1) + x^-$$

or $E(\bar{X}_1) = \mu$ which may approximately hold when the number of observations, m , in each run is large. In simulation studies, however, having a sufficiently long run length may be too costly.

Consider generalizing the determination of the initial conditions of the companion run as

$$x^+ = w(\bar{X}_1 - x^-) + x^- \tag{1}$$

where $w \geq 0$ is a weighting factor. $w = 2$ corresponds to the ABR approach. Using (1), in order to have x^+ and x^- equally biased on opposite sides of μ we need to have

$$x^- - \mu = \mu - w(\bar{X}_1 - x^-) - x^-$$

or taking expectations and rearranging

$$2(\mu - x^-) = w(E(\bar{X}_1) - x^-).$$

We would like to set $w = 2(\mu - x^-)/(E(\bar{X}_1) - x^-)$. If the convergence to steady state is monotonic then we would have either $x^- \geq E(\bar{X}_1) \geq \mu$ or $\mu \geq E(\bar{X}_1) \geq x^-$ and in

both cases $w \geq 2$. $w = 2$ corresponds to $E(\bar{X}_1) = \mu$ which, as discussed before, is the necessary condition for the ABR method to keep x^+ and x^- at equal distance from μ on opposite sides of μ .

Using w we can determine x^+ and as in the ABR method we can initialize a new simulation of m observations at x^+ and obtain \bar{X}_2 . Then using $X^* = (\bar{X}_1 + \bar{X}_2)/2$ we can obtain a point estimate for the mean. We will refer to this weighted version of the ABR method as the WABR method.

In a simulation study neither μ nor $E(\bar{X}_1)$ is known or otherwise there would be no need to use simulation for the estimation of μ . However, we may try to estimate w from the output of the simulation. We next discuss

finding w in conjunction with a first-order autoregressive process (AR(1)) and the M/M/s queueing systems.

2.2.1 AR(1) Process

Suppose that the output of simulation follows an AR(1) process which is defined by the following recursive relation:

$$X_i = \mu + \alpha (X_{i-1} - \mu) + \epsilon_i \text{ for } i = 1, 2, \dots$$

where $0 < \alpha < 1$, $E(\epsilon_i) = 0$ for all i ,

$$E(\epsilon_i \epsilon_j) = \begin{cases} \sigma^2 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

and X_0 is the initial condition.

For the AR(1) process, it can be shown that (see Turnquist and Sussman 1977) the expected value of the sample mean of first m observations, given initial conditions X_0 , is

$$E(\bar{X}) = \mu + (X_0 - \mu) \alpha (1 - \alpha^m) / (m(1 - \alpha)).$$

Applying this to \bar{X}_1 which is obtained by using $X_0 = x^-$ and to \bar{X}_2 which is obtained by using $X_0 = w(\bar{X}_1 - x^-) + x^-$ we obtain

$$E(\bar{X}_1) = \mu + (x^- - \mu) \alpha (1 - \alpha^m) / (m(1 - \alpha)) \text{ and}$$

$$E(\bar{X}_2) = \mu + (wE(\bar{X}_1) - (w-1)x^- - \mu) \alpha (1 - \alpha^m) / (m(1 - \alpha))$$

Letting $\gamma(m) = \alpha (1 - \alpha^m) / (m(1 - \alpha))$ we obtain

$$E(\bar{X}_1) = \mu + (x^- - \mu) \gamma(m) \tag{2}$$

$$E(\bar{X}_2) = \mu + \{w[\mu + (x^- - \mu)\gamma(m)] - (w-1)x^- - \mu\} \gamma(m)$$

$$E(\bar{X}_2) = \mu + \{(w-1)(\mu - x^-) - w(\mu - x^-) \gamma(m)\} \gamma(m) \tag{3}$$

Using $X^* = (\bar{X}_1 + \bar{X}_2) / 2$ and Bias (X^*) = $E(X^*) - \mu$ and substituting (2) and (3) we obtain Bias (X^*) = $\mu + (x^- - \mu + (w-1)(\mu - x^-) - w(\mu - x^-) \gamma(m)) \gamma(m) / 2 - \mu$ or

$$\text{Bias } (X^*) = \{(\mu - x^-)(w-2-w\gamma(m))\} \gamma(m) / 2$$

In order to eliminate the bias in X^* we need to have $w-2-w\gamma(m) = 0$ or $w^* = 2 / (1 - \gamma(m))$. Substituting the value of $\gamma(m)$ we obtain

$$w^* = \frac{2m(1 - \alpha)}{m(1 - \alpha) - \alpha(1 - \alpha^m)} \tag{4}$$

If we know the value of α for an AR(1) process then we can eliminate the bias of X^* for a value of m as small as 1. For $m = 1$, for example, w^* values are 20, 40 and 200 for $\alpha = 0.90, 0.95$, and 0.99 , respectively. Obviously, we would not know the value of α in advance. However, we can estimate α from a simulation run and use the estimated value to find w^* . We employ such a procedure and present some empirical results in section 3.

In the expression for $E(\bar{X}_2)$ we treated w as a constant but we actually need to estimate w and therefore it is a random variable. If w and \bar{X}_1 are obtained from the same run, there will be some correlation between them. This correlation was not accounted for in solving for $E(\bar{X}_2)$ and this may cause the bias to be somewhat different from 0. Obtaining w^* from an independent run would avoid such a problem.

The variance term for the sample mean for an AR(1) process that has fixed initial conditions is independent of the initial conditions and only depends on $V(\epsilon_i) = \sigma^2$, α , and the number of observations, m (see Deligonul 1987). However, in our case and for the case of the ABR approach, the initial conditions for the companion run, x^+ , is a random variable and the variance of the sample mean depends on the variance of the initial conditions. It can be shown that $V(\bar{X}_2) \geq V(\bar{X}_1)$. Since $X^* = (\bar{X}_1 + \bar{X}_2) / 2$, we have $V(X^*) = [V(\bar{X}_1) + V(\bar{X}_2) + 2\text{Cov}(\bar{X}_1, \bar{X}_2)] / 4$. The covariance term in the expression for $V(X^*)$ will be negative since we induce a negative correlation between \bar{X}_1 and \bar{X}_2 . In spite of the above properties, it is not clear if the variance of the sample mean increases or decreases with ABR and WABR methods compared to making a single run.

2.2.2 M/M/s Queue

We now consider the M/M/s queueing system where the time between arrivals is exponentially distributed with mean rate λ and each of the s identical parallel servers have exponentially distributed service times with mean rate μ . Kelton and Law (1985) analyze the transient behavior of this system. Specifically, they define $P_k(n, i)$ as the probability of having i customers in the system at the time of arrival of the n 'th customer given that k customers were present at time 0 (where i includes

the just arriving customer). They develop an iterative algorithm to find the values of $P_k(n,i)$ and using these values they calculate the expected delay of customer n .

Consider the approach we developed in section 2. We may adapt the approach to an M/M/s system by first making a run for m service completions starting the simulation with x^- customers in the system. Then using a suitable weight, w , and the average number of customers in the system obtained from the first run, say $\bar{N}_1(m)$, we can find the starting conditions, x^+ , for the companion run. Here, we will have to round x^+ to the nearest integer since we can only consider an integer number of customers present in the system at time 0. From the companion run we obtain the average number

of customers in the system, $\bar{N}_2(m)$. A point estimate, $N^*(2m)$, for the mean number of customers in the system can be found by $N^*(2m) = (\bar{N}_1(m) + \bar{N}_2(m))/2$. In order to find a suitable value for w in this procedure we approximate the number of customers in the system by an AR(1) process. Specifically, letting N_n denote the number of customers found in the system by the n 'th arriving customer, we approximate the $\{N_n, n=1,2,\dots\}$ process by an AR(1) process. We estimate the lag 1 correlation of the number in the system process using a jackknife estimator and substitute this value for α in expression (4) to find a value for w^* .

We demonstrate the performance of the above procedure analytically using the transient results developed by Kelton and Law (1985) for the M/M/s system. Using the $P_k(n,i)$ values we can compute the expected number of customers the n 'th arriving customer will find in the system given that originally there were k customers in the system. Denoting this value by $E_k(N_n)$ we have

$$E_k(N_n) = \sum_{i=1}^{k+n} (i-1)P_k(n,i).$$

Using the sequence $E_k(N_1)$,

$E_k(N_2), \dots, E_k(N_m)$, where $x^- = k$ in this case, we determine the lag 1 correlation, α , and obtain w^* from

(4). We then find $x^+ = w^*(E_k(\bar{N}_1(m)) - k) + k$ where

$$E_k(\bar{N}_1(m)) = \sum_{i=1}^m E_k(N_i)/m.$$

Rounding x^+ to the closest

integer we get k' , which is the initial condition for the companion run, and we find $E_{k'}(N_i)$ $i=1,2, \dots, m$ and

$E_{k'}(\bar{N}_2(m))$. Then the expected value of the estimator we suggested for the steady state mean number of customers in the system can be found as

$E_k(N^*(2m)) = (E_k(\bar{N}_1(m)) + E_{k'}(\bar{N}_2(m)))/2$. Denoting the steady state mean number in the system by N , the bias of $N^*(2m)$ is $E_k(N^*(2m)) - N$. The bias of $N^*(2m)$ is given in Table 1 for different values of m for M/M/1, M/M/3, and M/M/5 queueing systems having traffic intensity $\rho=0.90$ and arrival rate $\lambda=1$. In Table 1 we also present the bias for the estimator of the ABR method as well as the bias for the estimator that is obtained using a single run. Total number of observations in all cases is $2m$, where WABR and ABR methods make a pair of runs each having m observations. The results of Table 1 show that WABR method reduces the bias slightly more than the other methods in all cases. However, there remains a substantial bias in M/M/3 and M/M/5 systems. This probably indicates that the AR(1) approximation does not sufficiently capture the properties of the M/M/s models in terms of the initial bias. Table 1 also presents w^* values and as expected these values approach to 2 (the w value for the ABR method) as m increases.

Table 1: Percentage Bias of the Estimators for the Expected Number of Customers in the System

Model	2m	WABR	ABR	Single Run
M/M/1	200	-29.0 (2.36) ¹	-34.0	-34.9
	400	-16.1 (2.27)	-20.0	-22.0
	800	-7.6 (2.18)	-9.8	-12.2
	1600	-3.2 (2.10)	-4.0	-6.2
	2000	-2.4 (2.09)	-2.9	-5.0
M/M/3	4000	-1.0 (2.04)	-1.1	-2.5
	200	-42.6 (2.29)	-45.9	-46.2
	400	-31.7 (2.23)	-34.4	-35.5
	800	-24.2 (2.16)	-25.7	-27.3
	1600	-20.1 (2.09)	-22.5	-22.2
M/M/5	2000	-19.3 (2.08)	-19.7	-21.2
	4000	-17.9(2.04)	-18.1	-19.1
	200	-47.1 (2.37)	-51.3	-52.0
	400	-39.6 (2.28)	-42.5	-43.6
	800	-34.2 (2.19)	-35.8	-37.1
M/M/5	1600	-31.2 (2.11)	-31.8	-33.1
	2000	-30.7 (2.09)	-31.1	-32.3
	4000	-29.7 (2.04)	-29.8	-30.1

¹ Values in parenthesis are w^* values

3 EMPIRICAL RESULTS

We present some empirical results for the AR(1) and M/M/1 processes. We do not consider M/M/3 and M/M/5 processes since the bias of the estimator of the WABR method is high for reasonable run lengths for these processes.

In the experiments we used the first of a pair of runs to compute both w^* and the sample mean although this may cause some undesirable correlation between them. We used the average of the sample means obtained from the two runs as an estimator of the mean response. We repeated this procedure b times and obtained b i.i.d. estimators from which we constructed a 90% c.i. We computed a new w^* from the first run of each pair of runs. Notice that the only major source of error in this c.i. is the bias in the point estimators for the steady state mean.

For the AR(1) process we simulate, we use $\mu=8.12$, $V(\epsilon_j)=\sigma^2=1$, $\alpha=0.90$, and $X_0=0$. We use the c.i. coverage and average relative precision (ARP) (where relative precision is the c.i. half length divided by the absolute value of the estimator) as the performance measures. Table 2 gives the coverage (proportion of 50 c.i.'s that cover the mean) and the ARP for different sample sizes for the WABR and ABR methods as well as for the independent replications of a single run. For the experiments of Table 2 $b=10$ is used and each method uses a total of $2mb$ observations for each c.i. Notice that the WABR method produces satisfactory results even for small sample sizes. The results for the ABR method becomes satisfactory for larger sample sizes. Making independent replications of a single run yields poor results except for very large sample sizes. We also tried to reduce the ARP for smaller sample sizes by sequentially increasing the number of replications, b . In this case, the coverage for the sample size of 40 deteriorated while sample sizes of 80 and 100 still yielded satisfactory results. This indicates that there remains some bias in the estimator for only the sample size of 40. The results also compare well with the extensive test results reported by Sargent et al. (1988) using several methods. We cannot directly make comparisons since the parameters used for the AR(1) process are not the same.

We conducted similar experiments for the M/M/1 system having $\rho=0.90$ and constructed c.i.'s for the mean number of customers in the system. Table 3 gives the results of these experiments and the type of information given in this table is similar to that of Table 2. Each c.i. is constructed from 10 replications and coverages are out of 50 c.i.'s. WABR method's results again seem satisfactory even for small sample sizes. For example,

for the sample size of 200 we obtain a coverage of 0.84 and an ARP of 0.29 with a total sample size of 2,000 in 10 replications. Similarly, the coverage and ARP are 0.90 and 0.30 for a sample size of 400 which uses 4,000 total observations in 10 replications. Kelton and Law (1983) present results for the same problem where they make independent replications and truncate the heavily biased initial observations from each replication. They obtain a high relative precision and a 0.82 coverage using a total sample size of about 10,000.

Table 2: Coverages and ARP's of 90% C.I.'s for the AR(1) Process

2m	Method					
	WABR		ABR		Single Run	
	Cov.	ARP	Cov.	ARP	Cov.	ARP
40	0.82	0.19	0.48	0.17	0.02	0.11
80	0.92	0.12	0.80	0.11	0.08	0.08
100	0.94	0.12	0.94	0.10	0.20	0.07
200	0.86	0.08	0.88	0.07	0.54	0.05
400	0.96	0.05	0.94	0.05	0.60	0.03
800	0.86	0.03	0.86	0.03	0.82	0.02
1600	0.92	0.02	0.92	0.02	0.86	0.02
3200	0.92	0.02	0.92	0.02	0.90	0.01

Table 3: Coverages and ARP's of 90% C.I.'s for the M/M/1 Queueing System

2m	Method					
	WABR		ABR		Single Run	
	Cov.	ARP	Cov.	ARP	Cov.	ARP
200	0.84	0.29	0.62	0.25	0.36	0.30
400	0.90	0.30	0.78	0.23	0.60	0.27
800	0.86	0.26	0.90	0.21	0.68	0.23
1000	0.94	0.21	0.96	0.19	0.84	0.21

We also made some runs for the M/M/1 system having $\rho=0.95$. For sample sizes of 400 and 800 we obtained poor coverages. For a sample size of 1,600 however, we obtained a coverage of 0.86 (out of 50 c.i.'s) and an ARP of 0.28. We again used 10 replications for each c.i. and hence a total number of 16,000 observations. In their corresponding results, Kelton and Law (1983) obtained a high relative precision and a coverage of 0.70 using a total sample size of about 12,500.

We also used a sequential approach in conjunction

with the WABR method where we kept increasing the number of replications until the relative precision satisfied a preset upper bound. We applied this procedure to the M/M/1 system having $\rho=0.90$ and we tried several sample sizes. The coverages somewhat deteriorated when we forced a relative precision of 0.15 and 0.10. This indicates that there is some bias in the estimators used.

4 CONCLUSIONS

In this paper we addressed the problem of initialization bias in steady state simulations. We tried to balance the bias incurred in a run with a bias in the opposite direction so that an estimator obtained from such a pair of runs is approximately unbiased. We presented both analytical and empirical results to test the performance of our approach.

Our analytical results indicate that the approach yields satisfactory results for the AR(1) and M/M/1 processes but relatively poor results for the M/M/3 and M/M/5 processes. Our empirical results are also satisfactory for the AR(1) and M/M/1 processes though some bias seem to be left in the estimators for small sample sizes.

Overall, it seems that the idea of balancing negative and positive biases is promising and it may lead to obtaining valid c.i.'s using relatively short independent replications of a simulation. The work of Kelton (1989) is also based on a similar reasoning where he stochastically chooses different initial conditions for independent replications so that the biases in different runs would balance each other. An important issue in our approach is the determination of the weight which is used to find the initial conditions for the second run of a pair of runs. Further research in the determination of this weight may improve the performance of the approach and make it applicable for a variety of systems. It may also be worthwhile experimenting with estimating w^* from a relatively long independent run and then using the same value for all pairs of runs.

We demonstrated our approach for systems having a single state variable but it is directly applicable to systems having more than one state variables. The only requirement for any system to be applicable is to have the computed initial values for all state variables for the companion run to be feasible. Even if some of these values correspond to infeasible states, it is still possible to initialize these state variables to the closest feasible values.

It may be worthwhile investigating a combination of our approach with approaches that analyze simulation output using a single long run. Instead of making a single long run we may make two long runs initializing the second run as suggested in this paper. This may

improve the quality of the point estimator. Then, we may arrange the observations in these two runs in such a way that they will be suitable for applying methods that are developed for a single long run.

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AUTHOR BIOGRAPHIES

M. MURAT KÖKSALAN is an associate professor in the Industrial Engineering Department at the Middle East Technical University. His research interests are statistical analysis of simulation experiments and multiple criteria decision making. He was a visiting associate professor in the Krannert Graduate School of Management, Purdue University during the 1990-91 academic year when part of this research was conducted.

NAIL BASÖZ was a Master's student in the Industrial Engineering Department at the Middle East Technical University when this research was conducted. Currently he has a managerial position in a private company in Turkey.